Microeconomics

Oligopoly

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Structure

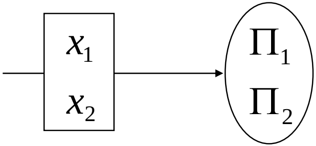
Introduction

- Household theory
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
    - Monopoly and monopsony
    - Game theory
    - **Oligopoly**
- External effects and public goods

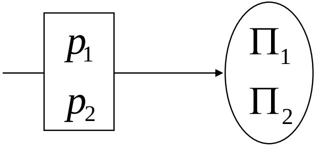
Pareto-optimal review

Price versus quantity competition

Cournot 1838 :



Bertrand 1883 :



Bertrand criticizes Cournot, but Kreps/Scheinkman 1983:

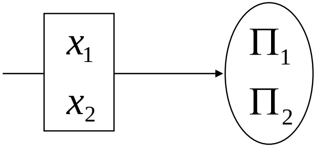
simultaneous capacity construction

$+$simultaneous price competition (Bertrand competition)

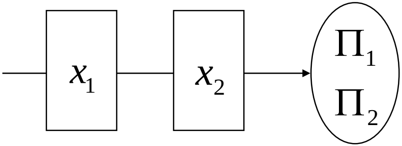
$=$Cournot results

Simultaneous versus sequential competition

Cournot 1838 :



Stackelberg 1934:



Profit function of firm 1 (analogously for firm 2):

$$\begin{aligned}\Pi_1(x_1, x_2) &= p\left(x_1 + x_2\right) x_1 - C_1\left(x_1\right) \\ &= \left(a - b\left(x_1 + x_2\right)\right) x_1 - c_1 x_1\end{aligned}$$

Overview

- Model overview
- The Bertrand model (simultaneous price competition)
- The Cournot model (simultaneous quantity competition)
  - The Cournot model with two firms
  - Best-response functions
  - Cournot-Nash equilibrium
  - The Cournot model with n firms
  - Comparative statics
    - Marketing activities
    - Quantity tax
    - Cost competition
- The Stackelberg model (sequential quantity competition)
- cartel

Homogeneous products and undercutting

Definition

Products are called homogeneous if consumers have no preferences with respect to

- features,
- time, or
- location.

Consequences:

- only prices are relevant for demand
- only one price, the lowest price, is relevant
- slight undercutting is highly profitable
- impending price competition

Homogeneous products are substitutes

Homogeneous products are extreme cases of substitutes

Definition

Goods 1 and 2 are substitutes if the following is true:
If the price of good 1 increases, then the demand for good 2 increases.

Examples: butter and margarine, cars and motorbikes

Definition

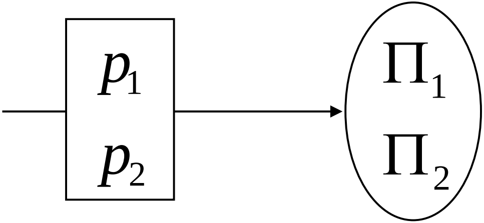
Goods 1 and 2 are complements if the following is true:
If the price of good 1 increases, then the demand for good 2 decreases.

Examples: cinema and popcorn, left and right shoe, hardware and software ...

Clever man: Joseph Bertrand

- Joseph Louis François Bertrand (1822 – 1900) was a French mathematician and pedagogue.
- In 1883 he developed price competition when dealing with the Cournot model.

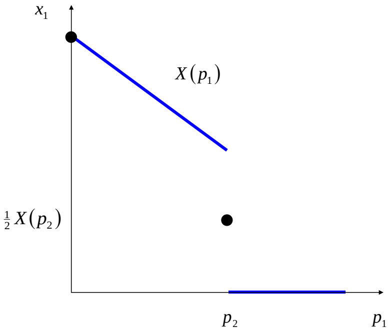
Simultaneous price competition = Bertrand model



Demand functions

- Assumptions:
  - homogeneous product
  - consumers buy best
  - linear demand
- Demand for firm 1:

$$x_1(p_1,p_2) = \begin{cases} d - ep_1, & p_1 < p_2 \\ \frac{d-ep_1}{2}, & p_1 = p_2 \\ 0, & p_1 > p_2 \end{cases}$$



- Unit cost c_1:

$$\Pi_1(p_1,p_2) = (p_1 - c_1)x_1(p_1,p_2)$$

Bertrand paradox

Lemma

For $c := c_1 = c_2 < \frac{d}{e}$ there is only one equilibrium:
$\left(p_1^B, p_2^B\right) = (c, c)$.

$$\begin{aligned} x_1^B &= x_2^B = \frac{1}{2}X(c) = \frac{d - ec}{2} \\ \Pi_1^B &= \Pi_2^B = 0 \end{aligned}$$

1. $\left(p_1^B, p_2^B\right) = (c, c)$ is an equilibrium
   - higher price $\Rightarrow$?
   - lower price $\Rightarrow$?
2. $\left(p_1^B, p_2^B\right) = (c, c)$ is the only equilibrium.
   - $\left(p_1^B + \Delta p_1, p_2^B\right)$?
   - $\left(p_1^B + \Delta p, p_2^B + \Delta p\right)$?
   - $\left(p_1^B - \Delta p_1, p_2^B\right)$?

Betrand paradox

Why paradox: for the Cournot model the result is different

Problem

Assume two firms with identical unit costs of 10. The strategy sets are $S_1 = S_2 = \{1, 2, ..., \}$. Determine both Bertrand equilibria.

Bertrand paradox

- Theory of repeated games
- Different average costs $\longrightarrow$ what happens?
- Price cartel $\longrightarrow$ agreement to charge monopoly price
- Products not homogeneous, but differentiated

The Cournot model with two firms

- Firms simultaneously choose their outputs
- Outputs: $x_1 \geq 0$ and $x_2 \geq 0$
- Solution procedure:
  - Determine best-response functions
  - Intersection of best-response functions
  - Cournot-Nash equilibrium $\left(x_1^C, x_2^C\right)$:

$$x_1^C \overset{!}{=} x_1^R\left(x_2^C\right) \text{ and } x_2^C \overset{!}{=} x_2^R\left(x_1^C\right)$$

Reminder: best response and Nash equilibrium

		hunter 2	
		stag	hare
hunter 1	stag	5, 5	0, 4
	hare	4, 0	4, 4

	stag	hare
stag	5, 5 $\boxed{1}$	0, 4
hare	4, 0	4, 4 $\boxed{1}$

	stag	hare
stag	5, 5 $\boxed{1}\boxed{2}$	0, 4
hare	4, 0	4, 4 $\boxed{1}\boxed{2}$

Best-response functions for quantity competition I

$$\frac{\partial \Pi_2(x_1, x_2)}{\partial x_2} = MR_2(x_2) - MC_2(x_2) = a - 2bx_2 - bx_1 - c_2 \overset{!}{=} 0$$

or equivalently

$$MR_2(x_2) = a - 2bx_2 - bx_1 \overset{!}{=} c_2 = MC_2(x_2)$$

Solving for x_2 yields the best-response function for firm 2:

$$x_2^R(x_1) = \frac{a - c_2}{2b} - \frac{1}{2}x_1$$

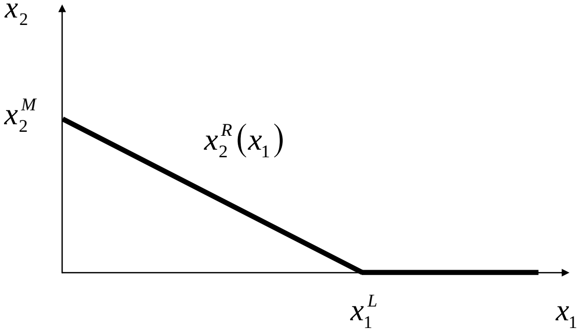
However, for $x_1 > \frac{a-c_2}{b} = x_1^L$ firm 2 wants to supply the quantity zero because

$$p\left(x_1^L\right) = a - bx_1^L = c_2$$

Best-response functions for quantity competition II

Hence,

$$x_2^R(x_1) = \begin{cases} \frac{a-c_2}{2b} - \frac{x_1}{2}, & x_1 < \frac{a-c_2}{b} = x_1^L \\ 0, & \text{otherwise} \end{cases}$$



Assumption: Unit costs c_1 and c_2 are sufficiently close such that in equilibrium both firms will supply a positive quantity.

Cournot-Nash equilibrium

- Best-response function of firm 1

$$x_1 = x_1^R(x_2) \stackrel{!}{=} \frac{a - c_1}{2b} - \frac{x_2}{2}$$

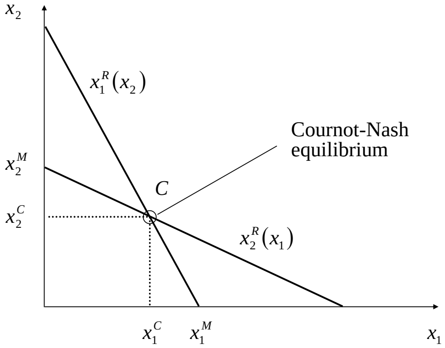
- Best-response function of firm 2

$$x_2 = x_2^R(x_1) \stackrel{!}{=} \frac{a - c_2}{2b} - \frac{x_1}{2}$$

- Solve two equations with two unknowns:
  Cournot-Nash equilibrium

$$\left(x_1^C, x_2^C\right) = \left(\frac{1}{3}\frac{a - 2c_1 + c_2}{b}, \frac{1}{3}\frac{a - 2c_2 + c_1}{b}\right)$$

Cournot-Nash equilibrium



Problem

$p(X) = 20 - X$. $c = 0$

Cournot-Nash equilibrium

Monopoly quantity?

Rates of concentration

- Output of firm i: x_i
- Total output of all firms at the market: X
- Market share of firm i :

$$s_i := \frac{x_i}{X},$$

The k-th rate of concentration C_k adds market shares of the k largest firms:
$s_1 \geq s_2 \geq ...$ and

$$C_k = \sum_{i=1}^{k} s_i$$

Rates of concentration

Problem

Determine the C_2 rate of concentration for the following examples:

1. Two firms with equal market shares.
2. Three firms with market shares of $s_1 = 0.8, s_2 = 0.1$, and $s_3 = 0.1$.
3. Three firms with market shares of $s_1 = 0.6, s_2 = 0.2$, and $s_3 = 0.2$.

Rates of concentration and market dominance

§ 19 (3) GWB (Gesetz gegen Wettbewerbsbeschränkungen) assumes, „dass ein Unternehmen marktbeherrschend ist, wenn es einen Marktanteil von mindestens einem Drittel hat. Eine Gesamtheit von Unternehmen gilt als marktbeherrschend, wenn sie

1. aus drei oder weniger Unternehmen besteht, die zusammen einen Marktanteil von 50 vom Hundert erreichen oder
2. aus fünf oder weniger Unternehmen besteht, die zusammen einen Marktanteil von zwei Dritteln erreichen [...]"

We can express these conditions by the rates of concentration C_1 or C_k. For example, a market share of at least one third is equivalent to $C_1 \geq 33.33\%$.

Problem

Use a rate of concentration to express the condition that four firms are market dominating!

Monopoly commission

The monopoly commission has the official mandate to regularly report the status of development of concentration in the German economy. This report (Anlageband zum Hauptgutachten) always concerns

- Total revenue in the industry (pX),
- Number of firms in the same area of business (n),
- C_3, C_6, C_{10}, C_{25}, C_{50}, and C_{100} rates of concentration,
- Herfindahl index (also called absolute Herfindahl-Hirschman index HHI), and
- Variation coefficient (also called relative Herfindahl index).

Herfindahl index I

Definition (Herfindahl index)

$$H = \sum_{i=1}^{n} \left(\frac{x_i}{X}\right)^2 = \sum_{i=1}^{n} s_i^2$$

Examples:

- Monopoly
  $\Rightarrow$ Herfindahl index 1
- Two firms with market shares 90% and 10%
  $\Rightarrow$ Herfindahl index $0.90^2 + 0.10^2 = 0.82$.

Herfindahl index II

Problem

Determine the Herfindahl index for n equally large firms!

Problem

Which market is more concentrated,

- a market with two equally large firms,
- a market with three firms and market shares 0.8, 0.1, and 0.1, or
- a market with three firms and market shares 0.6, 0.2, and 0.2?

Herfindahl index III

V = variance = normalized deviation from the mean value:

$$V = \frac{\text{standard deviation}}{\text{mean}} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \frac{X}{n}\right)^2}}{\frac{X}{n}}$$

The Herfindahl index is large if the variance is large and if the number of firms is small:

$$H = \frac{1 + V^2}{n}.$$

The Lerner index in oligopoly

- For n firms total output X is the sum

$$X = x_1 + x_2 + ... + x_n.$$

- Condition for profit maximization of firm i:

$$MR\left(x_i\right) = p\left(X\right) + x_i \frac{dp}{dX}\frac{\partial X}{\partial x_i} \overset{!}{=} MC\left(x_i\right)$$

- Since $\frac{\partial X}{\partial x_1} = 1$ holds, we can derive the Amoroso-Robinson relation:

$$MR\left(x_i\right) = p + x_i \frac{dp}{dX} = p\left(1 + \frac{x_i}{X}\frac{X}{p}\frac{dp}{dX}\right) = p\left(1 + \frac{s_i}{\varepsilon_{X,p}}\right).$$

- $\frac{\varepsilon_{X,p}}{s_i}$: firm-specific elasticity of demand

The Lerner index in oligopoly

Using the Amoroso-Robinson relation, the Lerner index for a single firm i in an oligopoly is given by

$$\frac{p - MC_i}{p} \stackrel{!}{=} \frac{p - MR\left(x_i\right)}{p} = \frac{p - p\left(1 - \frac{s_i}{\left|\varepsilon_{X,p}\right|}\right)}{p} = \frac{s_i}{\left|\varepsilon_{X,p}\right|}.$$

Hence, $p \stackrel{!}{=} MC$ holds if

- $s_i = 0$ (very small firm)
- $\left|\varepsilon_{X,p}\right| = \infty$ (horizontal demand)

- The Lerner index for the whole industry is given by

$$\sum_{i=1}^{n} s_i \frac{p - MC_i}{p} = \sum_{i=1}^{n} s_i \frac{s_i}{|\varepsilon_{X,p}|} = \frac{1}{|\varepsilon_{X,p}|} \sum_{i=1}^{n} s_i^2 = \frac{H}{|\varepsilon_{X,p}|}.$$

- Hence, the index of the whole industry is the higher,
  - the more inelastic market demand, i.e., the lower $|\varepsilon_{X,p}|$,
  - the higher concentration of market shares, i.e., the higher H.

Comparative statics: marketing activities

$$\begin{aligned} X^C &= x_1^C + x_2^C = \frac{1}{3b}(2a - c_1 - c_2), p^C = \frac{1}{3}(a + c_1 + c_2), \\ \Pi_1^C &= \frac{1}{9b}(a - 2c_1 + c_2)^2, \Pi_2^C = \frac{1}{9b}(a - 2c_2 + c_1)^2, \\ \Pi^C &= \Pi_1^C + \Pi_2^C < \Pi^M. \end{aligned}$$

Profits of both firms depend positively on a and negatively on b.
Joint marketing activities in a whole industry:

- Flowers as "the world's most beautiful language"
- CMA: „Die Milch macht's"
  (CMA, Central Marketing Association, is liquidated after a judgment of the Bundesverfassungsgericht on February, 3rd, 2009.)

Comparative statics: quantity tax I

Problem

Two firms sell gasoline with unit cost $c_1 = 0.2$ and $c_2 = 0.5$. Inverse demand is given by $p(X) = 5 - 0.5X$.

1. Determine the Cournot equilibrium and the resulting market price!
2. The government introduces a quantity tax t on gasoline. What is the effect on the price that consumers pay?

Comparative statics: quantity tax II

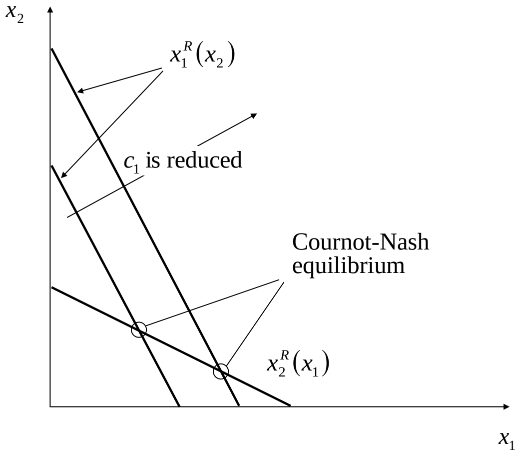
Problem

Two firms sell gasoline with unit cost $c_1 = 0.2$ and $c_2 = 0.5$. Inverse demand is given by $p(X) = 5 - 0,5X$.

1. Determine the Cournot equilibrium and the resulting market price!
2. The government introduces a quantity tax t on gasoline. What is the effect on the price that consumers pay?

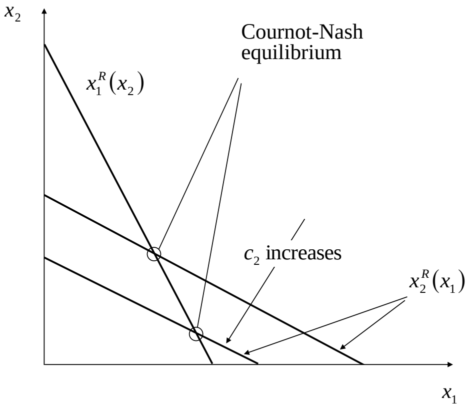
1. $x_1^C = 3.4$, $x_2^C = 2.8$ and $p^C = 1.9$
2. $p^C = 1.9 + \frac{2}{3}t$. Differentiating with respect to $t: \frac{dp^C}{dt} = \frac{2}{3}$, i.e., an increase in taxes by one Euro yields an increase in prices by 66 cents.

- Cost saving
- R&D



Comparative statics: increase in the competitor's cost I

- Sabotage
- Environmental regulations also for the competitor



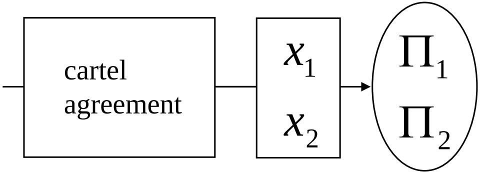
Comparative statics: increase in the competitor's cost II

- Long-distance bus services are prohibited in Germany since the 1930s to protect train services.
- Meanwhile long-distance bus services are allowed.
- Deutsche Bahn demands an obligation for buses to pay tolls on highways. The toll shall work as an instrument to achieve equality in competition between road and rail where track prices have to be paid.

Simultaneous quantity competition: consequences for firms

1. A duopoly can only be expected if market entry is blocked for other firms. Otherwise, positive profits would attract potential competitors. Market entry can be blocked due to uncompetitive cost structures or laws.
2. Uniform cost reductions for all firms, e.g., in case of
   - collective bargaining
   - deregulation claims
   - claims for state subsidies
3. Marketing activities (see above)
4. Opposing interests with respect to individual cost (see above)

Cartel agreement between duopolists I



Cartel agreement between duopolists II

A cartel agreement must specify at least three points:

1. *Distribution of cartel profit:*
   Every member of the cartel has to obtain a profit at least as high as the Cournot duopoly profit $(\Pi_i \geq \Pi_i^C;\ i = 1, 2)$. Beyond that the distribution results from the firms' negotiation skills.
2. *Production of cartel quantity:*
   Who produces which share of the cartel quantity?
3. *Control and sanction mechanisms*:
   ... have to be specified for the case of a breach of the agreement.

Cartel agreement between duopolists III

Cartel profit:

$$\begin{aligned}\Pi_{1,2}(x_1,x_2) \quad &: \quad = \Pi_1(x_1,x_2) + \Pi_2(x_1,x_2) \\ &= \quad p(x_1+x_2)\cdot(x_1+x_2) - C_1(x_1) - C_2(x_2).\end{aligned}$$

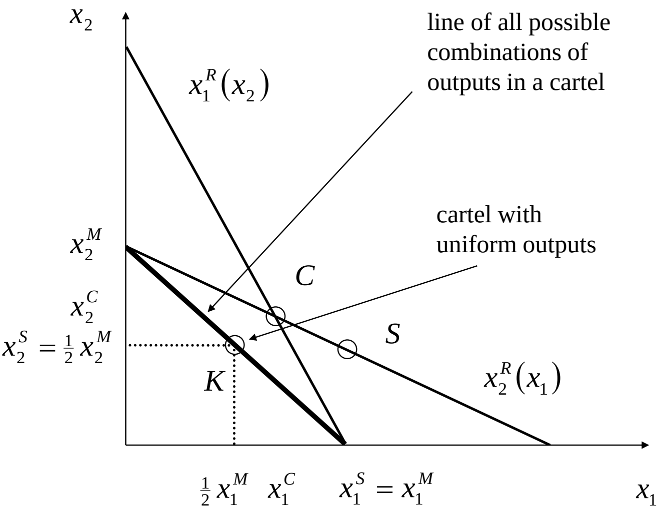
Maximization conditions:

$$\begin{aligned}\frac{\partial \Pi_{1,2}}{\partial x_1} \quad &= \quad p + \frac{dp}{dX}(x_1+x_2) - \frac{dC_1}{dx_1} \overset{!}{=} 0 \text{ and} \\ \frac{\partial \Pi_{1,2}}{\partial x_2} \quad &= \quad p + \frac{dp}{dX}(x_1+x_2) - \frac{dC_2}{dx_2} \overset{!}{=} 0\end{aligned}$$

- Equal marginal cost (as in "one market, two production sites")
- Negative externality $\frac{\partial \Pi_2}{\partial x_1} < 0$ in the Cournot model is taken into account in the cartel agreement $\longrightarrow \frac{dp}{dX}x_2 < 0$

Cartel

The cartel solution



- Optimality condition for firm 1

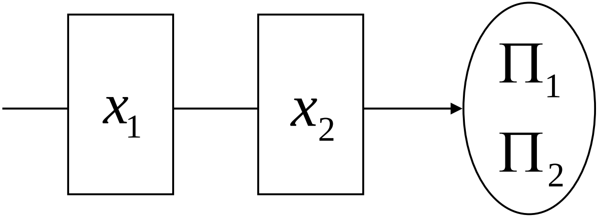
$$\underbrace{p\left(x_1+x_2\right)+x_1\frac{dp}{dX}-MC_1\left(x_1\right)}_{\text{marginal profit for unilateral quantity increase}} = -x_2\frac{dp}{dX} > 0$$

- Behavior according to the cartel agreement is no equilibrium of the considered game!
- Instability of the cartel (or the incentive for cartel fraud)

In the simple case of two players, the incentive for cartel fraud can be expressed as a prisoners' dilemma:

		firm 2: cooperate $x_2 = 2$	firm 2: cheat $x_2 = 3$
firm 1	cooperate $x_1 = 2$	$(8, 8)$	$(6, 9)$
	cheat $x_1 = 3$	$(9, 6)$	$(7, 7)$

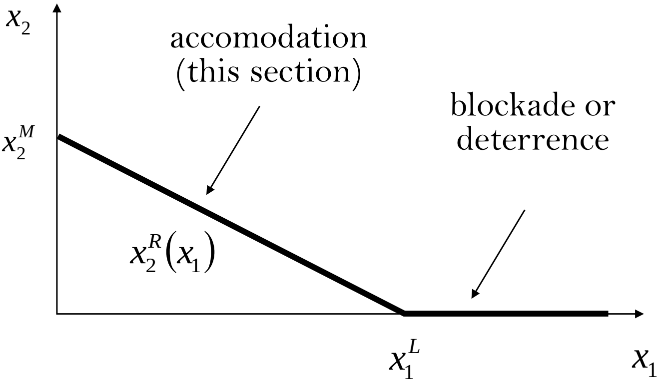
The Stackelberg model



Backward induction:

- Best-response function of firm 2
- Inserting best-response function in the profit function of firm 1
- Maximization for firm 1

Best-response function of firm 2



Now, the word "best response" makes sense.
Blockade or deterrence $\longrightarrow$ Pfähler/Wiese

The Stackelberg model

Problems

Problem

What is the value of $\frac{d\left(x_1+x_2^R(x_1)\right)}{dx_1}$ in the Cournot model and in the Stackelberg model?

Problem

How can you interpret $\frac{dx_2^R(x_1)}{dx_1}$ in the Stackelberg model? What value does this expression take for a linear inverse demand function $p(X) = a - bX$?

Recipe: how to solve the Stackelberg model I

$$\begin{aligned}\Pi_1(x_1,x_2) &= (a-b(x_1+x_2))\,x_1 - c_1x_1\\ \Pi_2(x_1,x_2) &= (a-b(x_1+x_2))\,x_2 - c_2x_2\end{aligned}$$

- Leader moves first, x_1.
- Follower observes x_1, chooses x_2

$$x_2^R(x_1) = \arg\max_{x_2} \Pi_2(x_1,x_2) = \frac{a-c_2}{2b} - \frac{1}{2}x_1$$

- Player 1 anticipates reaction, *reduced profit function*

$$\Pi_1(x_1) := \Pi_1\left(x_1, x_2^R(x_1)\right) = p\left(x_1 + x_2^R(x_1)\right)x_1 - c_1x_1$$

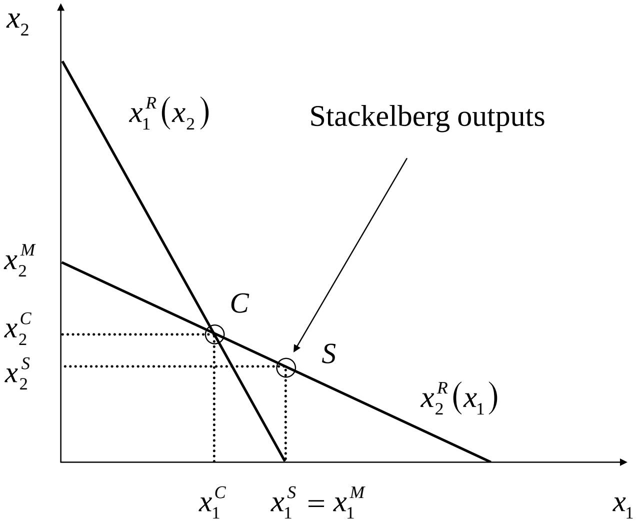
Recipe: how to solve the Stackelberg model II

- Backward-induction quantities:

$$\begin{aligned} x_1^S &: = \arg\max_{x_1} \Pi_1(x_1), \\ x_2^S &: = x_2^R\left(x_1^S\right) \end{aligned}$$

- Player 1 chooses profit-maximizing point on the follower's best-response function

Recipe: how to solve the Stackelberg model III



Recipe: how to solve the Stackelberg model IV

$$\Pi_1(x_1) := \Pi_1\left(x_1, x_2^R(x_1)\right) = p\left(x_1 + x_2^R(x_1)\right)x_1 - c_1 x_1$$

$$\begin{aligned}
& MR_1(x_1) \\
= \; & a - b\left(x_1 + x_2^R(x_1)\right) + x_1(-b) + x_1(-b)\left(-\frac{1}{2}\right) \\
= \; & a - b\left(x_1 + \frac{a-c_2}{2b} - \frac{1}{2}x_1\right) + x_1(-b) + x_1(-b)\left(-\frac{1}{2}\right) \\
= \; & a - bx_1 - \frac{b(a-c_2)}{2b} \overset{!}{=} c_1 = MC_1(x_1)
\end{aligned}$$

$$x_1^S = \frac{a - 2c_1 + c_2}{2b}, x_2^S := x_2^R\left(x_1^S\right) = \frac{a + 2c_1 - 3c_2}{4b}$$

Recipe: how to solve the Stackelberg model V

$$X^S := x_1^S + x_2^S = \frac{1}{4}\frac{3a - 2c_1 - c_2}{b}$$

$$p\left(X^S\right) = aX^S - b = \frac{1}{4}\left(a + 2c_1 + c_2\right)$$

$$\Pi_1^S = \frac{1}{8}\frac{\left(a + c_2 - 2c_1\right)^2}{b}, \Pi_2^S = \frac{1}{16}\frac{\left(a - 3c_2 + 2c_1\right)^2}{b}$$

The Stackelberg model

Problem

False or true? The profit of the Stackelberg leader cannot be smaller than the profit that would result in a Cournot duopoly.

Problem

$p(X) = 20 - X$. $c = 0$
Stackelberg quantities

Central tutorial

Problem Q.5.1.
Two firms A and B
$p(Q) = 48 - Q$
$c = 12$

a) Best-response function of both firms?
   Figure!
   Cournot quantities?
b) A is Stackelberg leader
   Stackelberg quantities?
c) Cartel solution?
d) Perfect competition quantity ($p \stackrel{!}{=} MC$)? Why are
   - Cournot results also called two-third solution and
   - Stackelberg results also called three-fourth solution?