Microeconomics

Preferences, indifference curves, and utility functions

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Structure

Introduction

- Household theory
  - Budget
  - **Preferences, indifference curves, and utility functions**
  - Household optimum
  - Comparative statics
  - Decisions on labor supply and saving
  - Uncertainty
  - Market demand and revenue
- Theory of the firm
- Perfect competition and welfare theory
- Types of markets
- External effects and public goods

Pareto-optimal review

Overview

- Preference relation
- Indifference curve
- Utility function

Preference relation

Mick Jagger: You can't always get what you want.

- Preference: appreciation
- Preference relation: special binary relation

Preference relation

Definition (Weak preference)

$$X = (x_1, x_2) \succsim (y_1, y_2) = Y$$

X is at least as good as Y

Definition (Indifference)

$$(x_1, x_2) \sim (y_1, y_2)$$

X is as good as Y

Definition (Strict preference)

$$(x_1, x_2) \succ (y_1, y_2)$$

X is better than Y

Preference relation

Problem

Derive the strict preference relation and the indifference relation from the weak preference relation!

- Completeness: Every individual can compare each pair of goods according to the weak preference relation $\precsim$:
  $A \succsim B$ or $B \succsim A$
- Transitivity: If $A \succsim B$ and $B \succsim C$ holds, then $A \succsim C$ is implied.(SH 65)

Problem

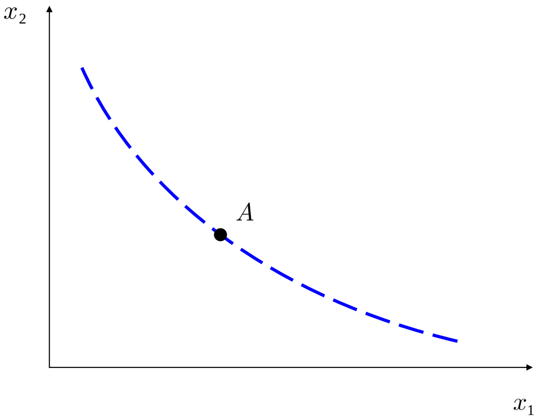
Do the axioms of completeness and of transitivity hold for the strict preference relation and the indifference relation?

Preference relation

Problem

Estefania spends all her monthly income on pizza and books.
$p_{pizza} = 9.00$, $p_{book} = 30.00$
$x_{pizza} = 30$, $x_{book} = 3$.
There is no other combination of pizza and books within her budget that makes her better off.
Assume that p_{pizza} drops to 8.70 and
p_{book} increases to 33.00.
Without any additional information on Estefania's preferences, can we know whether she is better off due to the change in prices? Hint: Can Estefania afford the old consumption bundle at the new prices?

The locus of all bundles of goods between which the individual is indifferent. (SH 65)



Indifference curve

Problem

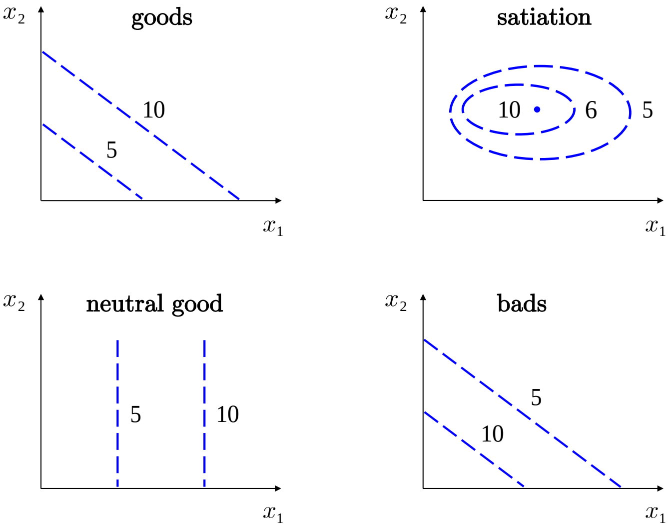
Sketch appropriate indifference curves:

1. Perfect substitutes:
   The bundle (x_1, x_2) is strictly preferred to (y_1, y_2) if and only if $x_1 + x_2 > y_1 + y_2$.
2. Perfect complements:
   Strict preference if and only if $\min(x_1, x_2) > \min(y_1, y_2)$.
3. Lexicographic preferences:
   Strict preference if and only if
   - $x_1 > y_1$ or
   - $x_1 = y_1$ and $x_2 > y_2$

   holds. (SH 63f)

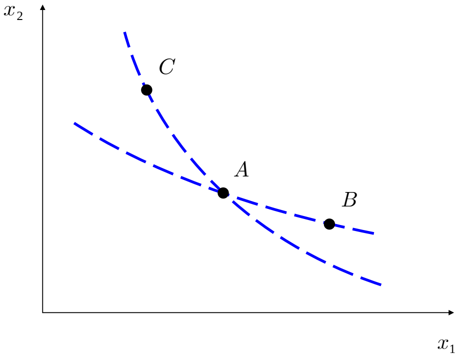
Indifference curve

Definition and examples



Indifference curves must not intersect!

$$C \sim A \wedge A \sim B \Rightarrow C \sim B$$



- Obi: more is more
- ,,Mehr ist besser" (SH 66)
- Non-satiation

$$(x_1 \geq y_1) \wedge (x_2 \geq y_2) \wedge X \neq Y \Rightarrow X \succ Y$$

Problem

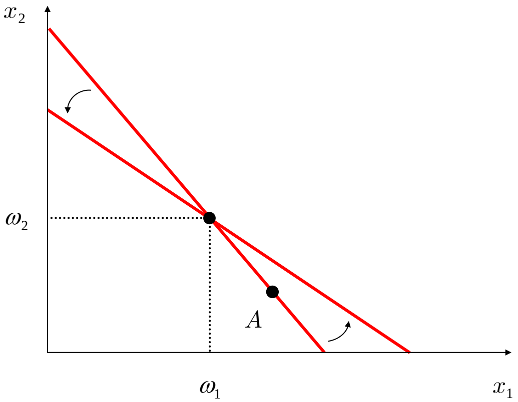
How would you illustrate monotonicity?

Problem

Monotonicity two slides ago?

Indifference curve

Monotonicity

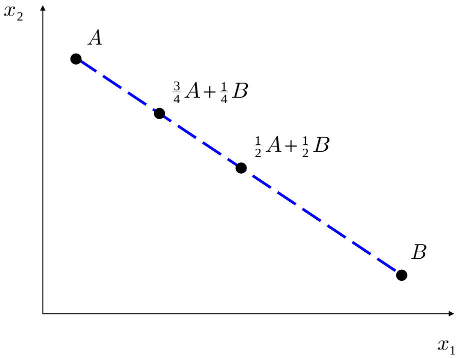


Problem

Would the household choose a consumption bundle to the left of the intersection?

Indifference curve

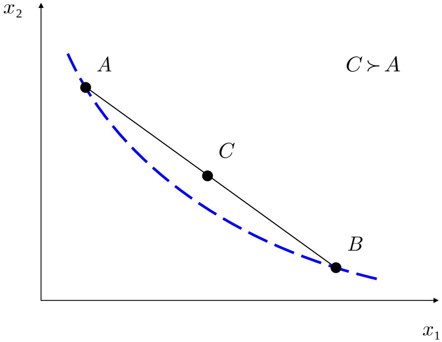
Convex linear combinations



- $0 \cdot A + (1-0)\,B = ?$
- Does $(3,7)$ represent a convex linear combination of the bundles $(3,6)$ and $(3,9)$?

Indifference curve

Convexity: "Extremes are bad"



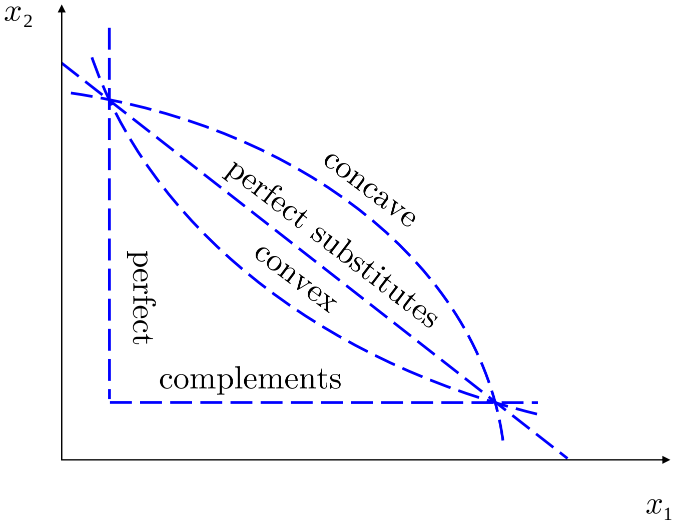
- Convexity
  = weak convexity:
  C is at least as good as A
- Strict convexity: C is better than A (SH 65)

Problem

Convexity or strict convexity in the figure above?

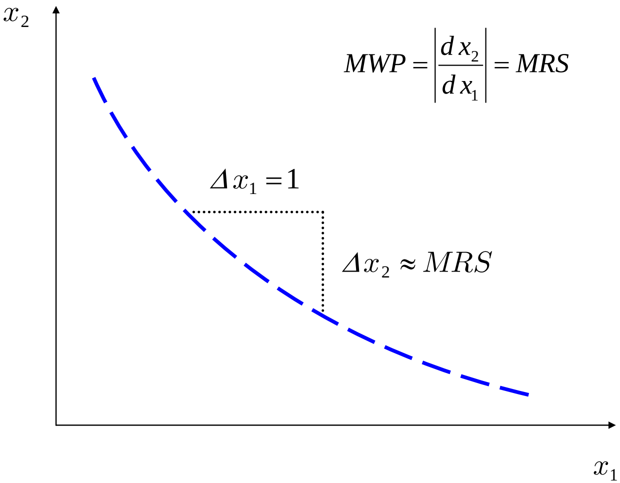
Indifference curve

Perfect complements, perfect substitutes, ...



Indifference curve

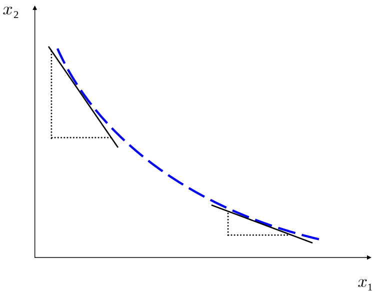
Marginal rate of substitution



Indifference curve

Marginal rate of substitution

- Convex and monotonic preference
  $\Rightarrow$ MRS decreases with increasing x_1



- Perfect substitutes $\Rightarrow$ constant
- Perfect complements $\Rightarrow$ in some points not well defined

Problem

$\left|\frac{dx_2}{dx_1}\right|^{\text{Mary}} = 2, \left|\frac{dx_2}{dx_1}\right|^{\text{Laura}} = 5$. Mary hands one unit of good 1 to Laura and receives one unit of good 2. Who is better off?

Indifference curve

MRS versus MOC

Problem

Assume that a household consumes two goods such that

$$MRS = \left| \frac{dx_2}{dx_1} \right| < \frac{p_1}{p_2} = MOC$$

holds. In which direction does consumption behavior of the household change? Begin your argument either like this: "If the household consumes one additional unit of good 1 ..." or like this: "If the household forgoes consumption of one unit of good 1 ..."

Utility functions

Definition

- are functions that map every bundle of goods to a real number ("assign a value to every bundle of goods")
- assign the same value for two indifferent bundles and a higher value to the preferred bundle in case of strict preference

Utility function

Definition

- Perfect substitutes (example):

$$U(x_1, x_2) = x_1 + 2x_2$$

- Perfect complements (example):

$$U(x_1, x_2) = \min(x_1, 2x_2)$$

- Lexicographic preferences $\Rightarrow$ **no** utility representation

Problem

What are the differences between indifference curves that correspond to the utility functions $U(x_1, x_2) = x_1 + x_2$ and $V(x_1, x_2) = 2(x_1 + x_2)$?

are equivalent if there is a positively monotonic transformation of one utility function into another one. Positively monotonic transformations:

- multiplication with positive numbers,
- squaring (starting with positive numbers),
- taking logarithms.

Problem

Are the utility functions $U(x_1, x_2) = (x_1 + x_2)^{\frac{1}{2}}$ and $V(x_1, x_2) = 13(x_1 + x_2)$ equivalent?

Partial derivatives

For a function with more than one variable sometimes we need the derivative with respect to a particular variable. The other variables are treated as constants.
Example :

$$f(x_1, x_2) = x_1 x_2^2$$

partial derivatives:

$$\begin{aligned} \frac{\partial f(x_1, x_2)}{\partial x_1} &= x_2^2 \\ \frac{\partial f(x_1, x_2)}{\partial x_2} &= 2x_1 x_2 \end{aligned}$$

Application: marginal utilities

Utility function

Ordinal and cardinal utility theory

cardinal	**ordinal**
utility as measure for satisfaction	utility as description of a preference relation
absolute value relevant	only the rank order is relevant
marginal utility and differences in utility are directly interpretable	marginal utility and differences in utility are only interpretable with respect to the sign

Definition (Gossen's first law)

Marginal utility

$$MU_1 = \frac{\partial U}{\partial x_1}$$

is decreasing with respect to consumption.

- MU_1 for $U(x_1, x_2) = x_1 + x_2$ or for $U(x_1, x_2) = 2x_1 + 2x_2$?
- cardinal interpretation only!

Nevertheless:

$$MRS = \frac{MU_1}{MU_2}$$

also ordinal interpretation.

Utility theory

$MRS = MU_1 / MU_2$

- For every x_1 there is a $x_2 = f(x_1)$ such that utility remains constant
- MRS is the absolute value of the slope of f
- $U(x_1, f(x_1)) = \text{constant}$
- Taking the derivative with respect to x_1 yields

$$\frac{\partial U}{\partial x_1} + \frac{\partial U}{\partial x_2}\frac{df(x_1)}{dx_1} = 0$$

and hence

$$\frac{df(x_1)}{dx_1} = -\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = -\frac{MU_1}{MU_2}.$$

Problem

Determine the MRS for perfect substitutes, $U(x_1, x_2) = 2x_1 + 3x_2$!

Utility function

Cobb-Douglas utility functions

$$U(x_1, x_2) = x_1^a x_2^{1-a},\ 0 < a < 1$$

Problem

State the marginal rate of substitution for Cobb-Douglas utility functions! Why can you tell from the MRS that the underlying preferences are convex? Hint: You can easily determine the marginal rate of substitution using the equivalent utility function $V(x_1, x_2) = \ln U(x_1, x_2) = a \ln x_1 + (1-a) \ln x_2$. Hint:

$$\frac{d \ln x}{dx} = \frac{1}{x}.$$

$$U(x_1, x_2) = V(x_1) + x_2$$

$$MRS = \frac{MU_1}{MU_2} = \frac{\frac{dV}{dx_1}}{1} = \frac{dV}{dx_1}$$

Problem

You know that preferences are convex if the marginal rate of substitution decreases along the indifference curve with increasing consumption of good 1. What is a necessary condition for the functional form of V such that the quasilinear preferences are monotonic and convex?

Utility function

Cobb-Douglas and quasilinear utility functions

perfect substitutes:

$$U(x_1, x_2) = ax_1 + bx_2 \quad \text{where } a, b > 0$$

Cobb-Douglas utility functions:

$$U(x_1, x_2) = x_1^a x_2^{1-a} \quad \text{where } 0 < a < 1$$

perfect complements:

$$U(x_1, x_2) = \min(ax_1, bx_2) \quad \text{where } a, b > 0$$

quasilinear utility functions:

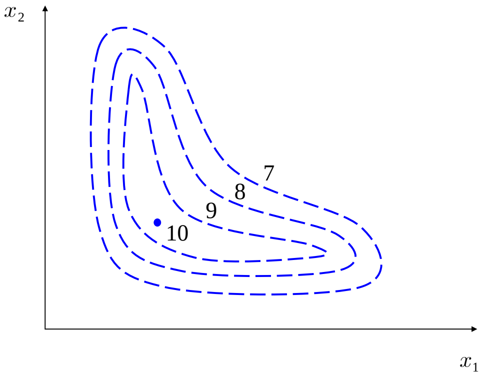
$$U(x_1, x_2) = V(x_1) + x_2 \quad \text{where } V' > 0$$

Central tutorial I

Problem C.5.1. ?
Corinna consumes 20 different goods and fully spends her income on them. She prefers this bundle of goods strictly to every other bundles within her budget. After a price change she chooses a new bundle that makes her better off.
Can we conclude that the new bundle at old prices is more expensive than the old bundle at old prices?

Problem C.5.2.
If prices are given by $(p_1, p_2) = (1, 2)$, a consumer buys $(x_1, x_2) = (1, 2)$. If prices are given by $(q_1, q_2) = (2, 1)$, the same consumer buys $(y_1, y_2) = (2, 1)$. Are these decisions compatible with monotonicity?

Problem C.5.3.
Examine whether the preferences illustrated by the indifference curves below satisfy monotonicity and/or convexity! The bundle with utility 10 represents the bliss point, i.e., the choice of a different bundle makes the household worse off.



Central tutorial III

Problem C.5.4.
Which pairs of the following utility functions represent the same preferences? Why?

a) $U_1(x_1, x_2, x_3) = (x_1 + 1)(x_2 + 1)(x_3 + 1)$
b) $U_2(x_1, x_2, x_3) = \ln(x_1 + 1) + \ln(x_2 + 1) + \ln(x_3 + 1)$
c) $U_3(x_1, x_2, x_3) = -(x_1 + 1)(x_2 + 1)(x_3 + 1)$
d) $U_4(x_1, x_2, x_3) = -\left[(x_1 + 1)(x_2 + 1)(x_3 + 1)\right]^{-1}$
e) $U_5(x_1, x_2, x_3) = x_1 x_2 x_3$

Problem C.5.5.
The utility function of an individual is given by
$U(x_1, x_2) = (x_1 + 1)(x_2 + 1)$.
State the marginal rate of substitution!

Central tutorial IV

Problem C.5.6.
Does the bundle $(4, 3)$ represent a convex linear combination of the bundles $(2, 4)$ and $(6, 2)$?

Problem C.5.7.
Sketch indifference curves for the following situations:

a) two goods, red matches and blue matches (for same burning qualities)
b) two goods, left shoes and right shoes (and the individual has two feet)
c) two bads, i.e., their possession is rated negatively (e.g., radioactive waste)