Unions and unemployment benefits: some insights from a simple three-player example

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Abstract

The aim of this paper is to analyze the interconnections between employment and unionization. We will also see how unemployment benefits drive the interplay of employment and unionization. The basic input into our model stems from cooperative game theory. Building on the Shapley value, several values for TU games with coalition structures have been presented in the literature, most notably by Aumann and Drèze (1974) and Owen (1977). We present a value that is capable of dealing with unemployment and unionization. We show that unemployment benefits increase wages but contribute to unemployment, that unemployment can be voluntary, and that unions tend to be beneficial for employed workers if there is overstaffing.

Keywords: Aumann-Drèze value, Owen value, Shapley value, outside option, unionization, unemployment benefits

JEL classification: C71, J51

1 Introduction

There is more structure on labor markets than supply and demand functions for labor reveal. First, some workers are employed while others are unemployed. Second, some workers, employed or not, form a union. The aim of this paper is to analyze the interconnections between employment and unionization. We will also see how unemployment benefits drive the interplay of employment and unionization.

The basic input into our model stems from cooperative game theory. In order to facilitate comparative statics, we need a single-valued solution concept

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like the very prominent Shapley (1953) value. The Shapley value can be used to assess the payoffs for players in an economy without any structure such as unions or unemployed agents. Formally, cooperative game theory presupposes some game or coalition function v that describes the economic possibilities open to various groups of people, called coalitions. The coalition function assigns a "worth" $v(K)$ to any subset K of the set of individuals (players) N. The Shapley value, then, is a function that maps coalition functions into payoffs for all players. The general idea is to divide the worth $v(N)$ according to some average of the "marginal contributions". A marginal contribution for a player indicates by how much the worth of a coalition increases when this player joins the coalition. While the Shapley value is often used for normative purposes (how to divide costs, for example), we argue that a positive interpretation is also possible. Indeed, without knowing the concrete bargaining protocol (which would make non-cooperative game theory possible), the Shapley value predicts the payoffs in a plausible manner.

However, in order to deal with unemployment and unions, we will need a richer structure. We will use two different partitions on the set of players. The sets making up a partition are called components. The first partition (called AD-partition because of the paper by Aumann and Drèze (1974)) allows to model employment and unemployment. Employed workers are in a component that hosts capitalists, too. To fix ideas, consider one capitalist (player 1) who may employ 1 or 2 workers (players 2 and 3). If both workers are employed, we are dealing with the (trivial) partition $\mathcal{P}_{AD} = \{\{1,2,3\}\}$. On the other hand, $\mathcal{P}_{AD} = \{\{1,2\},\{3\}\}$ reflects that worker 3 is unemployed.

Aumann and Drèze (1974) propose a partitional value that is component efficient, i.e., the firms divide their product (their worth) between capitalists and workers employed in that firm. While that is a sensible attribute for firms, the AD-payoffs for a component's players are dictated by the marginal contributions of players within this component. This implies that differing outside options of players do not bear on the payoff. However, the salary in a firm might well depend on those options. Therefore, we suggest a component-efficient value that captures outside options. This outside-option value is close to the one used by Wiese (2007) to model the power of parties within government coalitions. Meanwhile, Casajus (2009) suggests an attractive alternative outside-option value. However, since the Casajus value cannot be defined in terms of a rank-order definition, it is not clear how it can be extended in the manner proposed in this paper.

Besides the AD-partition (modelling unemployment), we will introduce the union partition which serves to model unionization. For our simple example, the relevant union partitions are $\mathcal{P}_u = \{\{1\},\{2\},\{3\}\}$ and $\mathcal{P}_u = \{\{1\},\{2,3\}\}$, the first indicating the absence of a union and the second unionization (workers 2 and 3 form a union). While the agents in an AD-partition work together, the union components bargain as a group. The corresponding value is called the union, or Owen value (see Owen, 1977; Hart and Kurz, 1983). As the Shapley and the AD-value, the union value is based on marginal contributions. However, it is the marginal contribution of whole components, not the marginal contribution of individual players, that go into the union value. Thus, the players who belong to a component are saying to the other players: If you take some of us, you'll have all of us.

Sometimes, unionization delivers higher payoffs to its members. Consider, for example, the three-player game v defined by $v(\{1,2,3\}) = v(\{1,2\}) = v(\{1,3\}) = 1$ which vanishes for $K \neq \{1,2,3\}, \{1,2\}$, or $\{1,3\}$. Here, player 1 (the capitalist) needs either of the workers 2 or 3 (or both) in order to produce the worth 1. If there is no union (of players 2 and 3), the Shapley payoffs are $\left(\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right)$. In case of unionization, i.e., in case of partition $\mathcal{P}_u = \{\{1\}, \{2,3\}\}$, the Owen value yields $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$ benefitting the unionized workers.

However, unity is not always strength. Take the three-player game v defined by $v(\{1,2,3\}) = 1$ and $v(K) = 0$ for $K \neq \{1,2,3\}$. Since the situation is totally symmetric, the Shapley payoffs are $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. If players 2 and 3 formed a union, again $\mathcal{P}_u = \{\{1\}, \{2,3\}\}$, they would suffer since the Owen payoffs are $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$. The reason is this. The component $\{2,3\}$ is as important to achieving the worth of 1 as is player 1. Therefore, the payoff to 1 is as high as the payoff to the component hosting players 2 and 3. This explains why these players are actually hurt by forming an Owen component.

In order to address the problems of unionization and unemployment, we need a value that depends on both the AD-partition and the union partition. Therefore, we will blend the outside-option value and the Owen value. The resulting value is called union outside-option value. In contrast to usual papers in cooperative game theory, an axiomatic treatment has not been attempted. Instead, we will present the value, explain its mechanics and apply it to the problem at hand.

Our paper is not the first to try a cooperative application to labor market issues. In a recent paper, Bae (2005) uses the Shapley value to analyze the merger incentives of firms and their unions. The merging of unions means that workers of both unions join the productive process. In terms of our paper, this union merger would be reflected in an appropriate AD- rather than Owen partition. Indeed, the author uses AD-payments without any outside-option argument.

In spirit, our setup is close to the modelling by Berninghaus et al. (2001). They consider the question of whether parties are better off bargaining on their own (decentralized bargaining) or together with others (collective bargaining). However, instead of a partitional approach, these authors use the Nash bargaining solution and propose the following procedure. If the two players merge (collective bargaining), there is only one Nash bargaining game; if they do not merge, two separate Nash bargaining games are considered.

In our approach, there is only one game to play. This, in our mind, allows for interdependencies between the "two" bargaining processes. For example, if two workers offer their services to a capitalist, one might expect that the payoff for each of them depends on the productivity of the other. This is indeed what we will find.

Together with unionization, we analyze unemployment benefits. These have to be understood in a broad sense and may include (the monetary equivalent of) the benefits of leisure. Of course, unemployment benefits can be negative if the financial hand-out is low and if unemployed agents suffer from boredom or the stigma of being unemployed.

While the outside-option value is the basic input, the outer structure of our model is non-cooperative and has three stages. 1. On the basis of the unemployment benefits, the workers decide on unionization. 2. The capitalist makes an employment offer to the workers, individually or to both. 3. The workers, who foresee the wages, decide on whether to accept employment or not. In contrast to principal–agent models, the principal does not propose wages. Instead, wages rest on the productivities of the workers, the outside options, and the unionization.

As might be expected, unemployment benefits do not only define the payoff for unemployed workers but influence the payoff for the employed ones. Although an employed worker does not receive unemployment benefits, his payoff ("wage") is a positive function of unemployment benefits. This fact has often been noted in the labor-market literature (see, for example, Snower, 1995, p. 626).

However, unemployment benefits also determine employment. We find that unemployment benefits may drive people out of work. In our paper, there are two reasons for this to happen. Either unemployment benefits drive up wages by increasing workers' threat point so that employment is not worthwhile for the capitalist. Or, unemployment benefits are so high that workers prefer not to work although the capitalist would be ready to offer employment (voluntary unemployment).

Our paper also looks into the question of how unionization influences wages and employment. The economic effects of trade unions have been analyzed for a long while, at least since the seminal works by Dunlop (1944) (the union as an economic organization maximizing the wage bill), Ross (1948) (the union as a political institution fighting for fairness and equity), and Freeman and Medoff (1984) (the union as a two-faced institution, provoking inefficiency (high wages and unemployment) on one hand and promoting productivity and better workplace conditions on the other hand). More recent appraisals are Blanchflower and Bryson (2004), Kaufman (2002), and Turnbull (2003). Of course, our paper cannot do justice to all the exhaustive theoretical and empirical work cited in these surveys. What we try to do is to shed some light on these issues from the point of view of cooperative game theory. The rather complex setup (outside options, two partitions) makes impossible a general approach. Rather, we will have to content outselves with a specific three-player example along the lines of the above partitions.

With respect to wages, we find that a worker will prefer to be part of a union if the other worker (also a union member!) is unemployed and outside options are important. Indeed, unions prevent the capitalist from exploiting the industrial reserve. Our model is supported by the often observed "union/nonunion relative wage differential" (for an early survey, see Lewis, 1986) only, if unemployed workers keep on being union members. If there is no unemployment, overstaffing (to be made precise later) makes unionization worthwhile for the employed.

Our paper's prediction about the effect of unionization on employment is ambiguous. If workers are free to choose whether to form unions or not, they will not unionize if doing so is detrimental to employment.

In summary, we argue that our approach is well-suited for the problem at hand. In particular, it is an improvement over the cooperative models cited above. It can address the complicated interlinkages between unionization, unemployment benefits, and unemployment (industrial reserve) in a novel framework.

The paper is organized as follows. In section 2 basic definitions are given. The relevant values for our approach are presented in section 3, among them the Shapley value, the AD-value, the outside-option value, and the union value. This section culminates in the union outside-option value. We apply this value in section 4. Section 5 concludes the paper.

2 Definitions

A game (in coalition function form) is a pair (N, v) (often abbreviated by v) where $N = \{1, 2, ..., n\}$ is a finite set and v a function $2^N \to \mathbb{R}$ such that $v(\emptyset) = 0$. The set of all games on N is denoted by G. A payoff vector x for N is an element of $\mathbb{R}^n$ or a function $N \to \mathbb{R}$.

Following Aumann and Drèze (1974), we define coalition structures: A coalition structure $\mathcal{P}$ on N (sometimes written as $(N, \mathcal{P})$) is a partition of N into components $C_1, ..., C_m$:

$$\mathcal{P} = \{C_1, ..., C_m\}, \bigcup_{j=1}^{m} C_j = N, C_j \cap C_k = \emptyset, j \neq k$$

The set of all partitions on N is denoted by $\mathfrak{P}$. For any player $i \in N$, the component containing i, is written $\mathcal{P}(i)$.

Permutations (rules of order, rank order) ρ on N are written as $(\rho_1, ..., \rho_n)$ where ρ_1 is the first player in the order, ρ_2 the second player etc. Formally, rank orders are bijective functions $N \to N$. The set of all rank orders on N is denoted by RO. For every $i \in N$ there exists a $j(i) \in N$ such that $\rho_{j(i)} = i$. Then, we define $K_i(\rho) := \left\{\rho_1, ..., \rho_{j(i)}\right\}$. Thus, $K_i(\rho)$ is the set of players up to and including player i (for a given rank order ρ).

A permutation $\rho \in RO$ is called consistent with $\mathcal{P}$, if for any $1 \leq j < k < \ell \leq n$ obeying $\mathcal{P}(\rho_j) = \mathcal{P}(\rho_\ell)$ we have $\mathcal{P}(\rho_k) = \mathcal{P}(\rho_j)$. The set of permutations on N that are consistent with $\mathcal{P}$ is denoted by $RO_{\mathcal{P}}$. We obtain $RO_{\mathcal{P}}$ from RO by deleting those permutations that "separate" players belonging to one component.

The Shapley value and other related values make heavy use of marginal contributions of players. For any coalition $S \subseteq N$ and any player $i \in N$ we define

$$MC_i^S(v) := v(S \cup i) - v(S \backslash i),$$

where the usual abuse of notation occurs. Imagine the n players standing outside a room and entering one after the other, in a given rank order ρ. After player i enters the room, the player set $K_i(\rho)$ is assembled in the room. Now, player i is attributed his marginal contribution $MC_i^{K_i(\rho)}(v)$.

3 Values

Values return payoff vectors for coalition functions and partitions. Formally,

- a value on N is a function $\psi : G \to \mathbb{R}^n$;
- a (partitional) value on $(N, \mathfrak{P})$ is a function $\psi : G \times \mathfrak{P} \to \mathbb{R}^n$;
- a (bi-partitional) value on $(N, \mathfrak{P}, \mathfrak{P})$ is a function $\psi : G \times \mathfrak{P} \times \mathfrak{P} \to \mathbb{R}^n$.

The most famous value on N is the Shapley value, written $\varphi(v)$ for $v \in G$. It is given by

$$\varphi_i(v) = \frac{1}{n!} \sum_{\rho \in RO} MC_i^{K_i(\rho)}(v), i \in N.$$

Player i's Shapley value is the average of his marginal contributions for all rank orders; the cardinality of RO is $n!$.

Our interpretation of the Shapley value is this: It mirrors the economic well-being of the individuals in the absence of firms or trade unions. All other values presented in this paper copy the basic structure of the Shapley value: The payoffs are, to some extent, some average of the marginal contributions. However, they are partitional values and reflect firms (by way of an AD-partition $\mathcal{P}_{AD}$) and unions (through a union partition $\mathcal{P}_u$).

The AD-value is the partitional value given by

$$\varphi_i^{AD}(v, \mathcal{P}_{AD}) = \frac{1}{n!} \sum_{\rho \in RO} MC_i^{K_i(\rho) \cap \mathcal{P}_{AD}(i)}(v)$$

According to the AD-value, the players' payoff depends on the marginal contributions to subsets of their own component. Since there exist no links to players outside the component $\mathcal{P}_{AD}(i)$, each component is an island. While the Shapley value is Pareto efficient, the AD-value is component efficient, i.e.

$$\sum_{i \in \mathcal{P}_{AD}(i)} \varphi_i^{AD}(v, \mathcal{P}_{AD}) = v(\mathcal{P}_{AD}(i)), i \in N$$

In our understanding, the AD-components stand for firms. People within firms/components can do business together, but there are no (hardly any) economic links between people belonging to different firms/components. Of course, this is a rather coarse characterization. The main point is that there are more important and deeper links within components/firms than between them.

For the AD-value, the components are islands not only with respect to production (component efficiency) but also with respect to distribution. The outside-option value (oo-value) is also component efficient with respect to $\mathcal{P}_{AD}$, but assumes that the payoff for player i depends on his outside opportunities, i.e. his marginal contribution to coalitions of players some of which do not belong to $\mathcal{P}_{AD}(i)$. For example, in case of unemployment, the capitalist has outside options which he can use against his employees.

The oo-value is given by

$$\begin{aligned}&\varphi_i^{oo}(v,\mathcal{P}_{AD},\lambda)\\ =\ &\frac{1}{n!}\sum_{\rho\in RO}\left\{\begin{array}{ll} v(\mathcal{P}_{AD}(i))-\sum_{j\in\mathcal{P}_{AD}(i)\backslash\{i\}}MC_j(\rho,\mathcal{P}_{AD},\lambda), & \mathcal{P}_{AD}(i)\subseteq K_i(\rho),\\ MC_i(\rho,\mathcal{P}_{AD},\lambda), & \text{otherwise,}\end{array}\right.\end{aligned}$$

$i\in N$, where

$$MC_i(\rho,\mathcal{P}_{AD},\lambda)=\lambda MC_i^{K_i(\rho)}(v)+(1-\lambda)MC_i^{K_i(\rho)\cap\mathcal{P}_{AD}(i)}(v).$$

An axiomatization of the outside-option value for $\lambda:=1$ is offered in Wiese (2007).

In looking at a permutation ρ, player i gets his marginal contribution inside and outside his component, $MC_i(\rho,\mathcal{P}_{AD},\lambda)$, if he is not the last player in his component in ρ, i.e., if $\mathcal{P}_{AD}(i)$ is not included in $K_i(\rho)$. If, however, i is the last player in his component, he gets the worth of this component minus the marginal contributions of the other players in his component, i.e., the players in $\mathcal{P}_{AD}(i)\setminus\{i\}$. This construction ensures component efficiency.

In case of $\lambda=0$, we get the AD-value as a special case. Positive values of λ reflect outside opportunities where marginal contributions to coalitions outside $\mathcal{P}(i)$ get a positive weight. Low values of λ reflect the inability to use outside options as a threat in bargaining. For example, employment of yet another worker may not be feasible or substitution of the presently employed by the presently unemployed may not be possible. For the trivial partition $\mathcal{P}_{AD}=\{N\}$ we obtain $\varphi_i^{oo}(v,\{N\},\lambda)=\varphi_i(v)=\varphi_i^{AD}(v,\{N\})$ for all $\lambda\in[0,1]$.

We now turn to the Owen, or union, value. It makes use of only those permutations consistent with some given partition $\mathcal{P}_u$. It is given by

$$\varphi_i^u(v,\mathcal{P}_u)=\frac{1}{|RO_{\mathcal{P}_u}|}\sum_{\rho\in RO_{\mathcal{P}_u}}MC_i^{K_i(\rho)}(v),i\in N.$$

Consider again the n players standing outside our room. The components queue outside the door and then the players enter, one after the other, without breaking the components. In a sense, the players in a component offer their total service or no service at all.

Finally, we can present the union outside-option value. It is obtained by merging the union and the outside-option values in the obvious manner:

$$\begin{aligned}&\varphi_i^{u-oo}(v,\mathcal{P}_{AD},\lambda,\mathcal{P}_u)\\ =\ &\frac{1}{|RO_{\mathcal{P}_u}|}\sum_{\rho\in RO_{\mathcal{P}_u}}\left\{\begin{array}{ll} v(\mathcal{P}_{AD}(i))-\sum_{j\in\mathcal{P}(i)\backslash i}MC_j(\rho,\mathcal{P}_{AD},\lambda), & \mathcal{P}_{AD}(i)\subseteq K_i(\rho),\\ MC_i(\rho,\mathcal{P}_{AD},\lambda), & \text{otherwise,}\end{array}\right.\end{aligned}$$

$i \in N$, where

$$MC_i(\rho, \mathcal{P}_{AD}, \lambda) = \lambda MC_i^{K_i(\rho)}(v) + (1-\lambda) MC_i^{K_i(\rho) \cap P_{AD}(i)}(v).$$

4 A simple labour market

4.1 Partitions and payoffs

Turning to the 3-player example from the introduction, we consider three AD-partitions, $\mathcal{P}_{AD} = \{\{1,2,3\}\}$, $\mathcal{P}_{AD} = \{\{1,2\},\{3\}\}$, and $\mathcal{P}_{AD} = \{\{1\},\{2\},\{3\}\}$. In the first, the capitalist (player 1) employs both workers (players 2 and 3), in the second, player 3 is unemployed, and in the third, both are unemployed. We also deal with two union partitions, $\mathcal{P}_u = \{\{1\},\{2\},\{3\}\}$ and $\mathcal{P}_u = \{\{1\},\{2,3\}\}$. The second indicates that workers 2 and 3 form a union. Thus, we have 6 partition combinations.

To fix ideas, we set $v(N) := 100$ (any positive value or a variable would do) and let $a_2 := v(\{1,2\})$ and $a_3 := v(\{1,3\})$. We assume $a_2 > a_3 \geq 0$, i.e., worker 2 is more productive than worker 3 in a one-worker firm. If workers are not employed, they receive unemployment benefit, $u \geq 0$. Hence, $v(\{2\}) = v(\{3\}) = u$ and $v(\{2,3\}) = 2u$. Since we want to concentrate on unionization and unemployment benefits, we let $v(\{1\}) := 0$, assuming zero normal profits for the capitalist. Superadditivity (which we do not, in general, assume) implies

$$\begin{aligned} 2u &\leq 100, \\ a_2 + u &\leq 100, \text{ and} \\ a_3 + u &\leq 100. \end{aligned}$$

We first report the values and then (see the next section) solve a three-stage model.

Result 1: For the six partition combinations, the union outside-option value yields the following payoffs:

$\mathcal{P}_{AD}$	$\mathcal{P}_u$	φ^{u-oo}
$\{\{1,2,3\}\}$	$\{\{1\},\{2\},\{3\}\}$	$\begin{pmatrix} A := \frac{100}{3}+\frac{a_2}{6}+\frac{a_3}{6}-u \\ B := \frac{100}{3}+\frac{a_2}{6}-\frac{a_3}{3}+\frac{u}{2} \\ C := \frac{100}{3}-\frac{a_2}{3}+\frac{a_3}{6}+\frac{u}{2} \end{pmatrix}$
$\{\{1,2,3\}\}$	$\{\{1\},\{2,3\}\}$	$\begin{pmatrix} D := 50-u \\ E := 25+\frac{a_2}{4}-\frac{a_3}{4}+\frac{u}{2} \\ F := 25-\frac{a_2}{4}+\frac{a_3}{4}+\frac{u}{2} \end{pmatrix}$
$\{\{1,2\},\{3\}\}$	$\{\{1\},\{2\},\{3\}\}$	$\begin{pmatrix} G := \frac{a_2}{2}+\frac{1}{6}\lambda(a_3-u)-\frac{u}{2} \\ H := \frac{a_2}{2}-\frac{1}{6}\lambda a_3+\frac{1}{6}u(3+\lambda) \\ = \frac{a_2}{2}-\frac{1}{6}\lambda(a_3-u)+\frac{u}{2} \\ I := u \end{pmatrix}$
$\{\{1,2\},\{3\}\}$	$\{\{1\},\{2,3\}\}$	$\begin{pmatrix} J := \frac{a_2}{2}-\frac{u}{2} \\ K := \frac{a_2}{2}+\frac{u}{2} \\ L := u \end{pmatrix}$
$\{\{1\},\{2\},\{3\}\}$	$\{\{1\},\{2\},\{3\}\}$ or $\{\{1\},\{2,3\}\}$	$\begin{pmatrix} M := 0 \\ N := u \\ P := u \end{pmatrix}$

In particular, we find:

Result 1a If the capitalist wants to employ one worker only, he will choose the more productive worker 2. For moderate unemployment benefits ($u < a_3$), worker 2's wage is higher in the presence of a union than without a union. In order to accept employment, worker 2 needs to be sufficiently productive and unemployment benefits need to be sufficiently low.

Result 1b The incentives of the capitalist to employ worker 3 on top of worker 2 depend on whether or not the workers form a union. If they do, worker 3 will be employed whenever his marginal contribution exceeds unemployment benefits. If there is no union, the capitalist might be prepared to employ a worker 3 even if that worker's marginal contribution is negative. In case of low average marginal contributions ($\frac{1}{2}(100-a_3)+\frac{1}{2}(100-a_2) < 50$) the workers prefer to be unionized. ■

The values in the above table are obtained by straightforward calculations. The first row corresponds to the Shapley value (all workers employed, no union). The Owen value (all workers employed, workers form a union) is seen in row 2. In the last row, the capitalist does not employ any worker so that no output is produced. Then the capitalist and the workers are paid their reservation payoff, a profit of 0 and the unemployment benefit u, respectively. Rows 3 and 4 refer to the case where only worker 2 is employed. For these cases, the (union) outside-option value has been devised. The appendix provides an example of how the payoff is to be calculated.

Note that the bargaining power of any agent is expressed by the two partitions and by λ. In particular, the capitalist cannot make a take-it-or-leave-it offer to the worker(s) in order to lower their payoffs to the reservation level.

We can use the values to theorize about the players' preferences. By simple comparisons (note the letters standing for the payoffs), and explicating Result

1a, we find:

- By $a_2 > a_3$, the capitalist prefers to have worker 2 rather than worker 3 as his only employee (compare profits G and J, respectively, with the symmetrical profits obtained by interchanging workers 2 and 3).
- If worker 2 is the only employee and if the workers are not unionized, worker 2's payoff
$$H = \frac{a_2}{2} - \frac{1}{6}\lambda\left(a_3 - u\right) + \frac{1}{2}u$$
reveals that the capitalist can use worker 3 to lower worker 2's wage. This mechanism will work,
  - if there is a high degree of flexibility and outside options (λ is high),
  - if worker 3 is productive (if he were employed), and
  - if unemployment benefits are moderate.

  In terms of Marx (1985, pp. 657), worker 3 forms the industrial reserve. If, however, unemployment benefits are not moderate ($u > a_3$), the capitalist suffers from the outside option (dealing with worker 3). Indeed, worker 2 might say to the capitalist that the capitalist would need to deal with worker 3 unless he, worker 2, would be prepared to put up employment.
  Interestingly, unionization prevents the use of the industrial reserve by the capitalist. This can be seen from worker 2's payoff $K = \frac{a_2}{2} + \frac{1}{2}u$. A comparison of H with K shows that unions make worker 2 more willing to accept employment in case of $a_3 > u$.

Turning to Result 1b, we find:

- If the workers form a union, the capitalist wants to employ worker 3 on top of worker 2 whenever worker 3's marginal contribution $100 - a_2$ exceeds unemployment benefit u ($D > J$).

  If there is not union, the capitalist might be willing to employ worker 3 even if that worker has a negative marginal contribution. Indeed, we find
$$A > G \Leftrightarrow 100 - a_2 > \frac{1}{2}\left[u\left(3 - \lambda\right) - a_3\left(1 - \lambda\right)\right]$$
  where moderate unemployment benefits can make the right-hand term negative. In that case, the third worker is not employed for his productiveness but is brought into the firm in order to increase the capitalist's bargaining power vis-a-vis worker 2. This function is especially important to the capitalist if he cannot use the unemployed worker 3 as industrial reserve. Formally, we have $\frac{\partial\left[\frac{1}{2}\left[u(3-\lambda)-a_3(1-\lambda)\right]\right]}{\partial\lambda} = \frac{1}{2}\left(a_3 - u\right)$, where $a_3 > u$ (moderate unemployment benefits) and low values of λ make employment of worker 3 more probable.

- Since both $E > B$ and $F > C$ are equivalent to $\frac{1}{2}a_2 + \frac{1}{2}a_3 > 50$, both employed workers prefer unionization if their average productivity in a one-worker firm is sufficiently high, or differently put, if the average marginal contribution of the additional worker is sufficiently low ($\frac{1}{2}(100 - a_3) + \frac{1}{2}(100 - a_2) < 50$). Thus, overstaffing makes unionization worthwhile for the employed. In contrast, the capitalist prefers unionization if the workers are relatively unproductive. A comparable result has been presented by Horn and Wolinsky (1988, p.488) in the context of a non-cooperative model. They assume $a_2 = a_3$ and find an incentive to unionize in case of $a_2 > 50$ which is a special case of our result. The reader is invited to consult the appendix for details.

4.2 The sequential model

4.2.1 Game sequence

We now turn to a model consisting of three stages. First, the workers decide on unionization. Here, we apply the Pareto principle so that one worker alone can decide about unionization if the other is indifferent. Second, the capitalist makes an employment offer to worker 2, worker 3, both, or none. (Wages are determined later.) Finally, the workers accept employment or decline. If any worker declines, no workers are employed. This is not restrictive. Since the capitalist can foresee the workers' payoffs and decisions, he will make acceptable offers. In order to maintain tractability, we assume $\lambda := 1$.

4.2.2 Solving for subgame-perfect equilibria

If the capitalist plans to employ one worker only (stage 2), he will choose the more productive worker 2 by $a_2 > a_3$ (see profits G and J). Thus, at stage 2, the capitalist chooses between

- employing worker 2, only,
- employing both workers, and
- employing none.

By $a_2 > a_3$, both workers will accept employment if worker 3 accepts (see payoffs $B > C$ and $E > F$). Solving the model requires simple but tedious case distinctions which we will relegate to the appendix.

4.3 Employment (stages 2 and 3)

4.3.1 Voluntary unemployment

From the point of view of social policy, it is an important question whether unemployment benefits affect unemployment and voluntary unemployment. As De Vroey (2004, pp. 13) points out, several alternative definitions of voluntary unemployment co-exist. Definitions of voluntary unemployment make use of counterfactual thought experiments: Would the unemployed worker like to be employed in lieu of another actually employed one? Since we deal with small numbers of heterogeneous workers, we propose the following working definition: an unemployed worker is voluntarily unemployed if employing him - on top of the actually employed workers - would lead to an unattractive wage rate, i.e., a wage rate lower than his unemployment benefit.

Result 2: Voluntary unemployment may happen. ■

In order to show Result 2, we look at the special case depicted in figure 1. “Nu” stands for “no union” and “Nu2” refers to one of several cases (see appendix). The agents preferences are indicated in their respective lines and are noted as a function of unemployment benefits u. For example, to the left of the first vertical line, the capitalist will prefer to employ worker 2 instead of employing no worker. To the left of the second vertical line, he prefers to employ both workers rather than none. To the left of the last vertical line, the capitalist would rather employ both workers than worker 2, only. The preferences for worker 2 (who will get an offer if only one worker gets an offer) and of both workers (identical to the preferences of worker 3) are depicted in a similar way. The fourth line (”accepted offer”) summarizes these lines into statements about employment.

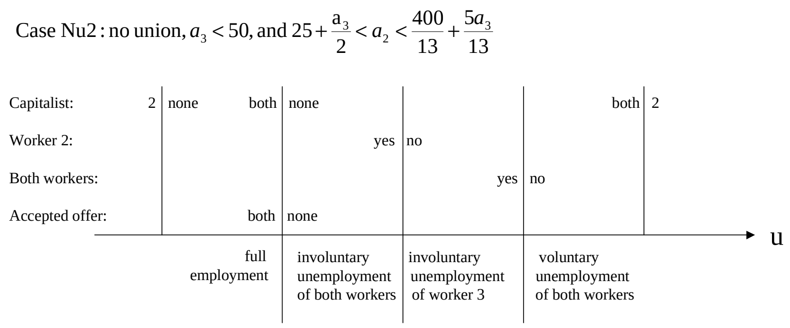

Figure 1: Preferences and outcomes in case Nu2

In the actual case, the capitalist is able to achieve his preferred outcome (both workers left of the second vertical line, none to the right of this line) because both workers are prepared to put up employment whenever the capitalist is ready to offer employment to both.

We see that involuntary and voluntary unemployment can well happen. Between the second and third vertical line, worker 2 would be prepared to accept employment of him alone and both workers would be prepared to accept employment of both. Here, we have involuntary unemployment of both workers. The area between the third and fourth vertical line is difficult to classify. Worker 3 is involuntarily unemployed. However, worker 2 is not prepared to be employed on his own while he is willing to be employed if both workers were active. To the right of the fourth line (at high levels of unemployment benefits), we have voluntary unemployment of both workers.

4.3.2 Employment and unionization

We now present the results about the effects of unemployment benefits and unionization on employment.

Result 3: As a general picture, unemployment is an increasing function of the level of unemployment benefits. Depending on parameters, unions can be harmful or beneficial for employment. ■

In order to show that unemployment benefits create unemployment, we refer to two figures. Figure 2 is based on $a_3 = 20$ (an example for $a_3 < 50$) and figure 3 on $a_3 = 60$ (an example for $a_3 > 50$). In these figures, unemployment benefit u is plotted against $a_2 \geq a_3$. Obviously, unemployment benefits are detrimental to employment.

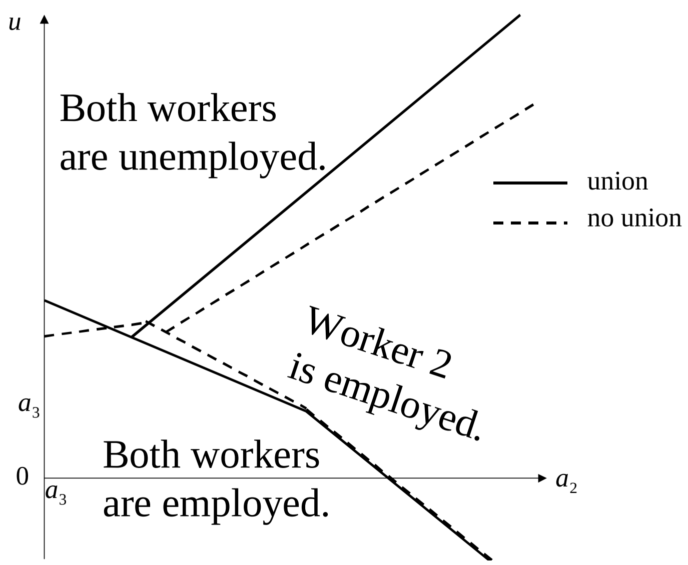

Figure 2: Employment and unions for $a_3 = 20$

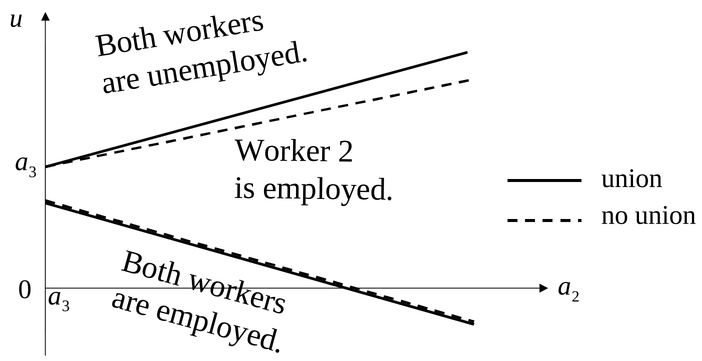

Figure 3: Employment and unions for $a_3 = 60$

The effect of unionization on employment is quite unclear. For example, in figure 2, in the leftmost triangle bordering the u-axis, we have unemployment without unions and full employment in case of unions. For the very small triangle to the right of this triangle, we have unemployment in case of, and full employment, without unions.

4.4 Union choice (stage 1)

We now turn to stage 1 of our model, i.e., to the question of whether workers will want to unionize. There are two somewhat distinct reasons for unionization (or for deciding against unions). Workers make their union-choices in order to be employed (i.e., in order to obtain a salary instead of unemployment benefits) or in order to increase their salary. Thus, we distinguish between the employment and the salary motive.

Result 4: Workers are unanimous in their union choice and unions can never be blamed for unemployment. Workers decide on unions for both employment and salary motives. Unions tend to be beneficial for (employed!) workers if there is overstaffing or unemployment. ■

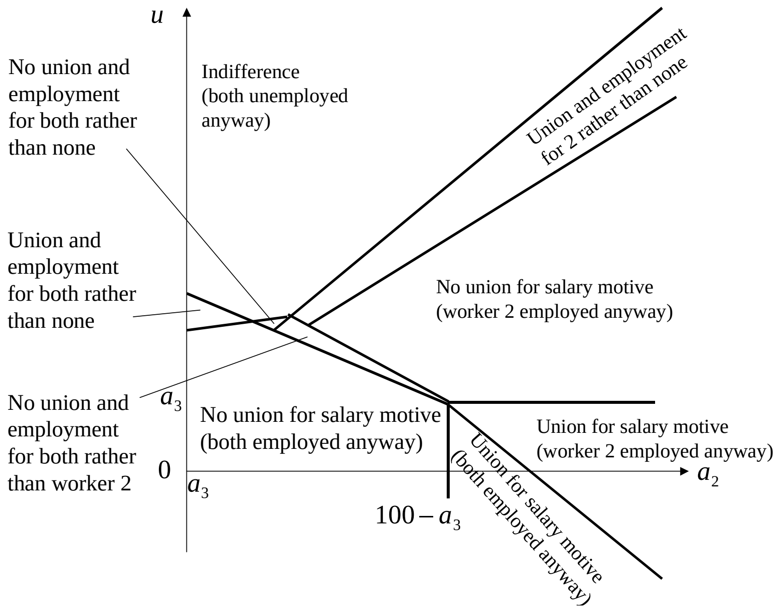

Figure 4: Union choice for $a_3 = 20$

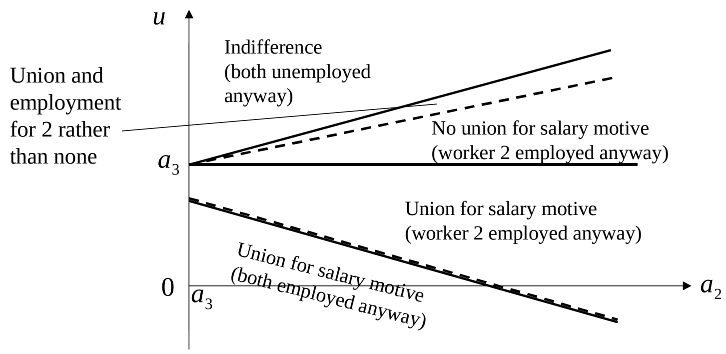

Figure 5: Union choice for $a_3 = 60$

Figures 4 (based on $a_3 = 20$) and 5 (based on $a_3 = 60$) inform about the choice of unions by the two workers. If one worker (worker 3) is indifferent towards unionization (because he is unemployed in either case), while the other (worker 2) has a definite preference, we assume that the latter one's preferences count. We also find that if both workers are employed, their preferences coincide.

Therefore, unions can never be blamed for unemployment from the point of view of stage 1.

In Result 1a, we note that moderate unemployment benefits ($u < a_3$) make worker 2 - as the only employee - prefer a union. This is reflected in both figures. According to Result 1b, overstaffing ($\frac{1}{2}(100 - a_3) + \frac{1}{2}(100 - a_2) < 50$) imply that both (employed!) workers prefer a union. Indeed, the equivalent formulation is $a_2 > 100 - a_3$ which holds everywhere in figure 5 (which is based on $a_3 > 50$) and to the right of $100 - a_3$ in figure 4.

5 Conclusions

The attraction of noncooperative (!) game theory has two sources. First, the pure theory of noncooperative games has made considerable progress and appeals to economists and other social scientists. Second, there exists a host of applications of game theory to diverse fields such as industrial organization, political science, evolutionary biology etc.

The state of affairs for cooperative game theory seems bleaker. While the axiomatic basis of cooperative game theory is sound and appealing, applications are scarce. This paper fruitfully applies cooperative game theory to a simple three-person game addressing questions of unionization, employment and unemployment benefits. In fact, we can use the model to draw some policy prescriptions. Of course, the recommendations following from our simple model (indeed, from any model) should be attached with an appropriate caveat.

With respect to employment maximization, the model takes a negative view on (high) unemployment benefits and a differentiated view on unionization. In fact, depending on the parameters, unionization may increase or decrease employment. If unions lead to higher wages, these wages may depress employment because the capitalist is not prepared to pay high wages. However, unemployment may also result from an unwillingness of workers to accept employment. In that case, high wages affected by a union may actually be beneficial for employment.

Interestingly, endogenous unionization has positive effects on employment. A worker who foresees that the existence of a union leads to his being unemployed will not join. Note, however, that in our model all workers a unionized or none. In real-world labor markets, some percentage of the workers (employed or unemployed) are union members, only. Then, union members may lobby for high wages that prove detrimental for the employment of other, non-union workers.

The model may also provide an indication of when obligatory unions (all the workers are obliged to join) can be expected to increase wages. If a substantial industrial reserve exists, a union provides protection against the potential competition by the unemployed. If (almost) all workers are employed, unions are beneficial if there is overstaffing, i.e., if there are some workers that might be laid off without much harm to productivity.

Our paper uses a non-core cooperative solution concept which can readily be criticized. Why do workers not earn their marginal product? Why do markets not clear? In our mind, there are two justifications for applying the union outside-option value. First of all, it encompasses a lot of social structure (employment, unions) that would be very difficult to model in a non-cooperative manner. Attempts in this direction have been presented by Horn and Wolinsky (1988) and Jun (1989), both using the Rubinstein bargaining procedure. These authors concentrate on the union aspect but do not take unemployment or unemployment benefits into account. Another interesting paper by Davidson (1988) assumes a Cournot oligopoly. Here, workers are homogeneous and outside options and unemployment benefits have no role to play.

Second, the use of non-core concepts may be taken to reflect labour market rigidities. While the industrial reserve does indeed lower wages, it cannot do so in a perfectly competitive fashion.

Future research could be persued along the following lines. In our paper, we have a single employer. Our method could also be used with several employers in order to provide a cooperative analogue to the above mentioned Cournot approach by Davidson (1988).

6 Appendix

6.1 Calculating G in Result 1

In order to confirm $G = \frac{a_2}{2} + \frac{1}{6}\lambda(a_3 - u) - \frac{u}{2}$ in the table of Result 1, assume $\mathcal{P}_{AD} = \{\{1,2\},\{3\}\}$ and $\mathcal{P}_u = \{\{1\},\{2\},\{3\}\}$. We then obtain

$$\begin{aligned}
MC_1((1,2,3),\mathcal{P}_{AD},\lambda) &= MC_1((1,3,2),\mathcal{P}_{AD},\lambda)\\
&= \lambda MC_1^{\{1\}}(v) + (1-\lambda) MC_1^{\{1\}\cap P_{AD}(1)}(v)\\
&= MC_1^{\{1\}}(v) = 0 - 0 = 0\\
MC_1((3,1,2),\mathcal{P}_{AD},\lambda) &= \lambda MC_1^{\{3,1\}}(v) + (1-\lambda) MC_1^{\{3,1\}\cap P_{AD}(1)}(v)\\
&= \lambda MC_1^{\{3,1\}}(v) + (1-\lambda) MC_1^{\{1\}}(v)\\
&= \lambda(a_3 - u) + (1-\lambda)(0-0) = \lambda(a_3 - u)\\
MC_2((2,1,3),\mathcal{P}_{AD},\lambda) &= MC_2((2,3,1),\mathcal{P}_{AD},\lambda)\\
&= \lambda MC_2^{\{2\}}(v) + (1-\lambda) MC_2^{\{2\}\cap P_{AD}(2)}(v)\\
&= MC_2^{\{2\}}(v) = u - 0 = u\\
MC_2((3,2,1),\mathcal{P}_{AD},\lambda) &= \lambda MC_2^{\{3,2\}}(v) + (1-\lambda) MC_2^{\{3,2\}\cap P_{AD}(2)}(v)\\
&= \lambda MC_2^{\{3,2\}}(v) + (1-\lambda) MC_2^{\{2\}}(v)\\
&= \lambda(2u - u) + (1-\lambda)(u-0) = u
\end{aligned}$$

and

$$\begin{aligned}
&\varphi_1^{u-oo}(v,\mathcal{P}_{AD},\lambda,\mathcal{P}_u)\\
=\ &\frac{1}{6}\Bigg(\underbrace{MC_1((1,2,3),\mathcal{P}_{AD},\lambda)}_{(1,2,3)} + \underbrace{MC_1((1,3,2),\mathcal{P}_{AD},\lambda)}_{(1,3,2)}\\
&+\underbrace{v(\{1,2\}) - MC_2((2,1,3),\mathcal{P}_{AD},\lambda)}_{(2,1,3)} + \underbrace{v(\{1,2\}) - MC_2((2,3,1),\mathcal{P}_{AD},\lambda)}_{(2,3,1)}\\
&+\underbrace{MC_1((3,1,2),\mathcal{P}_{AD},\lambda)}_{(3,1,2)} + \underbrace{v(\{1,2\}) - MC_2((3,2,1),\mathcal{P}_{AD},\lambda)}_{(3,2,1)}\Bigg)\\
=\ &\frac{1}{6}\Bigg(\underbrace{0}_{(1,2,3)} + \underbrace{0}_{(1,3,2)} + \underbrace{a_2 - u}_{(2,1,3)} + \underbrace{a_2 - u}_{(2,3,1)} + \underbrace{\lambda(a_3 - u)}_{(3,1,2)} + \underbrace{a_2 - u}_{(3,2,1)}\Bigg)\\
=\ &\frac{1}{6}(3a_2 - 3u + \lambda(a_3 - u)) = \frac{a_2}{2} + \frac{1}{6}\lambda(a_3 - u) - \frac{u}{2} = G
\end{aligned}$$

6.2 Stages 2 and 3 in case of no unions

For any given u, we first assume that workers decide against unions (stage 1). The capitalist

- prefers to employ both workers rather than worker 2, only, in case of

$$A > G \Leftrightarrow u < 100 - a_2 =: \gamma_{23>2},$$

- prefers to employ worker 2, only, rather than none in case of

$$G > M \Leftrightarrow u < \frac{3}{4}a_2 + \frac{1}{4}a_3 =: \gamma_{2>0},$$

- and prefers to employ both workers rather than none in case of

$$A > M \Leftrightarrow u < \frac{100}{3} + \frac{1}{6}(a_2 + a_3) =: \gamma_{23>0}.$$

Worker 2 is ready to accept employment as the only worker if

$$H > N \Leftrightarrow u < \frac{3}{2}a_2 - \frac{1}{2}a_3 =: \omega_2$$

holds. Both workers are prepared to put up employment if worker 3 is ready, i.e., if

$$C > P \Leftrightarrow u < \frac{200}{3} - \frac{2}{3}a_2 + \frac{1}{3}a_3 =: \omega_{23}.$$

We obtain the following partition in $a_2 - a_3$ space:

Nu1	$a_3 < 50, a_3 < a_2 < 25 + \frac{1}{2}a_3$	$\gamma_{2>0} < \omega_2 < \gamma_{23>0} < \omega_{23} < \gamma_{23>2}$
Nu2	$a_3 < 50, 25 + \frac{1}{2}a_3 < a_2 < \frac{400}{13} + \frac{5}{13}a_3$	$\gamma_{2>0} < \gamma_{23>0} < \omega_2 < \omega_{23} < \gamma_{23>2}$
Nu3	$a_3 < 50, \frac{400}{13} + \frac{5}{13}a_3 < a_2 < 40 + \frac{1}{5}a_3$	$\gamma_{2>0} < \gamma_{23>0} < \omega_{23} < \omega_2 < \gamma_{23>2}$
Nu4	$a_3 < 50, 40 + \frac{1}{5}a_3 < a_2 < \frac{800}{17} + \frac{1}{17}a_3$	$\gamma_{2>0} < \omega_{23} < \gamma_{23>0} < \gamma_{23>2} < \omega_2$
Nu5	$a_3 < 50, \frac{800}{17} + \frac{1}{17}a_3 < a_2 < \frac{400}{7} - \frac{1}{7}a_3$	$\omega_{23} < \gamma_{2>0} < \gamma_{23>0} < \gamma_{23>2} < \omega_2$
Nu6	$a_3 < 50, \frac{400}{7} - \frac{1}{7}a_3 < a_2 < 100 - a_3$	$\omega_{23} < \gamma_{23>2} < \gamma_{23>0} < \gamma_{2>0} < \omega_2$
Nu7	$a_2 + a_3 > 100$	$\gamma_{23>2} < \omega_{23} < \gamma_{23>0} < \gamma_{2>0} < \omega_2$

6.3 Stages 2 and 3 in case of unions

Assume that the workers have formed a union in stage 2. The capitalist

- prefers to employ both workers rather than worker 2, only, in case of

$$D > J \Leftrightarrow u < 100 - a_2 =: \gamma_{23>2}^{\text{union}},$$

- prefers to employ worker 2, only, rather than none in case of

$$J > M \Leftrightarrow u < a_2 =: \gamma_{2>0}^{\text{union}},$$

- and prefers to employ both workers rather than none in case of

$$D > M \Leftrightarrow u < 50 =: \gamma^{\text{union}}_{23>0}.$$

Worker 2 is ready to accept employment as the only worker if

$$K > N \Leftrightarrow u < a_2 =: \omega^{\text{union}}_2$$

holds. Both workers are prepared to put up employment if worker 3 is ready, i.e., if

$$F > P \Leftrightarrow u < 50 - \frac{1}{2}(a_2 - a_3) =: \omega^{\text{union}}_{23}.$$

We find the following partition in $a_2 - a_3$ space:

U1	$a_3 < 50, a_3 < a_2 < \frac{100}{3} + \frac{1}{3}a_3 < 50$	$\gamma^{\text{union}}_{2>0} = \omega^{\text{union}}_2 < \omega^{\text{union}}_{23} < \gamma^{\text{union}}_{23>0} < \gamma^{\text{union}}_{23>2}$
U2	$a_3 < 50, \frac{100}{3} + \frac{1}{3}a_3 < a_2 < 50$	$\omega^{\text{union}}_{23} < \gamma^{\text{union}}_{2>0} = \omega^{\text{union}}_2 < \gamma^{\text{union}}_{23>0} < \gamma^{\text{union}}_{23>2}$
U3	$a_3 < 50, a_2 > 50$ $a_2 + a_3 < 100$	$\omega^{\text{union}}_{23} < \gamma^{\text{union}}_{23>2} < \gamma^{\text{union}}_{23>0} < \gamma^{\text{union}}_{2>0} = \omega^{\text{union}}_2$
U4	$a_2 + a_3 > 100$	$\gamma^{\text{union}}_{23>2} < \omega^{\text{union}}_{23} < \gamma^{\text{union}}_{23>0} < \gamma^{\text{union}}_{2>0} = \omega^{\text{union}}_2$

6.4 Stage 1: Will the workers form a union?

We merge the Nu-partition with the U-partition to find out whether unionization is profitable for the workers. We find that a relatively simple partition suffices to answer this question:

1	$a_3 < 50, a_3 < a_2 < 25 + \frac{1}{2}a_3 < 50$	$\text{Nu1} \cap \text{U1}$
2	$a_3 < 50, 25 + \frac{1}{2}a_3 < a_2 < \frac{100}{3} + \frac{1}{3}a_3 < 50$	$(\text{Nu2} \cup \text{Nu3}) \cap \text{U1}$
3	$a_3 < 50, \frac{100}{3} + \frac{1}{3}a_3 < a_2 < 40 + \frac{1}{5}a_3 < 50$	$\text{Nu3} \cap \text{U2}$
4	$a_3 < 50, 40 + \frac{1}{5}a_3 < a_2 < \frac{800}{17} + \frac{1}{17}a_3 < 50$	$\text{Nu4} \cap \text{U2}$
5	$a_3 < 50, \frac{800}{17} + \frac{1}{17}a_3 < a_2 < 100 - a_3$	$(\text{Nu5} \cup \text{Nu6}) \cap (\text{U2} \cup \text{U3})$
6	$a_2 > 100 - a_3$	$\text{Nu7} \cap \text{U4}$

Comparing the payoffs for the two workers in all these parameter regions, we obtain figures 2 through 5 in the main text.

6.5 The Horn-Wolinsky model

Horn and Wolinsky (1988) assume to workers, workers A and B who jointly produce $x + y$ while either one of them alone produces x, only. Thus, we have

$$a_2 = a_3 = x.$$

The authors find that unionization pays in case of $y < x$. Letting $x + y = 100$ (which is not a serious assumption) and transferring this result into our notation yields

$$\begin{aligned} 100 - x &< x \text{ and} \\ x &> 50. \end{aligned}$$

This is exactly our result for the special case $a_2 = a_3$.

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