Microeconomics

Game theory

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Structure

Introduction

- Household theory
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
  - Monopoly and monopsony
  - **Game theory**
  - Oligopoly
- External effects and public goods

Pareto-optimal review

Overview „game theory"

- Decision theory and game theory/Nobel prices
- Games in strategic form
  - Examples
  - Solution concepts
- Three interesting games
  - Auction theory
  - The insurance game
  - Election campaign: parties and programs
- Illustrations
- Games in extensive form:
  game tree and backward induction
- Critical view on game theory

Decision theory and game theory

- Both theories are concerned with decisions.
- Decision theory: decisions of single agents that are confronted with an uncertain environment.
- Game theory is concerned with decisions of several agents:
  - Agents
  - Strategies
  - Payoffs

Nobel prices for game theory
1994 and 2005

1994

- $1/3$ John C. Harsanyi (University of California, Berkeley),
- $1/3$ John F. Nash (Princeton University), and
- $1/3$ Reinhard Selten (Rheinische Friedrich-Wilhelms-Universität, Bonn).

2005

- $1/2$ Robert J. Aumann (Hebrew University of Jerusalem), and
- $1/2$ Thomas C. Schelling (University of Maryland, USA).

The letters NASH are the initial letters of all these game theorists.

Clever man: John Forbes Nash, Jr. I

- John Forbes Nash, Jr. (1928-2015) was a US-American mathematician.
- After a promising start of his mathematical career he became ill with schizophrenia with thirty years and did not recover until the 1990s.

Clever man: John Forbes Nash, Jr. II

- Nash's story became known to a wider audience through the award-winning film "A beautiful mind".
- Nash graduated 1950 at the Princeton University with a thesis on game theory. Without knowing the works of Cournot he defines an equilibrium concept that became known as Nash equilibrium. He proves that a Nash equilibrium exists for all strategic games (that are extended by mixed strategies).

Examples

Stag hunt

		hunter 2: stag	hunter 2: hare
hunter 1	stag	5, 5	0, 4
	hare	4, 0	4, 4

To hunt down a stag, two hunters are needed.
Cooperation may pay, but may also fail.

Examples

Stag hunt

		player 2: head	player 2: tail
player 1	head	$1,-1$	$-1,1$
	tail	$-1,1$	$1,-1$

Police versus thief

- head $=$ break-in or control at location "head"
- tail $=$ break-in or control at location "tail"

Examples

Battle of the sexes

		he	
		theater	football
she	theater	$4, 3$	$2, 2$
	football	$1, 1$	$3, 4$

- Different standards
- Harmonizing laws in Europe

Examples

Game of chicken

		driver 2 continue	driver 2 swerve
driver 1	continue	0, 0	4, 2
driver 1	swerve	2, 4	3, 3

- 1 and 2 approach a crossing (a parking spot). One speeds on and "wins".
- 1 and 2 contemplate to open a pharmacy in a small town. The market is too small for both.

Game of chicken

Production game

		firm2 produce little	produce much
firm 1	produce little	$(100, 100)$	$(25, 150)$
	produce much	$(150, 25)$	$(-10, -10)$

Game of chicken?

Examples

Prisoners' dilemma

		player 2 deny	player 2 confess
player 1	deny	3, 3	1, 4
player 1	confess	4, 1	2, 2

- both deny: relatively small sentence (relatively high payoff)
- both confess: relatively high sentence
- one confesses, the other denies: leniency program

Prisoners' dilemma
Examples

1. Installing a catalyst
2. Stealing a car
3. Paying taxes

Often laws can be interpreted as solutions to prisoners' dilemma situations:

1. Environmental requirements or Pigovian tax
2. Criminal law
3. Tax law

Other solutions:

- Repeated games
- Reputation
- Altruism

		firm 2 high price	low price
firm 1	high price	$(100, 100)$	$(25, 150)$
	low price	$(150, 25)$	$(30, 30)$

Prisoners' dilemma?

Strategies and strategy combinations

- head or theater are strategies
- (head, tail) or (theater, theater) are strategy combinations

Example production game:
Strategy combination ($x_1 =$ produce little, $x_2 =$ produce much) yields 25 for firm 1.

Solution concepts

Which strategies will the players choose?

- Dominant strategy
  Independent of the strategy chosen by the other player, I have a best strategy
- Nash equilibrium
  Strategies for both such that unilateral deviation is not beneficial

Examples

Prisoners' dilemma

- Contradiction between
  - individual rationality $\Rightarrow$ Choose the dominant strategy!
  - collective rationality $\Rightarrow$ Realize Pareto improvements!
- There is no solution of this dilemma if the game is only played once
- There is a solution of this dilemma in the infinitely repeated game

Exercises

Production game with investment or subsidies

Firm 2 invests in a machine and can "produce much" less costly

		firm 2: produce little	firm 2: produce much
firm 1	produce little	$(100, 100)$	$(25, 200)$
	produce much	$(150, 25)$	$(-10, 40)$

Dominant strategies?
Nash equilibria?

- Software and processors have to be compatible.
- Intel will develop and produce faster processors if Microsoft develops and sells software for faster processors

		She (Microsoft) theater (faster)	football (fast)
he (Intel)	theater (faster)	$(40, 60)$	$(10, 10)$
	football (fast)	$(15, 10)$	$(60, 40)$

Dominant strategy?
Nash equilibria?

Best responses

Marking technique I

hunter 2

hunter 1		stag	hare
	stag	5, 5	0, 4
	hare	4, 0	4, 4

	stag	hare
stag	5, 5 $\boxed{1}$	0, 4
hare	4, 0	4, 4 $\boxed{1}$

	stag	hare
stag	5, 5 $\boxed{1}\boxed{2}$	0, 4
hare	4, 0	4, 4 $\boxed{1}\boxed{2}$

Best responses

Marking technique II

Problem

player 1		player 2: left	player 2: right
	up	$1, -1$	$-1, 1$
	down	$-1, 1$	$1, -1$

	player 2: left	player 2: right
up	$4, 4$	$0, 5$
down	$5, 0$	$1, 1$

Solution

Left game:

		player 2: left	player 2: right
player 1	up	$1,-1\,\boxed{1}$	$-1,1\,\boxed{2}$
	down	$-1,1\,\boxed{2}$	$1,-1\,\boxed{1}$

Right game:

	player 2: left	player 2: right
up	$4,4$	$0,5\,\boxed{2}$
down	$5,0\,\boxed{1}$	$1,1\,\boxed{1}\,\boxed{2}$

- left game: no dominant strategies and no Nash equilibrium
- right game: second strategy of player 2 is dominant and (down, right) is the Nash equilibrium

best responses = best-response function

- Best-response function for driver 1 in the game of chicken

  "Continue if driver 2 swerves,
  swerve if driver 2 continues"

- Best-response function for driver 2

  "Continue if driver 1 swerves,
  swerve if driver 1 continues"

Nash equilibrium:
Intersection of best-response functions, i.e., strategy combinations

- (continue, swerve) and
- (swerve, continue)

The second-price auction I

- Bidders $i = 1, 2$
- r_i – i's reservation price ($=$ willingness to pay)
- $S_i = [0, +\infty)$ – i hands in a (sealed) bid
- $s_2 < s_1$ makes 1 get the object at price s_2 :

$$u_1(s_1, s_2) = \begin{cases} 0, & s_1 < s_2, \\ \frac{1}{2}(r_1 - s_2), & s_1 = s_2, \\ r_1 - s_2, & s_1 > s_2 \end{cases}$$

Claim: $s_1 := r_1$ is a dominant strategy.

The second-price auction II

1. $r_1 < s_2$
   $s_1 = r_1 \Rightarrow$ payoff 0
   $s_1 > r_1$ and $s_1 < s_2 \Rightarrow$ payoff 0
   $s_1 > r_1$ and $s_1 \geq s_2 \Rightarrow$ payoff < 0
   $s_1 < r_1 \Rightarrow$ payoff 0
2. $r_1 = s_2$
   Expected payoff is 0, no matter how s_1 is chosen. Do you see why?

Problem

Show that $s_1 = r_1$ is a dominant strategy in case of $r_1 > s_2$.

$\Rightarrow$ The second-price auction is dominance solvable

The first-price auction

- Bidders $i = 1, 2$
- r_i – i's reservation price ($=$ willingness to pay)
- $S_i = [0, +\infty)$ – i hands in a (sealed) bid
- $s_2 < s_1$ makes 1 get the object at price s_1 :

$$u_1(s_1, s_2) = \begin{cases} 0, & s_1 < s_2, \\ \frac{1}{2}(r_1 - s_1), & s_1 = s_2, \\ r_1 - s_1, & s_1 > s_2 \end{cases}$$

Claim: Players will bid less than their reservation price.

The insurance game

- Two travelers, $i = 1, 2$, whose antique vase was destroyed by the airline. Value unclear
- $S_i = \{2, 3, ..., 100\}$
- Both get the lowest figure adjusted by an honesty premium/dishonesty punishment of 2

$$u_1(s_1, s_2) = \begin{cases} s_1 + 2, & s_1 < s_2, \\ s_1, & s_1 = s_2, \\ s_2 - 2, & s_1 > s_2; \end{cases}$$

The insurance game

The matrix

	Traveler 2 requests so many coins						
	2	3	4	$\cdots$	98	99	100
2	$(2,2)$	$(4,0)$	$(4,0)$	$(4,0)$	$(4,0)$	$(4,0)$	$(4,0)$
3	$(0,4)$	$(3,3)$	$(5,1)$	$(5,1)$	$(5,1)$	$(5,1)$	$(5,1)$
4	$(0,4)$	$(1,5)$	$(4,4)$	$(6,2)$	$(6,2)$	$(6,2)$	$(6,2)$
$\vdots$	$(0,4)$	$(1,5)$	$(2,6)$				
98	$(0,4)$	$(1,5)$	$(2,6)$		$(98,98)$	$(100,96)$	$(100,96$
99	$(0,4)$	$(1,5)$	$(2,6)$		$(96,100)$	$(99,99)$	$(101,97$
100	$(0,4)$	$(1,5)$	$(2,6)$		$(96,100)$	$(97,101)$	$(100,100$

Problem

Any dominant strategies?

The insurance game

Somewhat reduced

Trav.1 claims	Traveler 2 requests so many coins: 2	3	4	$\cdots$	98	99
2	$(2,2)$	$(4,0)$	$(4,0)$	$(4,0)$	$(4,0)$	$(4,0)$
3	$(0,4)$	$(3,3)$	$(5,1)$	$(5,1)$	$(5,1)$	$(5,1)$
4	$(0,4)$	$(1,5)$	$(4,4)$	$(6,2)$	$(6,2)$	$(6,2)$
$\vdots$	$(0,4)$	$(1,5)$	$(2,6)$			
98	$(0,4)$	$(1,5)$	$(2,6)$		$(98,98)$	$(100,96)$
99	$(0,4)$	$(1,5)$	$(2,6)$		$(96,100)$	$(99,99)$

The insurance game

More reduced

		traveler 2: 2	traveler 2: 3
traveler 1	2	$(2,2)$	$(4,0)$
	3	$(0,4)$	$(3,3)$

Problem

Do you know this game?

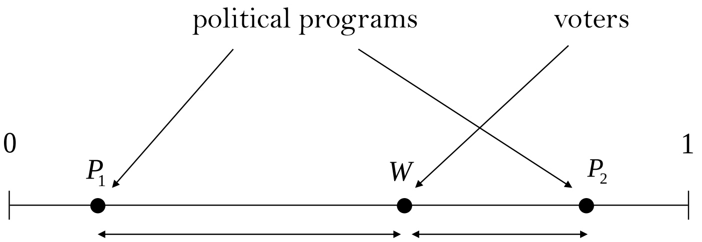
The insurance game

Find a (the?) equilibrium!

	2	3	4	$\cdots$	98	99	100
2	$(2, 2)$	$(4, 0)$	$(4, 0)$	$(4, 0)$	$(4, 0)$	$(4, 0)$	$(4, 0)$
3	$(0, 4)$	$(3, 3)$	$(5, 1)$	$(5, 1)$	$(5, 1)$	$(5, 1)$	$(5, 1)$
4	$(0, 4)$	$(1, 5)$	$(4, 4)$	$(6, 2)$	$(6, 2)$	$(6, 2)$	$(6, 2)$
$\vdots$	$(0, 4)$	$(1, 5)$	$(2, 6)$				
98	$(0, 4)$	$(1, 5)$	$(2, 6)$		$(98, 98)$	$(100, 96)$	$(100, 96)$
99	$(0, 4)$	$(1, 5)$	$(2, 6)$		$(96, 100)$	$(99, 99)$	$(101, 97)$
100	$(0, 4)$	$(1, 5)$	$(2, 6)$		$(96, 100)$	$(97, 101)$	$(100, 100$

Parties

Two parties/two programs



- One-dimensional political space (left - right)
- Voters prefer the program closest to their political preferences.
- Even distribution between 0 (extreme left) and 1 (extreme right).
- Parties choose programs P_1 and P_2, respectively.
- Equilibrium?

Theorem

In the above model, there exists exactly one equilibrium: both parties choose the middle position $\frac{1}{2}$.

Proof.

- In equilibrium, we have $P_1 = P_2$. Otherwise ...
- In equilibrium, we have $P_1 = P_2 = \frac{1}{2}$. Otherwise ...
- There is at most one equilibrium.
- $(P_1, P_2) = \left(\frac{1}{2}, \frac{1}{2}\right)$ is an equilibrium.
  - If party 1 deviates, ...
  - If party 2 deviates, ...

Parties

... and for three parties?

Theorem

There is no equilibrium with three political parties.

Proof.

There is no equilibrium at

- $P_1 \neq P_2 \neq P_3$
- $P_1 = P_2 \neq P_3$
- $P_1 = P_2 = P_3$
    - $= \frac{1}{2}$
    - $\neq \frac{1}{2}$

Instability of political programs

is a theoretical phenomenon with practical relevance:

- internal party strife
- median-voter orientation
- new parties at the left or right edge

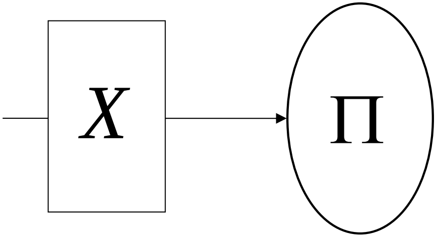
But: Political parties cannot change their programs arbitrarily.

Decision theory
Illustration

Decision situation = trivial game without opponent, e.g, monopoly situation

Model:
x : output or price
$\Pi(x)$: profit for output or price x

Simple illustration

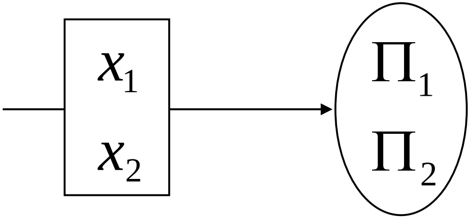


Model:

Two firms 1 and 2 with outputs x_1 and x_2, respectively

$$\begin{aligned} \text{Profit of firm 1} &: \quad \Pi_1(x_1, x_2), \\ \text{Profit of firm 2} &: \quad \Pi_2(x_1, x_2). \end{aligned}$$

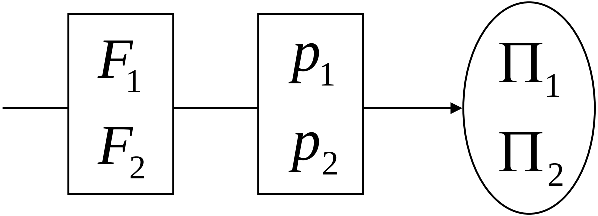
Simple illustration:



Two firms that

- first, choose expenditures for R&D simultaneously and
- second, choose prices simultaneously.

Simple illustration:

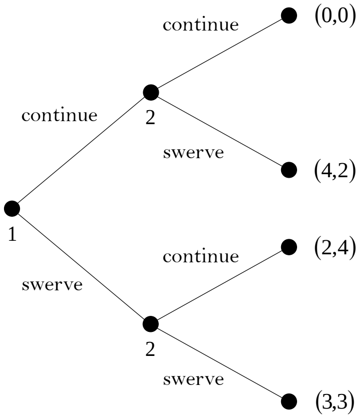


Game tree and backward induction

Definition

- so far: strategic games (simultaneous actions)
- now: game tree:
  - first, player 1 moves
  - second, player 2 moves knowing the action of player 1

Game of chicken:
First-mover advantage?



Player 2 knows the action chosen by player 1.
Player 1 can predict the reaction of player 2.

Problem

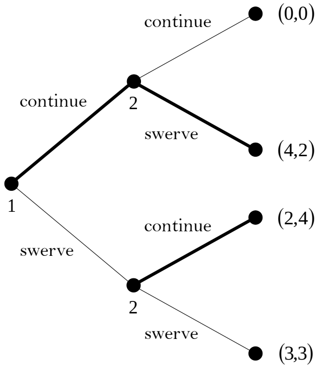
Simple illustration?

Problem

What will happen?

Game tree and backward induction

Chicken game II



If player 1 continues, player 2 has no choice: she has to swerve. Hence, player 1 obtains payoff 4, her best possible result.

Problem

Is there a first-mover advantage in "head or tail"?

Critical view on game theory

Equilibria serve to make theoretical predictions. However,

- equilibria are sometimes counterintuitive (insurance game) and
- there may be a couple of equilibria as in the following games

	continue	swerve
continue	0, 0	4, 2
swerve	2, 4	3, 3

	stag	hare
stag	5, 5	0, 4
hare	4, 0	4, 4

	theater	football
theater	4, 3	2, 2
football	1, 1	3, 4

Exercises

Problem 1
Is there a first-mover advantage in "battle of the sexes"? Draw a game tree and apply backward induction.

Problem 2
Apply the marking technique to this game:

		player 2	
		s_2^1	s_2^2
player 1	s_1^1	$1, 1$	$1, 1$
	s_1^2	$1, 1$	$0, 0$

Exercises

Problem 3

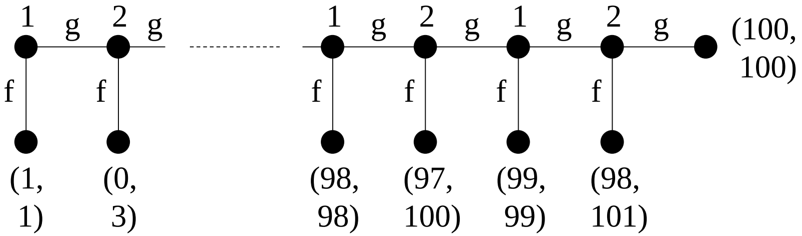
Find all equilibria of the following game:

		player 2		
		l	c	r
player 1	o	$(4, 5)$	$(2, 1)$	$(4, 4)$
	m	$(0, 1)$	$(1, 5)$	$(3, 2)$
	u	$(1, 1)$	$(0, 0)$	$(6, 0)$

Problem 4

For the centipede game, players choose alternately f (finish) or g (go on).

- What would you do if you were player 1?
- Solve the game by backward induction.
- Do you want to revise your answer?



Central tutorial I

Problem P.5.1.
Players 1 and 2
Two strategies: "cooperation" or "confrontation"

Both choose "cooperation" $\Rightarrow$ payoff €100
Both choose "confrontation" $\Rightarrow$ payoff €0
One chooses "cooperation", the other "confrontation" $\Rightarrow$ the first obtains €P, the second €F

For which payoffs P and F is "confrontation" a dominant strategy for both players?

Central tutorial II

Problem P.5.2.
Strategy combination (down, right) is a Nash equilibrium. What do we know about the constants a, b, c, and d?

		player B	
		left	right
player A	up	$1, a$	$c, 1$
	down	$1, b$	$d, 1$

Central tutorial III

Problem P.5.3.

Adam and Eve meet for the first time under an apple tree. After exchanging their preferences for fruit, they agree on another meeting under one of the other fruit trees in the area and bid farewell. They are emotionally shaken, so they forget to agree on a particular fruit. Fortunately, there is only one very old plum tree and a less old cherry tree. Both know that Adam prefers plums, while Eve prefers cherries. Their payoffs are the following: If they meet under the plum tree, Adam has utility 3 and Eve 2. If they meet under the cherry tree, the payoffs are reversed. If they go to different trees both obtain 0 utility. Determine all Nash equilibria!