Backward induction in Indian animal tales

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Abstract

With regard to Indian animal tales, the indologist Heinrich Zimmer finds a "cool exactness in a sort of political algebra". We translate "political algebra" by noncooperative game theory and analyze three well-known fables in game-theory terms. We show that backward induction has already been applied in Indian animal tales.

The tragedy that follows a wrong plan,
The Triumph that results from the right plan,
To the rules of Polity both are linked;
so the wise can point them out,
as if displayed in advance.
(from the Panchatantra, translated by Olivelle 2006, p. 77, verse 1.60)

1. Introduction

The famous indologist Heinrich Zimmer (1969, p. 89) observes that Indian political thought was characterized by "cold-blooded cynical realism and sophistication". He also finds that "ancient Hindu political wisdom" brings about "the cold precision of a kind of political algebra, certain fundamental natural laws that govern political life, no matter where" (p. 90).

Meanwhile, Zimmer's political algebra has been developed by economists and mathematicians under the heading of "game theory". The interested reader can consult one of the many textbooks on game theory, for example parts 1 and 2 in Gibbons (1992) or chapter 3 in Dixit & Skeath (1999). The aim of this paper is to show that the reasoning employed by human and animal actors in some Indian fables can be analyzed by a powerful method developed by game theorists, backward induction. Sometimes these actors employ backward induction and sometimes, very much to their detriment, they fail to do so. In the stories presented in this paper, the didactic purpose of teaching forward-looking behavior seems very obvious. Thus, we may credit Indian political thought with the early invention and application of backward induction.

It is not an easy question whether or not the Indians share this achievement (independent invention and application of backward induction) with other ancient cultures. For example, when Brams (2003) analyzes stories from the Hebrew bible, he also uses backward induction. In our mind, this does not necessarily mean that the bible authors also apply backward induction. In contrast to the Indian fable tellers, their focus is not on strategic thinking, but rather on telling the history of the Israelis and on the relationship between God and His people. (Of course, the fact that Brams (2003) and, in more detail, Brams (2011), apply the Theory of Moves developed by that author to biblical stories, does not imply that biblical story tellers had any idea about this recent branch of game theory.)

Apart from biblical stories, Brams (2011) shows how non-cooperative game theory can be used to analyze, inter alia, jury selection, Aritophanes's play Lysistrata, Shakespeare's Macbeth, or the Cuban Missile Crisis. Similarly, in an as yet unpublished manuscript, Chwe (2010) argues that "folk game theory" can "take the perspective of outsiders", such as slaves or Jews. To the best of our knowledge, this essay is the first to provide a game-theoretic analysis of some Indian fables. However, this paper is not the first one to point out early advances of Indian scholarship in the fields of economics and game theory. In particular, Sihag (2007) claims that Kautilya's arthashāstra already "knew" about game-theoretic niceties such as time inconsistency and asymmetric information.

We will explain the political algebra of game theory by way of three animal tales, (i), the tiger and the traveller, (ii), the lion, the mouse, and the cat, and (iii), the cat and the mouse. Zimmer himself cites the second and third fable. While the Indian fable tellers did not have the formal instrument of backward induction at their disposal, the stories and morals of the stories clearly show that they understood backward induction very well. This is obvious from all three stories

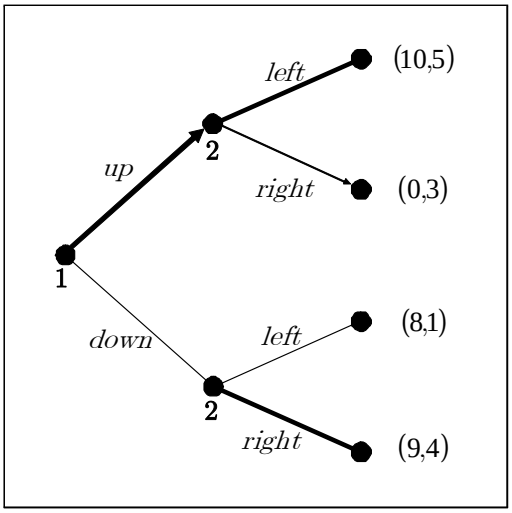

Figure 2.1: A game tree

although only the last one has the players act according to backward induction. In the first two examples, backward induction is violated and it is this very violation that the fable tellers want to point out to their readers.

2. Noncooperative game theory and backward induction

In this section, we present as much game theory as needed for the purposes of this paper. Instead of formal definitions, we try to bring across the basic reasoning by way of a simple example. Consider the game between the players 1 and 2 depicted in figure 2.1. You see that some nodes are indexed by the player names (1 or 2). At these nodes players 1 and 2 have to make a choice. Player 1 moves first, at the initial node (the leftmost node). He chooses up or down. Next, it is player 2's turn who chooses between left and right. When both players have chosen their actions, they obtain the appropriate payoffs or "utilities". The payoff information is noted near the terminal nodes (the rightmost nodes). The first number indicates the payoff for player 1 and the second number is the payoff for player 2. For example, if player 1 chooses up and player 2 chooses right, player 1 obtains the payoff of 0 and player 2 the payoff of 3.

Backward induction means "looking ahead" by "proceeding backwards". Before player 1 can decide on his move, he needs to know how player 2 will react to up, or down, chosen by player 1. Thus, backward induction starts with the

players that move last. Consider the node where player 2 has to make a decision after player 1 chose up. Comparing the payoffs 5 and 3, player 2 chooses left. The corresponding edge has been reinforced. In contrast, player 2 will choose right if he learns that player 1 has chosen down (this follows from $4 > 1$).

Now, after knowing the choices of player 2, we can look at player 1's decision. If he chooses up, player 2 will choose left so that player 1 obtains a payoff of 10. If, however, player 1 chooses down, player 2 will choose right so that player 1 obtains 9. Comparing 10 and 9, it is obvious that player 1 should choose up.

Thus, player 1 choosing up and player 2 choosing left is the predicted outcome. However, this may not be the observed outcome. For example, player 1 choosing up and player 2 choosing right is indicated by the arrows. In that sequence of events, player 2 would have made a mistake. By $5 > 3$ he could have done better.

3. The tiger and the traveller

The first example is the tale of the tiger and the traveller known from the Hitopadesha collection of fable-based advice (see, for example, Kale & Kale (1967, pp. 7-9) or the comic book by Chandakant & Fowler (1975, pp. 14-18)).

This is the story: A tiger that finds himself on one side of a lake sees a traveller passing by on the opposite side. The tiger attempts to catch and eat the traveller by offering a golden bracelet to him. Since the traveller is suspicious of the tiger's intentions, the tiger argues that he would not (he claims to have profoundly changed his former evil behavior) and could not (he claims to be old and weak) do any harm to the traveller. Finally, the traveller is convinced, gets into the murky waters where he gets stuck. Immediately, the tiger takes advantage of the traveller's misfortune and kills him as planned.

Consider the payoffs in figure 3.1. The first number at the final nodes refers to the tiger, the second one to the traveller. The tiger's payoffs are -2 for giving away the bracelet and not eating the traveller, 10 for keeping the bracelet and enjoying a good meal, and 0 for the status quo of keeping the bracelet but staying hungry. The corresponding traveller's payoffs are 5, -100, and 0.

The tragic sequence of events sketched above is indicated by the arrows. The tiger (ti) moves first by promising the bracelet (upper branch). The traveller (tr) enters the lake (upper branch) and then the tiger kills the traveller (lower branch).

The game tree of this story has three stages. First, the tiger offers the bracelet and talks about his guru who has convinced him to lead a more virtuous life or the tiger refrains from offering the bracelet and/or from talking convincingly. Then,

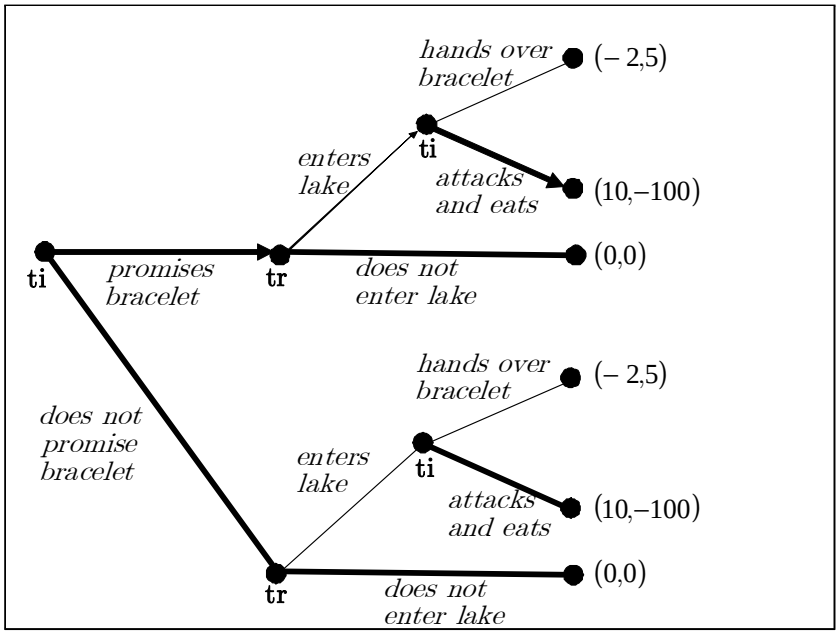

Figure 3.1: The tiger and the traveller

the traveller needs to decide on whether or not to accept the tiger's invitation to join him by crossing the lake. Finally, the tiger fulfills his promise or reneges on it.

One may of course speculate why the traveller is so "stupid". Did "greed cloud the mind" or did he act on some probability assessment about the lion telling the truth? Indeed, the tiger claims to have studied the Vedas to lend credibility to his peaceful intentions. However, it seems obvious that the fable writer does not think of this example under the heading of "better safe than sorry". Instead he argues that the tiger's preferences being as they are the traveller should have known his fate in advance. Before being killed, the traveller has time for some wise insights to share with the readers (see Kale & Kale 1967, p. 8):

> That he reads the texts of religious law and studies the Vedas, is no reason why confidence should be reposed in a villain: it is the nature that predominate [sic] in such a case: just as by nature the milk of cows is sweet.

Knowledge of backward induction would also have led the traveller to avoid

the lake. By $10 > -2$, he should have foreseen his being eaten after entering the lake so that keeping clear of the lake is best by $0 > -100$.

Interestingly, the traveller should refrain from entering the lake independent of whether or not the tiger talks about his guru who advised the tiger to pursue a more virtuous life. In game-theory parlance, the tiger's arguments, the first step in our game tree, are just "cheap talk". Both a mischievous and a benevolent tiger could claim their benevolence without any cost. Therefore, this claim is not credible.

Pious appearances are also used by the cat in an animal tale from the Panchatantra (see, for example Olivelle 2006, pp. 393-399). The cat is chosen as a judge in a dispute between a partridge and a hare. Although wary of the danger, the two contestants finally approach the cat who kills them without much ado.

4. The lion, the mouse, and the cat

The second animal tale is also taken from the Hitopadesha (see Kale & Kale 1967, p. 51). A lion that lives in a cave is infuriated by a mouse that also lives in his cave. The mouse regularly gnaws at the sleeping lion's mane. Since the lion does not succeed in catching the mouse, he invites a hungry and desperate cat to live and eat in his cave.

The arrangement between the lion and the cat works out well. The mouse does not dare to show up while the cat is present. Therefore, the lion is happy to share his food with the cat as promised although he does not particularly like the cat's company by itself. One day, the mouse is detected by the cat who catches and kills it. The lion does not see any reason to extend his hospitality and makes the cat leave his cave. Soon, the cat returns to its former miserable state.

The moral to be drawn from this fable is obvious: Do your work but see to it that you are also needed in the future.

The reader is invited to have a look at figure 4.1. The first number at the final nodes refers to the lion, the second to the cat. Both players obtain a payoff of 0 if the lion does not invite the cat to stay so that the lion's mouse problem is not solved and the cat cannot eat the food provided by the lion. The lion's payoff is 5 if the mouse does not annoy him and increases up to 7 if, on top, the cat does not stay in the cave. The cat in the cave has a payoff of 3 if it can stay in the cave and an increased payoff of 4 for eating the mouse and staying in the cave.

The arrows indicate the story as told in the Hitopadesha. This is not the backward-induction result which, again, is indicated by the thickened lines. The

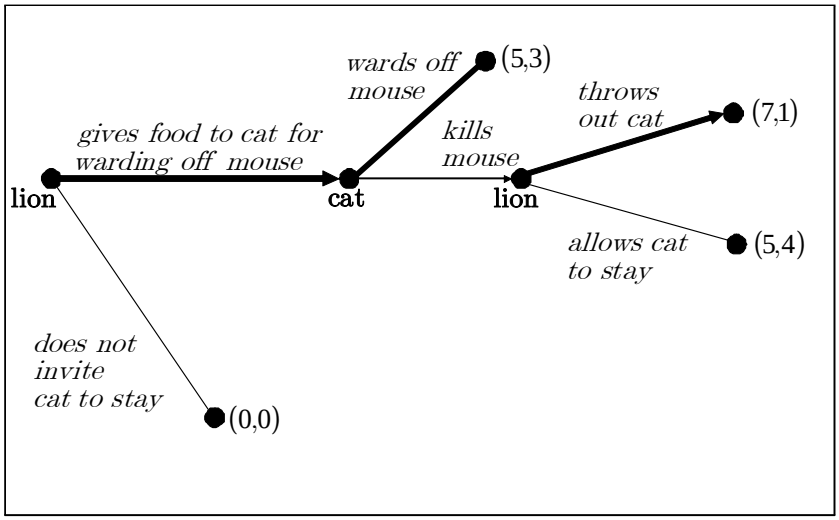

Figure 4.1: The lion, the mouse, and the cat

wise cat would foresee that it is in the best interest of the lion to get rid of it after the mouse is killed ($7 > 5$). Therefore, the cat should have kept on warding off the mouse (payoff 3) rather than killing the mouse and be thrown out of the convenient cave (payoff 1). Working backwards one final step, we see that the lion was right to invite the cat into his cave ($5 > 0$). Indeed, because of the cat's mistake, the lion is even better off obtaining 7 rather than 5.

Again, one may ask the question whether there are defensible reasons for the violation of backward induction. Did the cat think that another mouse would show up promptly so that the lion would need the cat's services again? It seems that the fable's author did not think along these lines, but had the more straightforward didactic aim of teaching the forward-looking behavior the cat did not master.

A second possibility comes to mind: The cat may have entertained the hope that the lion would show thankfulness to the cat for freeing the lion of the mouse for good. However, in line with the cynical realism observed by Zimmer, we would rather not follow this line of thought, but insist on the lesson that friendship has no worth and that the behaviors of humans or animals are dictated by future gains and losses, rather than by friendly acts in the past.

5. The cat and the mouse

In the previous animal tale, the lion profited from the opponent's mistake. Sometimes, however, players hope that opponents react rationally. To show this, we finally present a fable from book 12 of the grand epic Mahabharata (see Fitzgerald 2004, pp. 513-518). A he-cat is caught in a net laid out by a trapper. The mouse is happy to see her enemy in this difficult situation when she realizes that an owl is about to attack from above and a mongoose is sneaking up on her. She offers the cat to destroy the net if the cat gives her shelter. The mouse realizes that her plan needs a good deal of rationality and foresight on the cat's part (p. 514):

> So I will make use of my enemy the cat. I shall contribute to his welfare ... And now may this enemy of mine happen to be smart.

Fortunately, the cat agrees to the bargain. When seeing the mouse under the cat's protection, owl and mongoose look out for other prey. The cat is dismayed to find that the mouse is in no hurry to fulfill her promise. Indeed, the mouse realizes that freeing the cat immediately makes her an easy victim of the cat. In a long dialogue, the logic of the situation is explicitly spelled out. As the mouse remarks (p. 517):

> No one is really an ally to anyone, no one is really a friend to anyone ... When a job has been finished, no one pays any attention to the one who did it; so one should make sure to leave all his tasks with something still to be done. At just the right time, sir, you will be filled with fear of the [trapper] and intent on getting away, and you won't be able to capture me.

Thus, the mouse waits until the trapper approaches. At the very last moment, the mouse liberates the cat that now has better things to do than mouse hunting. Both manage to find a safe place to hide, but certainly not the same.

Figure 5.1 shows the game tree of this animal tale. The first payoff accrues to the mouse (m), the second one to the cat. The mouse obtains 0 for escaping unharmed and suffers the payoff of -100 for being killed by owl, mongoose, or cat. The cat's payoff is zero for escaping unharmed, 2 for escaping and eating the mouse, -50 for being killed by the trapper and -48 for being killed by the trapper after eating the mouse.

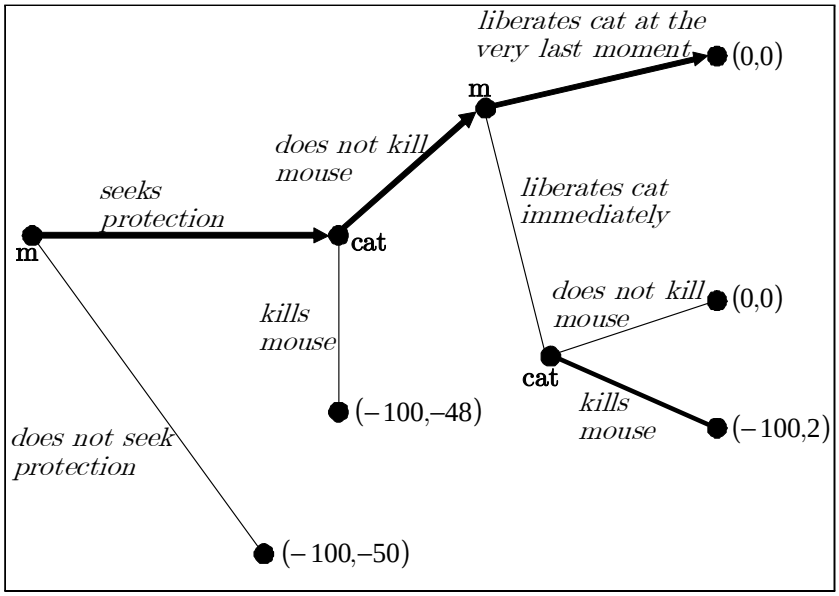

Figure 5.1: The cat and the mouse

Foreseeing that the cat will kill the mouse if liberated well before the trapper arrives ($2 > 0$), the mouse prefers to wait until the trapper approaches ($0 > -100$). The cat is clever enough not to kill the mouse before he is liberated ($0 > -48$). Thus, indeed, the mouse made a clever move to seek the cat's protection ($0 > -100$).

Unlike the first two stories, in this story, the sequence of events is the one predicted by backward induction. Neither the mouse nor the cat makes a mistake.

6. Conclusions

As noted in the introduction, Indian political thought was cold-blooded and cynical. From the point of view of virtue ethics (see, for example, McCloskey 2006, pp. 63), one may note that Indian fables and also a good deal of economics stress the virtue of prudence at the expense of other virtues, such as justice, hope, love, faith, etc. Indeed, Indian animal tales often have a clear didactic purpose – to teach future kings how to exercise prudence by paying heed to basic tricks in strategic thinking.

Schwalbe & Walker (2001) trace the "early (sic) history of game theory" and note (on p. 126) that the first time "a proof by backward induction is used seems to be in von Neumann and Morgenstern (1953)". We do not mean to contradict these authors when we say that the application (rather than the use for a proof) is definitely much older, at least going back to some hundred years BCE, in India and may-be also in other ancient cultures.

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