Decision theory and probability theory: Pascal's wager and pre-modern Indian lotteries

Harald Wiese, University of Leipzig*

February 2021

Abstract

Pascal's wager is considered an important stepping stone for the emergence of probability that expresses subjective belief. Pascal is also sometimes credited with having invented decision theory. This paper analyzes the pre-modern Indian contributions to both decision theory and probability-as-subjective-belief theory. For that purpose, I have a close look at selected passages from the birth-story of Brahma, the Hitopadeśa, the Arthaśāstra, and the Mahābhārata. The main thesis is that the lottery offered by the king to the Buddha-to-be may be the world-first use of a lottery in order to find out about the strength of a decision maker's belief.

Keywords:

decision theory, probability, lottery, expectation, normalization, Pascal's wager, birth-story of Brahma, actions and states of the world, fate and human effort

*Postfach 920, 04009 Leipzig, Germany, tel.: 49 341 97 33 771, e-mail: wiese@wifa.uni-leipzig.de.
Paper presented at the Roundtable on Re-Thinking Indology at the Indian Institute of Advanced Studies, Shimla, on March 13, 2020. Thanks are due to Monika Zin-Oczkowska and Roland Steiner.

Dieu est, ou il n'est pas. Mais de quel côté pencherons-nous?
(Sellier (1991, p. 469))

If the next world is not a bogey man for children,
and if you think I should believe in it,
then give me five hundred nishkas
and I'll return you a thousand in another life!
(Meiland (2009b, p. 279))

1 Introduction

For the purpose of this paper, "objective" probability is assumed to derive from chance experiments like tossing a die several times. A decision under risk means choosing between several actions in the presence of objective probability information about the states of the world. In contrast, "subjective" probability means a degree of belief in a proposition that may reasonably differ between decision makers. Then, one can glean information about the decision maker's belief by confronting him with decisions involving these states of the world. Thus, decision theory and probability theory have close interlinkages.

In a much noted monography on the "emergence of probability", Hacking (2006, p. 11) opines: "The decade around 1660 is the birthtime of probability. In 1657 Huygens wrote the first probability textbook to be published. At about that time Pascal made the first application of probabilistic reasoning to problems other than games of chance, and thereby invented decision theory." Hacking's remarks concern subjective probability and refer to the European history of thought. Referring to the first edition from 1975, Hacking has come under some attack for arguing for a rather abrupt emergence of probability ideas, see Garber & Zabell (1979), Hald (1990), and Hacking (2006, Introduction 2006). Hacking did not mean to deny earlier more isolated instances of remarks that come somewhat close to Pascal and later scientists. With respect to India, he cites some relevant literature and adds that "a good deal of Indian probability lore is at present unknown to us." Meanwhile, this state of affairs has improved on account of Raju (2011). Probability theory is not addressed by Plofker (2009).

The current paper aims to add a paragraph to our knowledge on how probability was conceived in pre-modern India. At the same time, it adds two paragraphs on how decision theory seemed to be deeply ingrained in Indian thinking. For the purpose of this paper, one might start the discussion with Pascal's wager. While Pascal was most likely not influenced by much older Indian developments, there are interesting parallels to be observed.

Hacking (2006, pp. 63-72) provides a very sympathetic reconstruction of Pascal's argument. Hacking argues that Pascal developed logically valid (if theologically dubious) arguments for believing (indeed for striving to believe) in God. Pascal did not develop a very formal argument, nor did he provide a matrix like fig. 1. His argument can be sketched as follows: Any person has the option of believing in God (or acting so that this belief comes about) or of not believing in God. If God exists, the belief in Him (and perhaps the good actions that ensue) will lead to heaven, i.e., to eternal bliss after death, while disbelief carries the ultimate punishment of eternal damnation after death. If God does not exist, the pious life would have been led in vain, while disbelief leads to a hedonist life on earth, with nothing to come after death.

		state of the world	
		God exists	God does not exist
action	belief in God	eternal bliss	pious life in vain
	no belief in God	eternal damnation	hedonist life

Figure 1: Pascal's wager

Assume that a pious life might still be a happy one. Then, Hacking imputes the concept of a so-called "dominant" action to Pascal. If God does not exist, there is nothing wrong with believing in Him. If, however, he does exist, belief is clearly better than disbelief. Hence, belief in God dominates disbelief. Assume, however, that–in the absence of God–a hedonist life is to be preferred to a pious life. Then an expected-value argument would again lead to belief as the better action for any strictly positive probability for God's existence. This holds because Pascal attributes an infinite payoff to eternal bliss. The current author has no issues with this sympathic "reconstruction" of Pascal's ideas into modern decision-theoretic parlance.

Remember that Hacking attributes not only the concept of degree-of-belief probability to Pascal, but also the invention of decision theory. This is not unreasonable in view of the reconstruction sketched above. After providing a primer on probability and decision theory, I argue that both decision theory and probability-as-subjective-belief theory was "present" in pre-modern Indian texts. This paper defends two arguments: First, decision theory in terms of actions and states of the world (in modern parlance) is deeply rooted in pre-modern Indian thought. Second, a lottery found in the Buddha's birth-story of Brahma may be the world-first use of a lottery in order to find out about the strength of a decision maker's belief.

2 A primer on probability and decision theory

2.1 Probability theory

Consider a set of events or propositions. The events may be

- event F : "the casting of a die yields two pips" or
- event G : "the casting of a die yields an odd number of pips".

The propositions may relate to the weather in Shimla tomorrow (rain, no rain, warmer than 12 degrees, etc.), to God's existence, or the like. Depending on the problem to be analyzed one needs an all-encompassing set E that contains the interesting events or propositions. Whatever may be the case, E is certain to occur. For example, the number of pips after casting a die lies between 1 and 6. In the case of propositions, E may contain the two propositions "it will rain in Shimla tomorrow" and "it will not rain in Shimla tomorrow".

Formally, a probability p is a function defined on all the subsets (called events) or on all the propositions of the all-encompassing set E. p obeys two properties:

- normalization:
  for all events/propositions $F \subseteq E$: $0 = p(\varnothing) \leq p(F) \leq 1 = p(E)$

- additivity:
  for mutually exclusive events/propositions F and G: $p(F \cup G) = p(F) + p(G)$

Thus, every probability lies between 0 and 1, the probability of the empty set $\varnothing$ is zero (it cannot happen that nothing occurs), and the probability of the all-encompassing set E is one. Furthermore, the sum of the probabilities for event F (1/6) and event G (1/2) is 2/3 which is the probability of event $F \cup G$, i.e., of obtaining 1, 2, 3, or 5 pips.

Importantly, probabilities can be interpreted in two different manners. First, they "approximate" the frequency of occurrence of an event. For example, if you toss a die 1000 times, you should expect to record six pips about 167 times. Second, they express a degree of belief without statistical background.

2.2 Decision theory

2.2.1 Models

Let us now turn to basic decision theory. Consider a simple example (see fig. 2). A firm can produce umbrellas or sunshades. Umbrellas lead to a profit of 7, sunshades yield 5.

action	production of umbrellas	7
	production of sunshades	5

Figure 2: Payoffs depend on actions

In this most basic microeconomic decision model we have

- a set of actions A (production of umbrellas, production of sunshades)
- a set of consequences C (profits)
- a consequence function $f : A \to C$ that attributes a consequence $c \in C$ to every action $a \in A$ (the production of umbrellas leads to the profit of 7)

In the standard decision model, an agent chooses an action $a \in A$, earns the consequence $f(a)$ which may be better or worse than consequences obtained from other actions. The theoretical prediction is an action a^* that obeys

$$\underbrace{f(a^*)}_{\in C} \geq \underbrace{f(a)}_{\in C} \text{ for all } a \in A.$$

Differently put, the decision maker chooses an action a^* with consequence $f(a^*)$ such that no other action a exists that leads to a consequence $f(a)$ which is better than $f(a^*)$. In our example, the firm will and should produce umbrellas.

In more involved models, a set of states of the world is also added. Reconsider the firm that produces umbrellas or sunshades. The firm's profits now depend on the weather. There are two states of the world, good or bad weather. The matrix of fig. 3 indicates the profit as a function of the firm's decision (action) and of the state of the world.

		state of the world	
		bad weather	good w.
action	umbrellas	6	3
	sunshades	2	8

Figure 3: Payoff matrix

The highest profit is obtained if the firm produces sunshades and the weather is good. However, the production of sunshades carries the risk of a very low profit, in case of rain. The payoff matrix examplifies important concepts in our basic decision model: actions, states of the world, payoffs and payoff functions.

- The firm has two actions, producing umbrellas or producing sunshades.
- There are two states of the world, bad and good weather.
- The payoffs are 2, 3, 6, or 8.

- The payoff function determines the payoffs resulting from actions and states of the world. For example, the firm obtains a profit of 8 if it produces sunshades and it is sunny.

Let us translate this example into a somewhat more formal model. By W, we denote the set of states of the world. We always assume that A and W are set up so that the decision maker can choose one and only one action from A and that one and only one state of the world from W can actually happen.

Since the outcomes (the payoffs or profits in the umbrella-sunshade example) depend on both actions and states of the world, we need to consider tuples (a, w) with $a \in A$ and $w \in W$. The set of these tuples is denoted by $A \times W$. Instead of a consequence function f, we then deal with an uncertain-consequence function $g : A \times W \to C$, i.e., a consequence $c \in C$ is determined by both an action $a \in A$ and a state of the world $w \in W$.

2.2.2 Best response

For the analysis of decision situations, it is helpful to ask the following question: Given a specific state of the world, which action is best? We also say: Which action is a best response to a state of the world. In our example, if the weather is bad, the production of umbrellas yields a higher profit than the production of sunshades. This is indicated by $\boxed{\text{R}}$ in the matrix of fig. 4.

		state of the world	
		bad weather	good w.
action	umbrellas	6 $\boxed{\text{R}}$	3
	sunshades	2	8 $\boxed{\text{R}}$

Figure 4: Payoff matrix with best responses

2.2.3 Dominance

Sometimes, one action is better than another one for all states of the world. In the example of fig. 5 the production of umbrellas dominates the production of sunshades because we have

$$\begin{aligned} g\,(\text{umbrella, bad weather}) &> g\,(\text{sunshade, bad weather}) \text{ and} \\ g\,(\text{umbrella, good weather}) &> g\,(\text{sunshade, good weather}) \end{aligned}$$

In terms of best responses, umbrella production dominates sunshade production because we have the $\boxed{\text{R}}$ everywhere in the umbrella row.

		state of the world	
		bad weather	good w.
action	um-brellas	6 $\boxed{\text{R}}$	11 $\boxed{\text{R}}$
	sun-shades	2	8

Figure 5: Payoff matrix with a dominant action

2.2.4 Lotteries and expected value

Dominance makes deciding easy. If there is no dominant action, the decision maker may have information about the probabilities for the states which help him to come to a conclusion. Let us revisit the producer of umbrellas and sunshades whose payoff matrix is given below (fig. 6), but this time we add the belief (probability assessment) that bad weather occurs with probability $\frac{1}{3}$ (and hence good weather with with probability $\frac{2}{3}$).

Then, the action "produce umbrellas" yields the payoff 6 with probability $\frac{1}{3}$ and 3 with probability $\frac{2}{3}$. Thus, the probability distribution on the set of states of the world leads to a probability distribution for payoffs, in this example denoted by

$$L_{\text{umbrella}} = \left[6, 3; \frac{1}{3}, \frac{2}{3}\right]$$

Here, L stands for "lottery". One is often interested in the expected value $E\,(L)$ of a lottery L. In order to calculate the expected value (also called

		state of the world	
		bad w., $\frac{1}{3}$	good w., $\frac{2}{3}$
action	umbrellas	6	3
	sunshades	2	8

Figure 6: Payoff matrix with probabilities

mean), we multiply the probability for each state of the world with the corresponding payoff and sum over all states. In our example, we have

$$E\left(L_{\text{umbrella}}\right) = \frac{1}{3} \cdot 6 + \frac{2}{3} \cdot 3 = 4$$

3 Decision models in the Hitopadeśa

3.1 Investment and duty in short and long lifes

The Hitopadeśa is a fable collection that might have been assembled 800-950 CE. We find the following short remark on investments (in financial and human capital) and fulfillment of religious duties to be considered for people who live short or long lives:

> A wise man should think about knowledge and money as if he were immune to old age and death; but he should perform his duties as if Death had already seized him by the hair.
> (Törzsök (2007, p. 57))

Apparently, this saying deals with two different decision situations. One is concerned with investment in "knowledge and money", the other with performing duties. At first sight, the advice seems to be contradictory. Why work with different assumption when dealing with these different problems?

Let us try decision-theoretic analyses. With respect to the first decision, we propose the actions

$$\begin{aligned} \text{invest} &= \text{save money/increase knowledge,} \\ \text{do not invest} &= \text{spend money/do not labour for education} \end{aligned}$$

and the states of the world

short life,

long life.

The consequences can be seen in the decision matrix of fig. 7. In case of a short life, investments do not pay and the decision maker would rather like to enjoy his money and leisure as long as he lives (see R). In contrast, if the agent lives for a long time, investments pay off (see R). Indeed, the very bad outcome **VB** of poverty (material and spiritual one) occurs if the agent neglects investments and lives for a long time.

		state of the world	
		short life	long life
action	invest	no use for cap./knowl.	long use of cap./knowl. R
	do not invest	enjoym. money/leis. R	material pov./ spiritual pov. **VB**

Figure 7: The investment payoff matrix

The Hitopadeśa's advice of imagining a very long life amounts to the advice of investing. In this manner, the very bad outcome is avoided. Alternatively, one may reconstruct the advice by way of a "better safe than sorry" attitude.

Let us now model the second decision problem with the actions

fulfill dharma now

fulfill dharma later

and the states of the world as above. We then obtain the decision matrix of fig. 8. The highest payoff is g (fulfill dharma later, long life) and described by

- long life
- enjoyment of life in youth
- fulfill dharma later and
- heaven or good karma.

However, postponing the focus on *dharma* is risky. If his life is short, the agent suffers eternal damnation in hell or bad *karma* for his future lifes. In order to prevent this very bad outcome (**VB**), it is best to choose "fulfill dharma now" which is also the best action for "short life". This is the recommendation given in the Hitopadeśa!

		state of the world	
		short life	long life
action	dharma now	good karma R	good karma, little enjoym. in youth
	dharma later	bad karma **VB**	good karma, some enjoym. in youth R

Figure 8: The duty-now-or-duty-later matrix

3.2 Fate and human effort

In the Hitopadeśa's prologue, king Sudarśana muses about the relationship of fate and human effort. Criticizing lazy people who just rely on fate, he makes his point of view clear:

> One should not give up one's efforts, even when acknowledging the role of fate; without effort, one cannot obtain oil from sesame seeds.
>
> And there is another verse on this:
> Fortune gravitates towards eminent men who work hard;
> only cowards say it depends on fate.
> Forget about fate and be a man–use your strength!

> Then, if you don't succeed inspite of your efforts, what is there to blame?
> (Törzsök (2007, p. 69))

Thus, according to king Sudarśana, effort and fate co-determine the outcomes. We offer this interpretation: First, the decision maker may be lazy or busy, i.e., we have

$$A = \{\text{lazy, busy}\}.$$

Second, fate may be favorable or unfavorable:

$$W = \{\text{favorable, unfavorable}\}$$

The outcomes for all pairs from $A \times W$ should obey these rankings:

$$\begin{aligned} g(\text{busy, favorable}) &> g(\text{lazy, favorable}) > g(\text{lazy, unfavorable}), \\ g(\text{busy, favorable}) &> g(\text{busy, unfavorable}) > g(\text{lazy, unfavorable}) \end{aligned}$$

It is not clear how to rank

$$g(\text{lazy, favorable}) \text{ versus } g(\text{busy, unfavorable}).$$

The numerical payoffs as indicated in the matrix of fig. 9 reflect the above preferences. Thus, a payoff of 10 may result from lazyness and luck or else from high effort and ill fate. As king Sudarśana says, the payoff of 10 is no reason for reproach if it accrues to a person who has used his strength.

		state of the world (fate)	
		favorable	unfavorable
action	lazy	10	2
	busy	50	10

Figure 9: Lazy and lucky?

3.3 Action versus state of the world, effort versus fate

The matrix interpretation seems well in line with Sudarśana's thinking. Indeed, he adds to the above verse this line: "Just as a cart cannot move on one wheel, so fate itself cannot be fulfilled without human effort." (Törzsök (2007, p. 71)) To my mind, this is a nearly formal expression of our function g where action (the wheel "human effort") and state of the world (the wheel "fate") co-produce the outcome.

I claim that this way of thinking was very natural to the pre-modern Indian mind. Consider the Mahābhārata (300-500 CE) XIII.6.7:

> Just as seed will be fruitlessly sown without a field,
> so 'divine [power]' will not succeed without human activity.
> (Slaje (1998))

A similar dual approach to outcomes is taken by the Arthaśāstra (100 BCE-100 CE) 6.2.6-10:

> Good and bad policy pertain to the human realm, while good and bad fortune pertain to the divine realm. Divine and human activity, indeed, makes the world run. The divine consists of what is caused by an invisible agent. ... The human consists of what is caused by a visible agent.
> (Olivelle (2013))

4 Lotteries

4.1 The loan lottery

One of the Buddha's birth-stories[1], the birth-story of Brahma, is about a king who does not believe in the afterworld (*paraloka*) and holds other *Cārvāka* views. The former Buddha was a Brahma deity (a god) who tried to benefit the king by converting him to virtuous attitudes and behavior:

[1] The literary Buddhist genres of jātakas (birth-stories) and closely related avadānas are very well described by Straube (2020).

"If convinced that good and bad deeds have
happy and unhappy results in the next life,
one avoids evil and strives for purity.
But non-believers follow their whims.

The king's pernicious false view was an affliction that spelled ruin, bringing calamity on the world. As a result, the Great One, that divine seer, felt compassion for the king. So, one day, while the monarch was staying in a beautiful and secluded grove, caught up in sense pleasures, the Great One descended from the Brahma Realm and blazed in front of him." (Meiland (2009b, p. 271))

The god and the king engage in an argument on whether the "next world" exists. At first, the Buddha is not successful in convincing the king of *paraloka*. The king then comes up with a clever proposal:

> Well. great seer!
> If the next world is not a bogey man for children,
> and if you think I should believe in it,
> then give me five hundred nishkas
> and I'll return you a thousand in another life!
> (Meiland (2009b, p. 279))

		state of the world	
		paraloka exists	paraloka not exist
action	accept to give loan	1000 − 500 [R]	−500
	reject loan	0	0 [R]

Figure 10: Should the Buddha give a loan to the king?

Thus, the king proposes that he obtains a loan. Consider the matrix of fig. 10. The following argument rests on the premise that the Buddha-to-be has notions of both probability and expected value. This is not self-evident. Alternatively, the Buddha may reject the loan because he does not want to carry any risk, and in partical avoid the risk of losing 500 Nishkas. But bear with me and accept that the king and the Buddha might know about these modern-day concepts.

If the former Buddha accepts to give this loan, a lottery results. Let p_{asti} be the probability that the next world exists and $p_{n\bar{a}sti} = 1 - p_{asti}$ the probability for non-existence. Then, the loan lottery is given by

$$\begin{aligned} L_{\text{loan}} &= \left[\underbrace{1000 - 500}_{\substack{\text{loan is repaid} \\ \text{in other world} \\ \text{that exists}}}, \underbrace{-500}_{\substack{\text{loan is not repaid} \\ \text{in other world} \\ \text{as other world does not exist}}}; p_{asti}, \; p_{n\bar{a}sti} \right] \\ &= [500, -500; p_{asti}, 1 - p_{asti}] \end{aligned}$$

If the god does not accept the lottery, he obtains the payoff of 0. Now, so the king seems to argue, the lottery is worth accepting whenever its expected value

$$\begin{aligned} E\left(L_{\text{loan}}\right) &= p_{asti} \cdot 500 + (1 - p_{asti}) \cdot (-500) \\ &= - \underbrace{500}_{\substack{\text{loan given} \\ \text{in both cases}}} + p_{asti} \cdot \underbrace{1000}_{\substack{\text{repayment} \\ \text{if other world exists}}} \end{aligned}$$

is larger than zero, i.e., if

$$p_{asti} > \frac{500}{1000} = \frac{1}{2}$$

This seems a good test of whether the god himself believes in the other world. If he assumes a probability larger than $\frac{1}{2}$, he should accept the lottery.

If the king's lottery is cleverly constructed, the god's answer is surely ingenious. The god has no doubt about the other world, but does not think it realistic that he will get repaid:

> Even in this world, wealth seekers
> do not offer money to the wicked,

nor to the greedy, fools or indolents.
For whatever goes there comes to ruin.

But if they see someone who is modest,
naturally calm and skilled in business,
they will give him a loan, even without witnesses.
For money entrusted to such a man brings reward.

The same procedure for giving a loan
should be used for the next world, king.
But it would be improper to entrust money to you;
for your conduct is corrupted by wicked views.

Who would harrass you for a thousand nishkas
when you lie in hell, senseless, sick with pain,
brought there by your own actions
caused by the evil of your false views?

...

In the next world, where nihilists [*nāstika*, HW] live
a thick darkness and icy wind tortures
people by tearing through their very bones.
What prudent man would go there to get money?

...

When blazing iron nails fasten your body
to the ground red with smokeless flames
and you wail pitifully as your body burns,
who would ask you for your debt then?
(Meiland (2009b, pp. 279-281, 287))

Understandably, the god (if a "wealth seeker" at all) does not find this lottery attractive:

$$\begin{aligned} L_{\text{loan, not paid back}} &= \left[\underbrace{-500}_{\substack{\text{loan is not (!) repaid} \\ \text{in other world} \\ \text{that exists}}}, \underbrace{-500}_{\substack{\text{loan is not repaid} \\ \text{in other world} \\ \text{that does not exist}}} ; p_{asti}, p_{n\bar{a}sti} \right] \\ &= [-500; 1] \end{aligned}$$

4.2 The hell lottery

As seen in the previous subsection, the god predicts the king a dire future. The horrors of hell are not specific to the birth-stories. In the "Life of Buddha by Ashvaghosah", we are treated to similarly uncomfortable descriptions (see Olivelle 2009, pp. 407, 409). Returning to the birth-story of Brahma, let us quote some additional horrors:

> Some are wrapped in blazing iron turbans.
> Others are boiled to a broth in iron pots.
> Others are cut by showers of sharp weapons,
> their skin and flesh ripped by hordes of beasts.
>
> Tired, others enter Váitarani's acrid waters,
> which scroches them on contact like flames.
> Their flesh wastes away but not their lives,
> for they are sustained by their evil deeds.
>
> (Meiland (2009b, p. 285))

Understandably, the king is convinced:

> My mind almost runs wild with fear
> at learning of the punishments in hell.
> It practically burns with blazing thoughts

regarding my plight on meeting that fate.

Shortsightedly I trod the wrong path,
my mind destroyed by evil views.
Be then my path, recourse of the good!
By my resort and refuge, sage!

As you dispelled the darkness of my views
like the rising sun dispels night,
so tell me, seer, the path I should follow
to avoid a bad rebirth after this life.

(Meiland (2009b, p. 285))

The god is prepared to give this advice:

Conquer vice, so difficult to vanquish!
Pass beyond greed, so difficult to overcome!
You will thus reach the gleaming gold-gated city
of the king of heavens, ablaze with fine gems.

May your mind, which once praised evil views,
firmly cherish the creeds valued by good men.
Abandon immoral beliefs proclaimed
by those eager to pleasure fools.

...

With glory as its banner,
pity as its retinue
and tranquility as
its lofty flag, king,
if you travel in this chariot
glittering with wisdom
to benefit others and yourself,
you will certainly not enter hell.

(Meiland (2009b, pp. 291, 295))

An interpretation in terms of lotteries is plausible. After all, the king has shown to understand this concept in the previous subsection. The two lotteries are

- $L_{C\bar{a}rv\bar{a}ka}$, standing for his usual *nāstika* views and conduct, versus
- L_{virtue} standing for his new life of pity, tranquility, and wisdom.

Thus, we have

$$\begin{aligned} L_{C\bar{a}rv\bar{a}ka} &= \left[\underbrace{\text{pleasures in this life, but hell with endless horrors}}_{\text{other world exists}}, \right. \\ &\qquad \left. \underbrace{\text{pleasures in this life}}_{\text{other world does not exist}} ;\ p_{asti},\ p_{n\bar{a}sti} \right] \end{aligned}$$

or in utility numbers

$$L_{C\bar{a}rv\bar{a}ka} = [-100\ 000,\ 10; p_{asti}, p_{n\bar{a}sti}]$$

with expected payoff

$$\begin{aligned} E\left(L_{C\bar{a}rv\bar{a}ka}\right) &= p_{asti} \cdot (-100\ 000) + (1 - p_{asti}) \cdot 10 \\ &= 10 - 100\,010 p_{asti} \end{aligned}$$

and

$$L_{\text{virtue}} = [\text{life of pity, tranquility, and wisdom, no hell};\ 1]$$

with (expected) payoff

$$E\left(L_{\text{virtue}}\right) = 2.$$

Which choice maximizes expected payoff? The virtuous life is the better choice if

$$E\left(L_{\text{virtue}}\right) > E\left(L_{C\bar{a}rv\bar{a}ka}\right),$$

i.e., if the probability of the existence of the other world is sufficiently high. For our numbers, the above inequality is equivalent to

$$p_{asti} > \frac{8}{100010} \approx \frac{8}{10000} = 0.0008.$$

The king may not (really) believe in the other world, but prefers to play it safe.

Thus, Blaise Pascal was not the first to present an argument for believing in God that is based on very good or very bad outcomes.

5 Probabilies?

5.1 Using decisions to extract subjective probabilities

The king's lottery can be generalized. Let F stand for the non-existence of the other world and let $\alpha = \beta = 500$ (fig. 11). As we have seen, the Buddha rejects the loan. Leaving the Buddha who is not prepared to do any business with a bad person, accept the numbers in the general case at face value. For any event F, numbers $\alpha \geq 0$, $\beta \geq 0$ with $\alpha + \beta > 0$ can be found such that the agent is indifferent between rejecting and accepting. If the agent rejects for some specific numbers, one just needs to make accepting more attractive by increasing β and/or by decreasing α.

		state of the world	
		event F	event $\neg F$
action	reject	0 R	0
	accept	$-\alpha$	β R

Figure 11: For which numbers is the agent indifferent?

Indifference between the two actions leads to the equality $0 = p(F) \cdot (-\alpha) + [1 - p(F)](\beta)$ which then implies

$$\begin{aligned} p(F) &= \frac{\beta}{\alpha+\beta} \text{ and} \\ p(\neg F) &= \frac{\alpha}{\alpha+\beta} \text{ with} \end{aligned}$$

5.2 Normalization

I claim that $[-\alpha, \beta]$ (i.e., the above matrix with these specific payoffs) amounts to the probability $p(F)$. In order to check whether this is indeed the case, we need to confirm normalization and additivity (see subsection 2.1). By $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta > 0$, we find

$$0 \leq p(F) = \frac{\beta}{\alpha+\beta} \leq 1$$

Furthermore, note that $p(F) = 0$ is compatible with indifference only for $\beta = 0$ (check the matrix and $p(F)$). $p(F) = 1$ is compatible with indifference only for $\alpha = 0$ (and $\beta > 0$).

Since $[-\alpha, \beta]$ defines a lottery and hence the probability $\frac{\beta}{\alpha+\beta}$, the same holds for $[-5\alpha, 5\beta]$, or any multiplication with a non-zero constant, in particular $\frac{1}{\alpha+\beta}$. Therefore, for any event F, $0 \leq p(F) \leq 1$ can be found such that the agent is indifferent between rejecting and accepting (fig. 12). Thus, one can immediately obtain the probability for an event F by requiring $\alpha + \beta = 1$.

		state of the world	
		event F	event $\neg F$
action	reject	0 R	0
	accept	$-p(\neg F)$	$p(F)$ R

Figure 12: Indifference and probabilities

Unsurprisingly, one finds $p(F) + p(\neg F) = 1$.

5.3 Additivity

Turning to additivity, consider three events F, G, and H. The probabilities for each of the three events can be determined as above. Assume that the three events are mutually exclusive and obey $F \cup G \cup H = E$ and hence $H = \neg(F \cup G)$. Then, as in $p(F) + p(\neg F) = 1$, we obtain

$$p(F \cup G) + p(H) = 1$$

We now employ a Ramsey-type argument (see, for example, Jeffrey (1983, pp. 60-61)). Assume that the agent is offered the three lotteries $[-p(\neg F), p(F)]$, $[-p(\neg G), p(G)]$, and $[-p(\neg H), p(H)]$. Since he is indifferent between rejecting and accepting each single one, he should also be indifferent between rejecting and accepting all three of them. Assume that the agent accepts all three lotteries. If event F materializes, neither G nor H come to pass, and the agent obtains the payoff $-p(\neg F) + p(G) + p(H)$. If, however, H happens, the payoff is

$$-p(\neg H) + p(F) + p(G).$$

It is not difficult to see that these two payoffs are the same. Indeed, the agent's payoff does not depend on which of the three events happen.

Now, by indifference between rejecting all three lotteries and accepting all three of them, we then find $0 = -p(\neg H) + p(F) + p(G)$ and

$$p(F \cup G) \underset{p(F \cup G)+p(H)=1}{=} 1 - p(H) \underset{p(H)+p(\neg H)=1}{=} p(\neg H) \underset{0=-p(\neg H)+p(F)+p(G)}{=} p(F) + p(G)$$

This confirms additivity.

6 Conclusion

Let me summarize the findings with respect to decision theory and probability theory. Pascal may have been the inventor of decision theory for Europeans. Indian sources can claim priority by more than 1000 years.[2]

The lottery $[500, -500]$ suggested by the king is meant to trick the Buddha. If the believes in the existence of *paraloka* with a degree of belief of at least $1/2$, he should indeed offer the loan. We have seen how the Buddha evades this problem. Our main interest lies elsewhere: what does this birth-story tell about the author's knowledge of probability (and, again, decision theory)? To my mind, the loan offered by the *Cārvāka* king may well be the world-first invention of using a lottery in order to find out about the strength of a decision maker's belief. More boldly, and more risky, one may try to defend the birth-story of Brahma as exhibiting the world-first quantitatively defined probability. But, of course, $[-\alpha, \beta]$ is a somewhat clumsy expression in comparison to just a number between 0 and 1. This is not a serious objection. As we have seen above, any $[-\alpha, \beta]$ can be replaced by $\left[-\frac{\alpha}{\alpha+\beta}, \frac{\beta}{\alpha+\beta}\right] = [-p(\neg F), p(F)]$).

[2] Above, I have cited from the "Garland of the Buddha's Past lives" that is due to Aryashura. According to Meiland (2009a, pp. xviii-xix) and Steiner (2019), Aryashura seems to have lived in the fourth century C.E. His Sanskrit collection of birthstories has older Pāli parallels that have been edited and assembled in the collection entitled Jātakaṭṭhavaṇṇanā, translated by Rouse (1901). Here, within Jātaka No. 544 (entitled Mahānāradakassapa), the story about the king who denies any afterworld is found on pp. 121-126. According to von Hinüber (1998, p. 1), this Pāli collection dates from about 500 C.E. Von Hinüber has tried to trace the predecessors of these Pāli jātakas, partly reaching to pre-Buddhist times. However, the author has not been able to trace any relevant predecessors of Jātaka No. 544.

References

Garber, D. & Zabell, S. (1979). On the emergence of probability, *Archive for History of Exact Sciences* **21**: 33–53.

Hacking, I. (2006). *The Emergence of Probability*, 2 edn, Cambridge University Press.

Hald, A. (1990). *A History of Probability and Statistics and their Applications before 1750*, Wiley.

Jeffrey, R. C. (1983). *The Logic of Decision*, 2 edn, University of Chicago Press.

Meiland, J. (2009a). *Garland of the Buddha's past lives, volume one (by Aryashura)*, New York University Press and JJC Foundation.

Meiland, J. (2009b). *Garland of the Buddha's past lives, volume two (by Aryashura)*, New York University Press and JJC Foundation.

Olivelle, P. (2009). *The Law Code of Vishnu*, Harvard University Press.

Olivelle, P. (2013). *King, Governance, and Law in Ancient India: Kautilya's Arthasastra*, Oxford University Press.

Plofker, K. (2009). *Mathematics in India*, Princeton University Press.

Raju, C. K. (2011). Probability in ancient india, *in* D. M. Gabbay, P. Thagard & J. Woods (eds), *Philosophy of Statistics*, Vol. 7 of *Handbook of the Philosophy of Science*, Elsevier, pp. 1175–1195.

Rouse, W. H. D. (1901). Mahānāradakassapa (Jātaka 544), *in* E. Cowell (ed.), *The Jātaka or Stories of the Buddha's Former Births. Volume VI*, Cambridge University Press, pp. 114–126.

Sellier, P. (1991). *Pensées de Pascal*, Classiques Garnier.

Slaje, W. (1998). Nāsti daive prabhutvam. Traces of demythologisation in indian epic thought, *Journal of Indian Philosophy* **26**: 27–50.

Steiner, R. (2019). Āryaśūra, *Brill's Encyclopedia of Buddhism. Volume II: Lives*, Brll, Leiden, pp. 70–72.

Straube, M. (2020). Narratives: South Asia, *in* J. A. Silk, O. von Hinüber & V. Eltschinger (eds), *Encyclopedia of Buddhism Online*, Brill Academic Publisher. Consulted online on 30 October 2020.

Törzsök, J. (2007). *"Friendly Advice" by Narayana and "King Vikrama's Adventures"*, Vol. 32, NYU Press.

von Hinüber, O. (1998). *Entstehung und Aufbau der Jātaka-Sammlung*, Akademie der Wissenschaften und der Literatur, Mainz.