Microeconomics

Uncertainty

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Structure

Introduction

- Household theory
  - Budget
  - Preferences, indifference curves, and utility functions
  - Household optimum
  - Comparative statics
  - Decisions on labor supply and saving
  - **Uncertainty**
  - Market demand and revenue
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods

Pareto-optimal review

Description of the initial situation

Decisions under uncertainty

- Certainty: perfect information on every parameter relevant for the decision
- Uncertainty: the result also depends on the state of the world
  - Risk: probability distribution is known
  - Unmeasurable uncertainty: probability distribution is not known

Description of the initial situation

Payment (amount of money or utility) depends on

- the chosen action and
- the state of the world

		state of the world	
		bad weather	good wheather
action	umbrella production	100	81
	parasol production	64	121

Overview

- Description of the initial situation
- Decisions under unmeasurable uncertainty
- Decisions under risk
  - Bayes' rule and Bernoulli principle
  - St. Petersburg paradox (excursus)
- Justification of the Bernoulli principle
- Risk averse, risk neutral, and risk loving decision makers
- Demand for insurance
- Certainty equivalent and risk premium

Decisions under unmeasurable uncertainty

- Maximin rule
- Maximax rule
- Hurwicz rule
- Rule of minimal regret
- Laplace rule

Decisions under unmeasurable uncertainty

Maximin rule

- For every alternative determine the worst result (minimum in every row)!
- Choose the alternative with the highest minimum!

Problem

Which product (umbrella or parasol) is chosen according to the maximin rule?

Decisions under unmeasurable uncertainty

Maximax rule

- For every alternative determine the best result (maximum of the row)!
- Choose the alternative with the highest maximum!

Problem

Which product (umbrella or parasol) is chosen according to the maximax rule?

Decisions under unmeasurable uncertainty

Hurwicz rule

- The row's maximum is weighted with factor γ and the row's minimum with factor $1-\gamma$, where $0 \leq \gamma \leq 1$.
- Choose the alternative with the highest weighted average!

Problem

For $\gamma = 1$ the Hurwicz rule is equivalent to the ... rule.
For $\gamma = 0$ the Hurwicz rule is equivalent to the ... rule.

Problem

Which product (umbrella or parasol) is chosen according to the Hurwicz rule with optimism parameter $\gamma = \frac{3}{4}$?

- The payoff matrix is transformed into the matrix of regret.
- The elements of the matrix of regret measure the disadvantage that results from the misjudgment of the state of the world: Every element of a column is replaced by its absolute difference to the column's maximum.
- Choose the alternative that minimizes the maximal regret!

Problem

Which product (umbrella or parasol) is chosen according to the rule of minimal regret?

- The unmeasurable uncertainty is treated as a situation of risk; every state of the world is assumed to be equally likely.
- Choose the alternative with the highest expected value!

Problem

Which product (umbrella or parasol) is chosen according to the Laplace rule

Assume that the probability for good weather is $\frac{3}{4}$
Umbrella production leads to
probability distribution on payoffs = lottery

$$L_{\text{umbrella}} = \left[100, 81; \frac{1}{4}, \frac{3}{4}\right].$$

What about parasol production?
General notation for lotteries:

$$L = [x_1, ..., x_n; p_1, ..., p_n].$$

where

- $p_i \geq 0$ and
- $p_1 + ... + p_n = 1$

hold.

- Probability distributions can contain probability distributions as "payoffs".
- Compound distribution:

$$\left[L_1, L_2; p_1, p_2\right]$$

Problem

Let $L_1 = \left[0, 10; \frac{1}{2}, \frac{1}{2}\right]$ and $L_2 = \left[5, 10; \frac{1}{4}, \frac{3}{4}\right]$. Express $L_3 = \left[L_1, L_2; \frac{1}{2}, \frac{1}{2}\right]$ as a simple distribution!

- Expected value of a distribution $L = [x_1, ..., x_n; p_1, ..., p_n]$:

$$E(L) = p_1 x_1 + ... + p_n x_n.$$

- Bayes' rule:
  From the set of all possible probability distributions choose the one with the highest expected value.

Decisions under risk

Problem

Which production good is chosen by an entrepreneur who follows Bayes' rule and assumes that the probability for good weather is $\frac{3}{4}$?

Problem

Would you prefer $L_1 = \left[100, 0; \frac{1}{2}, \frac{1}{2}\right]$ over $L_2 = [50; 1]$?

Problem

Would you prefer L_1 over $L_3 = [40; 1]$?

Decisions under risk

Bayes' rule

- Plus: easy to calculate
- Minus: no consistency with typical behavioral patterns
  $\Rightarrow$ Application of the Bernoulli principle

- Expected utility for a given vNM utility function $u(x)$ and a distribution $L = [x_1, ..., x_n; p_1, ..., p_n]$:

$$E_u(L) = p_1 u(x_1) + ... + p_n u(x_n)$$

- Bernoulli principle:
  Choose the probability distribution with the highest expected utility.

Problem

Which good is produced if the vNM utility function is given by $u(x) = \sqrt{x}$ and if the probability for good weather is $\frac{3}{4}$?

- Peter tosses a fair coin
  until head appears for the first time.
- If Peter tossed the coin n times,
  he pays 2^n to Paul.
- Stochastic independence is given,
  hence, the probability for head at the n-th toss is $\left(\frac{1}{2}\right)^n$.

Problem

Write down the St. Petersburg lottery! Do the probabilities add up to 1?

- The expected value of the lottery L is given by

$$E(L) = \sum_{n=1}^{\infty} 2^n \cdot \left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \left(2 \cdot \frac{1}{2}\right)^n = 1 + 1 + \ldots = \infty.$$

- Bayes' criterion: Paul would accept every price that Peter requests for playing the game.
- Surveys show that very few people are willing to offer an amount of 10 or 20.

- Solution: Bernoulli principle with the natural logarithm as utility function:

$$E_{\ln}(L) = \sum_{n=1}^{\infty} \ln(2^n) \cdot \left(\frac{1}{2}\right)^n = \ln 2 \sum_{n=1}^{\infty} n \cdot \left(\frac{1}{2}\right)^n \underset{\text{difficult}}{=} 2 \ln 2$$

- In this case the St. Petersburg lottery has a value CE that ... (explained later)

Justification of the Bernoulli principle

- Assumption: The individual has a preference relation $\succsim$ on the set of probability distributions.
- In the following: Restriction of preference relations by axioms $\Longrightarrow$ derivation of the Bernoulli principle

Problem

Explain the axioms of completeness and transitivity for bundles of goods in preference theory!

Preference axioms

- **Completeness axiom:** Two lotteries L_1, L_2. $\Rightarrow$

$$L_1 \succsim L_2 \text{ or } L_2 \succsim L_1$$

- **Transitivity axiom:** Let $L_1 \succsim L_2$ and $L_2 \succsim L_3$. $\Rightarrow$

$$L_1 \succsim L_3$$

- **Continuity axiom:** Let $L_1 \succsim L_2 \succsim L_3$. $\Rightarrow$ There is a $p \in [0, 1]$ such that

$$L_2 \sim [L_1, L_3; p, 1-p]$$

- **Independence axiom:** Let L_1, L_2, L_3 and $p > 0$. $\Rightarrow$

$$[L_1, L_3; p, 1-p] \precsim [L_2, L_3; p, 1-p] \Leftrightarrow L_1 \precsim L_2.$$

Assumption:

- L_1 – payoff of 10 €
- L_2 – payoff of 0 €
- L_3 – certain death

$$L_1 \succ L_2 \succ L_3$$

Determine p such that

$$L_2 \sim [L_1, L_3; p, 1-p]$$

$p = 1 \Rightarrow [L_1, L_3; 1, 0] = L_1 \succ L_2.$

Preference axiom

Criticism of the independence axiom

Consider the lotteries

$$L_1 = \left[12 \cdot 10^6, 0; \tfrac{10}{100}, \tfrac{90}{100}\right] \qquad L_3 = \left[1 \cdot 10^6; 1\right]$$

$$L_2 = \left[1 \cdot 10^6, 0; \tfrac{11}{100}, \tfrac{89}{100}\right] \qquad L_4 = \left[12 \cdot 10^6, 1 \cdot 10^6, 0; \tfrac{10}{100}, \tfrac{89}{100}, \tfrac{1}{100}\right]$$

- Do you prefer L_1 over L_2?
- Do you prefer L_3 over L_4?

see lecture notes Advanced Microeconomics

A utility function for lotteries

vNM utility functions

Theorem

Preferences on lotteries obey the four axioms if and only if there is a vNM utility function $u : \mathbb{R}_+ \to \mathbb{R}$ *such that*

$$L_1 \succsim L_2 \Leftrightarrow E_u(L_1) \geq E_u(L_2)$$

holds for all lotteries L_1 *and* L_2*.* E_u *(or:* u*) represents preferences* $\succsim$ *on the set of lotteries;*

- u – vNM utility function with domain: payoffs
- E_u – expected utility with domain: lotteries

A utility function for lotteries

Transformations

Definition

u vNM utility function. v is called an affine transformation of u if $v(x) = a + bu(x)$ holds for $a \in \mathbb{R}$ and $b > 0$.

Lemma

If u represents preferences $\succsim$ for lotteries, this is also true for every affine transformation of u.

Problem

Find a simple affine transformation of $u(x) = 100 + 3x + 9x^2$!

Risk averse, risk neutral, and risk loving decision makers

Let L be a non-trivial lottery. Preferences are

- risk neutral if

$$L \sim [E(L); 1] \text{ or } E_u(L) = u(E(L))$$

- risk averse if

$$L \prec [E(L); 1] \text{ or } E_u(L) < u(E(L))$$

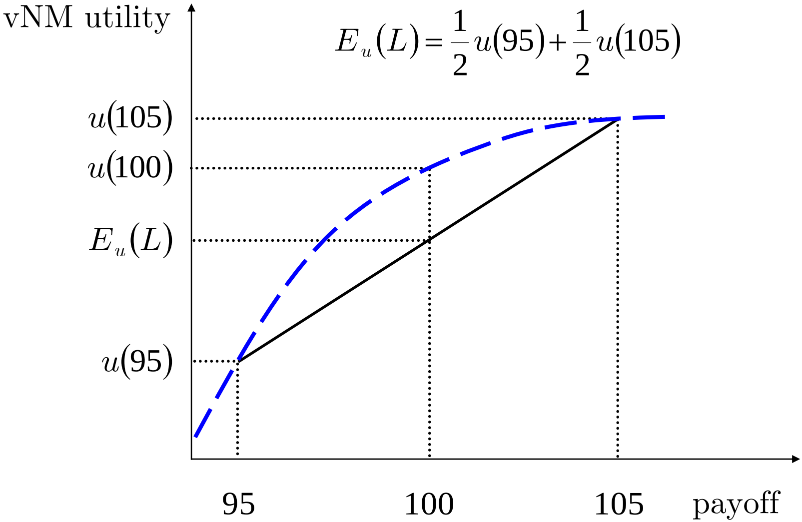
- risk loving if

$$L \succ [E(L); 1] \text{ or } E_u(L) > u(E(L))$$

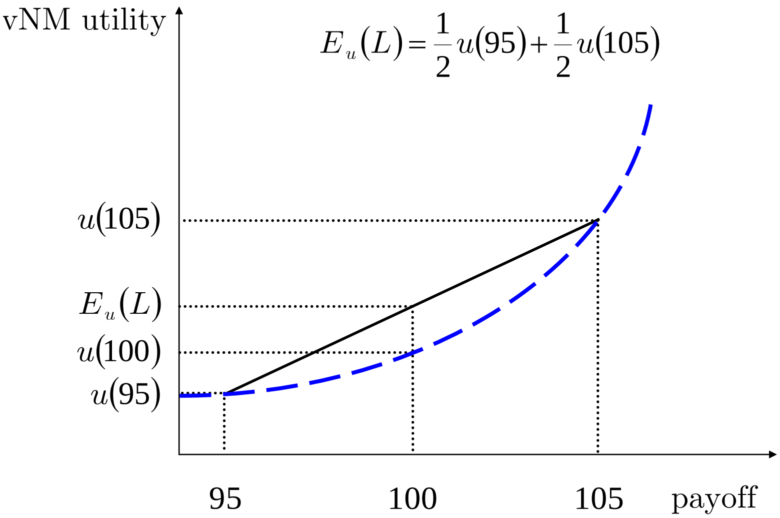
Problem

Is Bayes' rule a special case of the Bernoulli principle?

Risk averse decision makers



Risk loving decision makers



Demand for insurance

- A household has an initial wealth of A.
- With probability p the household looses an amount D with $D \leq A$.
- An insurance pays K in case of damage.
- The insurance premium is equal to $P = \gamma K$ with $0 < \gamma < 1$.
- Which insurance payment K should the household choose?

Demand for insurances

Budget line

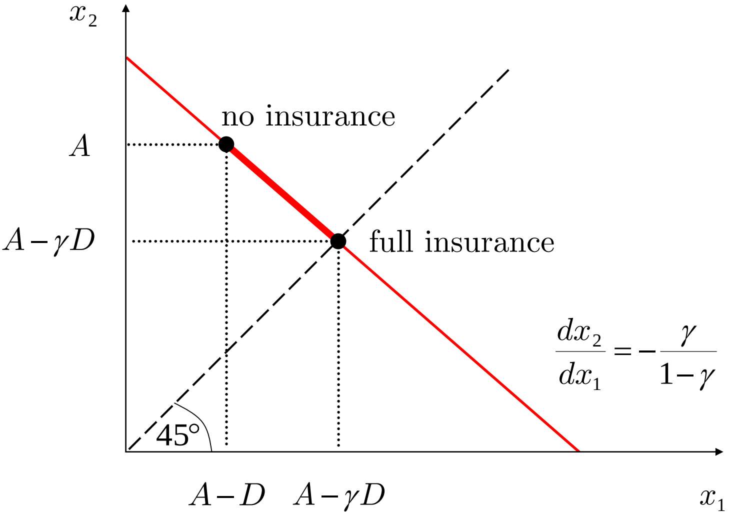
- x_1 = wealth in case of damage:
$$x_1 = A - D + K - P = A - D + (1-\gamma) K$$
- x_2 = wealth without damage
$$x_2 = A - P = A - \gamma K$$
- Budget line:
$$\frac{\gamma}{1-\gamma} x_1 + x_2 = \frac{\gamma}{1-\gamma} (A - D) + A.$$

Problem

Determine the slope of the budget line! What is the economic interpretation?

Demand for insurance

Budget line



Demand for insurance

Expected utility and indifference curves

$$U(x_1, x_2) = E_u([x_1, x_2; p, 1-p]) = pu(x_1) + (1-p)\,u(x_2)$$

$$MRS = \frac{MU_1}{MU_2} = \frac{p}{1-p}\frac{u'(x_1)}{u'(x_2)}.$$

Problem

risk averse $\Rightarrow$ convex preferences?

Problem

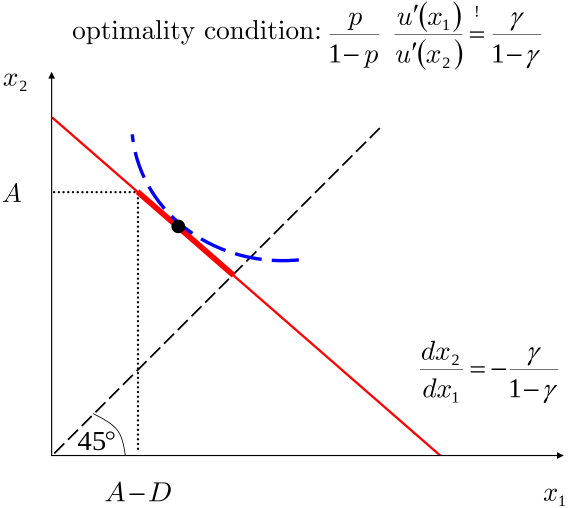
MRS along the 45° line?

Problem

The higher p, the ... the indifference curves.

Demand for insurance

Household optimum



Demand for insurance

Household optimum

- In the household optimum the following equation must hold:

$$\frac{p}{1-p}\frac{u'\left(x_1\right)}{u'\left(x_2\right)} \overset{!}{=} \frac{\gamma}{1-\gamma}.$$

- Reformulating yields

$$\frac{u'\left(A-D+\left(1-\gamma\right)K\right)}{u'\left(A-\gamma K\right)} = \frac{\gamma}{1-\gamma}\frac{1-p}{p}.$$

Problem

Benjamin owns a yacht worth € 10,000.00.
$p = 0.01$
$\gamma = 0.02$
vNM utility function $u\left(x\right) = \ln\left(x\right)$
Optimal insurance sum?

Demand for insurance

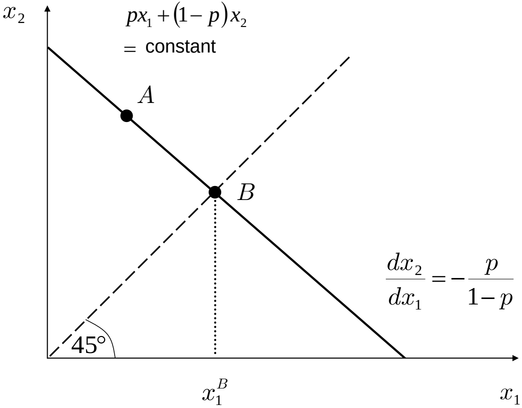
Fair insurance: expected payment of the insurance company equals the insurance premium:

$$pK = P.$$

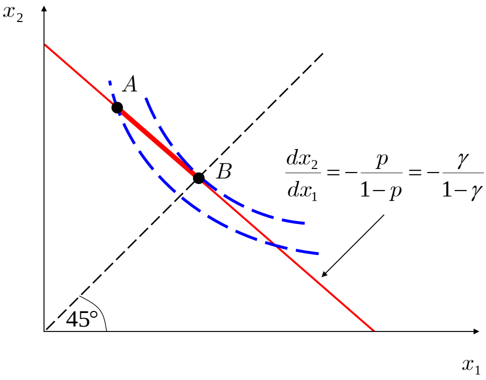
Hence,

$$\gamma = ?$$

The curve of constant expected value



$$\begin{aligned} E\left(\text{lottery } A\right) &= px_1^B + (1-p)\,x_2^B \\ &= px_1^B + (1-p)\,x_1^B = x_1^B. \end{aligned}$$

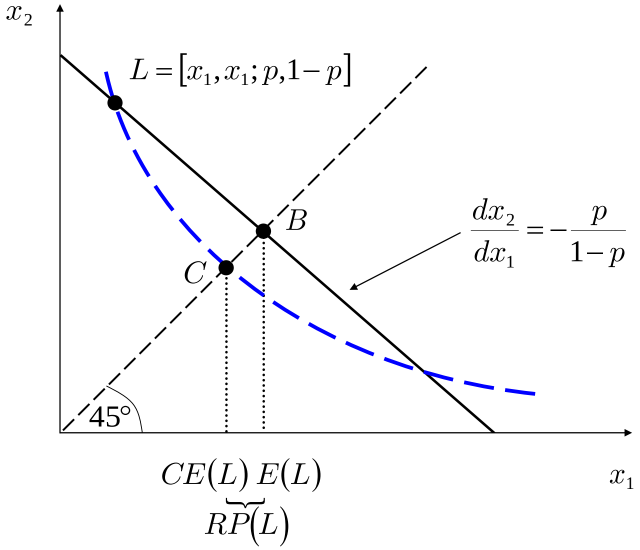


- Fair insurance:
  $\gamma = p$, i.e.,
  curve of constant expected value
  $=$ budget line
- Risk aversion means:
  Preferring the expected value of the lottery over the lottery itself

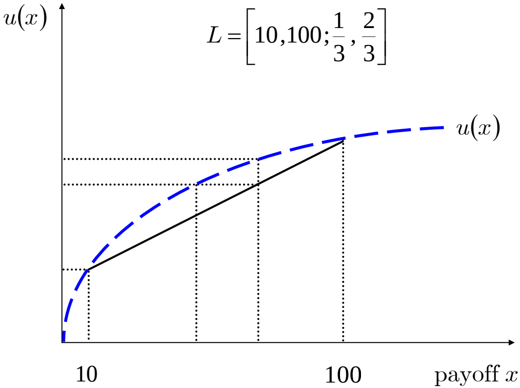
Certainty equivalent and risk premium

- Lottery L
  Certainty equivalent $CE(L)$:
  $$L \sim [CE(L); 1]$$
  or
  $$E_u(L) = u(CE(L)).$$
  Certainty equivalent: Which risk-free amount is worth the same as the lottery for the individual?
- Risk premium $RP(L)$:
  $$RP(L) = E(L) - CE(L)$$
  Risk premium:What is the individual's willingness to pay for removing the risk?

Certainty equivalent and risk premium



Certainty equivalent and risk premium



Problem

Expected value?
Expected utility?
Utility of the expected value?
Certainty equivalent?
Risk premium?

Certainty equivalent and risk premium

St. Petersburg paradox

- Using the natural logarithm as utility function, the expected utility of St. Petersburg lottery (see above) is given by

$$E_{\ln}(L) = (\ln 2) \cdot 2$$

- Then, the St. Petersburg lottery's CE is implicitly given by

$$1 \cdot \ln(CE) = E_{\ln}([CE; 1]) \overset{!}{=} (\ln 2) \cdot 2$$

and explicitly given by

$$CE = e^{\ln(CE)} \overset{!}{=} e^{(\ln 2) \cdot 2} = \left(e^{(\ln 2)}\right)^2 = 2^2 = 4.$$

Central tutorial I

Problem G.9.1.

		state of the world: left	right
action	up	10	8
	down	4	12

Which action would a decision maker choose according to

a) the maximin rule?

b) the maximax rule?

c) the Hurwicz rule with optimism parameter $\gamma = \frac{3}{4}$?

d) the rule of minimal regret?

e) the Laplace rule?

Central tutorial II

Problem G.9.2.
Two lotteries

$$L_1 = \left[100, 0; \frac{3}{5}, \frac{2}{5}\right]$$
$$L_2 = \left[100, 25; \frac{2}{5}, \frac{3}{5}\right].$$

$L_3 = \left[L_1, L_2; \frac{1}{2}, \frac{1}{2}\right]$ as simple lottery?

Problem G.9.3.
Risk aversion?

a) $u(x) = x^2$ for $x > 0$;
b) $u(x) = 2x + 3$;
c) $u(x) = \ln(x)$ for $x > 0$;
d) $u(x) = -e^{-x}$.

Central tutorial III

Problem G.9.4.

vNM utility function $u(x) = x^{\frac{1}{2}}$ with payoff x

income 10

gain/loss of 6 with probability $\frac{1}{2}$

- Express the situation with a lottery!
- Expected value of the lottery?
- Certainty equivalent?

Problem G.9.5.

vNM utility function $u(x) = x^{\frac{1}{2}}$

Two lotteries

$$L_1 = \left[100, 0; \frac{3}{5}, \frac{2}{5}\right], L_2 = \left[100, 25; \frac{2}{5}, \frac{3}{5}\right].$$

a) Which lottery is preferred?

b) Certainty equivalent of the second lottery?

Central tutorial IV

Problem G.9.6.
vNM utility function $u(x) = x^{\frac{1}{2}}$
Initial wealth 144€
Damage of 108€ with probability $\frac{1}{3}$

- Expected utility?
- Risk premium?