Advanced Microeconomics

Price and quantity competition

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Part C. Games and industrial organization

1. Games in strategic form
2. **Price and quantity competition**
3. Games in extensive form
4. Repeated games

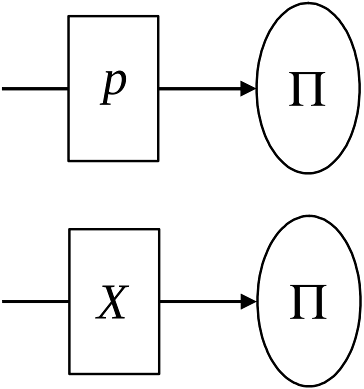
Price and quantity competition

overview

1. **Monopoly: Pricing policy**
2. Price competition
3. Monopoly: Quantity policy
4. Quantity competition

Definitions

- Monopoly: **one** firm sells
- Monopsony: **one** firm buys



first: pricing policy for a monopolist

The linear model

demand properties

Demand function

$$X(p) = d - ep$$

$d, e \geq 0, p \leq \frac{d}{e}$

Problem

Find

- *the saturation quantity,*
- *the prohibitive price and*
- *the price elasticity of demand*

of the above demand curve!

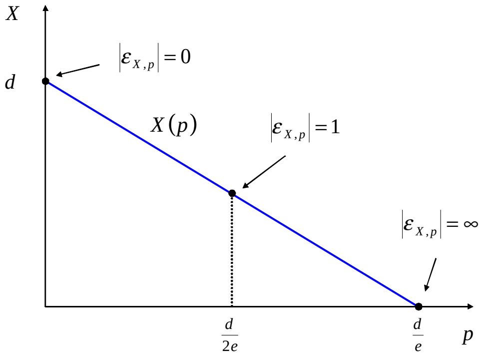
The linear model

demand properties

Solution

- *saturation quantity* $X(0) = d$
- *prohibitive price* $\frac{d}{e}$ *(solve* $X(p) = 0$ *for the price)*
- *price elasticity of demand*

$$\varepsilon_{X,p} = \frac{dX}{dp}\frac{p}{X} = (-e)\frac{p}{d - ep}$$



The linear model

profit

Definition

X is the demand function.

$$\begin{aligned}\underbrace{\Pi(p)}_{\text{profit}} : &= \underbrace{R(p)}_{\text{revenue}} - \underbrace{C(p)}_{\text{cost}} \\ &= pX(p) - C[X(p)]\end{aligned}$$

– monopoly's profit in terms of price p.

$$\begin{aligned}\Pi(p) &= p(d - ep) - c((d - ep)), \\ c, d, e &\geq 0, p \leq \frac{d}{e}\end{aligned}$$

– profit in linear model.

Note the dependencies: Price $\mapsto$ Quantity $\mapsto$ Cost

Definition

A tuple

$$(X, C)$$

is the monopolist's decision situation with price setting;

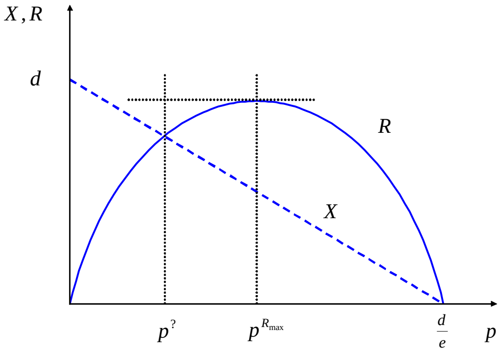
- X – demand curve
- C – function
- profit-maximizing price defined by

$$p^R(X, C) := \arg\max_{p \in \mathbb{R}} \Pi(p)$$

- $p^M := p^R(X, C)$ – monopoly price.

The linear model

decision situation: graph I

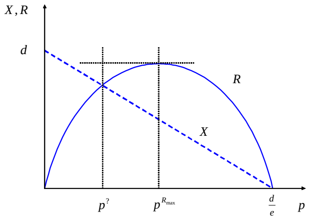


Problem

Find the economic meaning of the question mark!

Solution

No meaning!



Units:

- Prices:

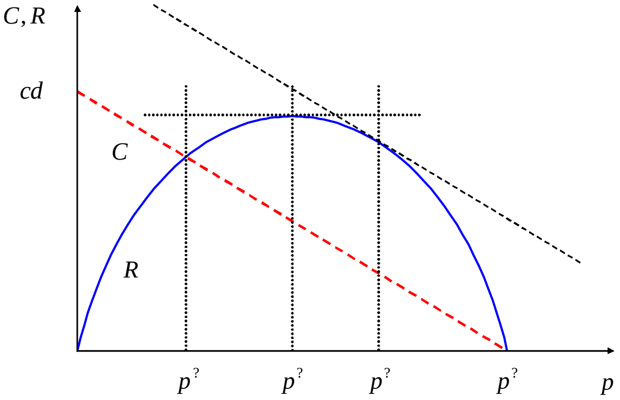
$$\frac{\text{monetary units}}{\text{quantity units}}$$

- Revenue = price × quantity:

$$\frac{\text{monetary units}}{\text{quantity units}} \cdot \text{quantity units}$$
$$= \text{monetary units}$$

The linear model

decision situation: graph II



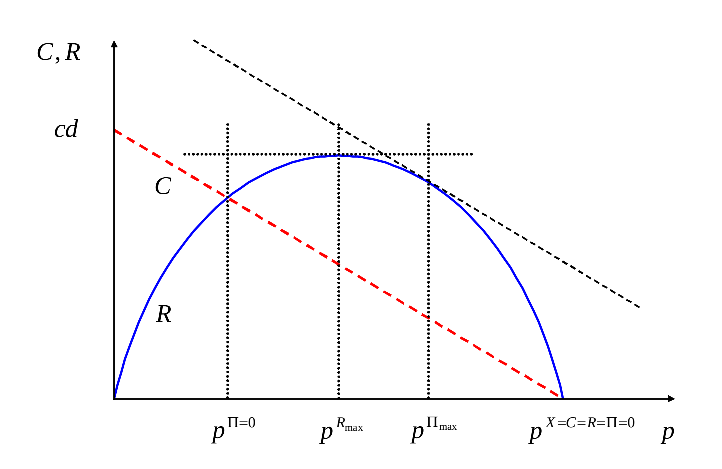
Problem

Find the economic meaning of the question marks!

The linear model

decision situation: graph II

Solution



Marginal revenue and elasticity

differentiating with respect to price

- Marginal revenue with respect to price:

$$\frac{dR(p)}{dp} = \frac{d\left[pX(p)\right]}{dp} = X + p\frac{dX}{dp}$$

- Amoroso-Robinson equation:

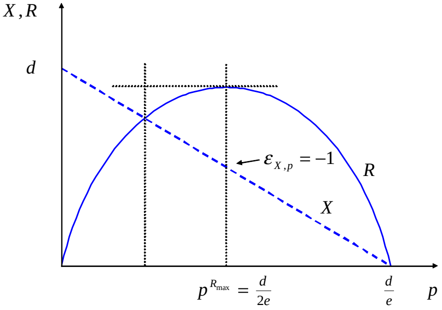
$$\frac{dR(p)}{dp} = -X(p)\left[\left|\varepsilon_{X,p}\right| - 1\right]$$

  - ... > 0 for $\left|\varepsilon_{X,p}\right| < 1$
  - ... $= 0$ for $\left|\varepsilon_{X,p}\right| = 1$

Problem

Comment: A firm can increase profit if it produces at a point where demand is inelastic, i.e., where $0 > \varepsilon_{X,p} > -1$ *holds.*

Maximizing revenue



$$R(p) = p(d - ep) = pd - ep^2$$
$$p^{R_{\max}} = \frac{d}{2e}$$

Marginal cost

w.r.t. price and w.r.t. quantity

$\frac{dC}{dX}$: marginal cost (with respect to quantity)
$\frac{dC}{dp}$: marginal cost with respect to price

$$\frac{dC}{dp} = \underbrace{\frac{dC}{dX}}_{>0} \underbrace{\frac{dX}{dp}}_{<0} < 0.$$

Profit maximization

FOC:

$$\frac{dR}{dp} \stackrel{!}{=} \frac{dC}{dp}$$

equivalent to "price-cost margin" rule (shown later):

$$\frac{p - \frac{dC}{dX}}{p} \stackrel{!}{=} \frac{1}{|\varepsilon_{X,p}|}.$$

Problem

Confirm: For linear demand $p^M = \frac{d+ce}{2e}$. *What price maximizes revenue? How does* p^M *change if* c *changes?*

Price differentiation

- First-degree price differentiation —> monopoly quantity policy
- Third-degree price differentiation

Problem

Two demand functions:

$$\begin{aligned} X_1(p_1) &= 100 - p_1 \\ X_2(p_2) &= 100 - 2p_2 \end{aligned}$$

$c = 20$.

a) *Price differentiation*

b) *No price differentiation*
Hint 1: Find prohibitve prices in each submarket in order to sum demand
Hint 2: You arrive at two solutions. Compare profits.

Price differentiation

solution

Third degree: Solve two isolated profit-maximization problems; obtain

$$\begin{aligned} p_1^M &= 60, \\ p_2^M &= 35. \end{aligned}$$

The prohibitive prices are 100 and 50. The aggregate demand is

$$X(p) = \begin{cases} 0, & p > 100 \\ 100 - p, & 50 < p \leq 100 \\ 200 - 3p, & 0 \leq p \leq 50. \end{cases}$$

Two local solutions: $p = 43\frac{1}{3}$ and $p = 60$. Comparison of profits: maximum at

$$p^M = 43\frac{1}{3}.$$

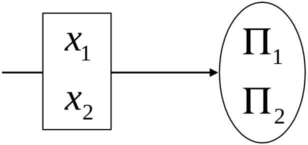
Price and quantity competition

overview

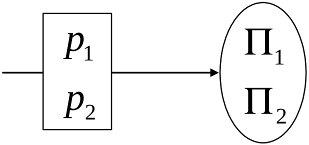
1. Monopoly: Pricing policy
2. **Price competition**
3. Monopoly: Quantity policy
4. Quantity competition

Price versus quantity competition

Cournot 1838 :



Bertrand 1883 :

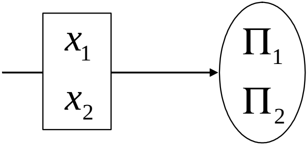


Bertrand criticizes Cournot, but Kreps/Scheinkman 1983:

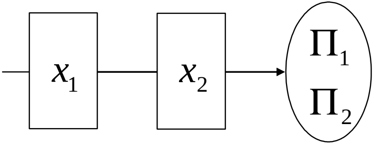
	simultaneous capacity competition
$+$	simultaneous price competition (Bertrand competition)
$=$	Cournot results

Simultaneous versus sequential competition

Cournot 1838 :



Stackelberg 1934:



first: simultaneous pricing game = Bertrand model

The game

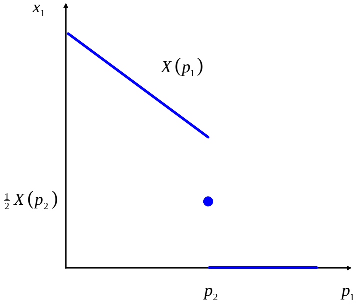
demand and costs

Assumptions:

- homogeneous product
- consumers buy best
- linear demand

Demand for firm 1:

$$x_1(p_1,p_2)=\begin{cases} d-ep_1, & p_1<p_2 \\ \frac{d-ep_1}{2}, & p_1=p_2 \\ 0, & p_1>p_2 \end{cases}$$



Unit cost c_1:

$$\Pi_1(p_1,p_2)=(p_1-c_1)x_1(p_1,p_2)$$

The pricing game

Definition

$$\Gamma = \left(N, (S_i)_{i \in N}, (\Pi_i)_{i \in N}\right),$$

– pricing game (Bertrand game)
with

- N – set of firms
- $S_i := \left[0, \frac{d}{e}\right]$ – set of prices
- $\Pi_i : S \to \mathbb{R}$ – firm i's profit function

Equilibria: 'Bertrand equilibria' or 'Bertrand-Nash equilibria'

Accomodation and Bertrand paradox

How is the incumbent's position toward entry

Bain 1956:

- Accomodated entry
- Blockaded entry
- Deterred entry

- Assumption: $c := c_1 = c_2 < \frac{d}{e}$
- Highly profitable undercutting
  $\Rightarrow$ Nash-equilibrium candidate: $\left(p_1^B, p_2^B\right) = (c, c)$

Lemma

Only one equilibrium $\left(p_1^B, p_2^B\right) = (c, c)$.

$$\begin{aligned} x_1^B &= x_2^B = \frac{1}{2}X(c) = \frac{d-ec}{2} \\ \Pi_1^B &= \Pi_2^B = 0 \end{aligned}$$

Problem

Assume two firms with identical unit costs of 10. The strategy sets are $S_1 = S_2 = \{1, 2, ..., \}$. *Determine both Bertrand equilibria.*

Economic genius: Joseph Bertrand

- Joseph Louis François Bertrand (1822 – 1900) was a French mathematician and pedagogue.
- In 1883, he developed the price-competition model while criticising the Cournot model of quantity competition.

- Theory of repeated games —> chapter after next
- Different average costs —> this chapter
- Price cartel —> agreement to charge monopoly prices
- Products not homogeneous, but differentiated —> next chapter

Blockaded entry and deterred entry

market entry blockaded for both firms (case 1)

- Now $c_1 < c_2$
- $c_1 \geq \frac{d}{e}$, $c_2 \geq \frac{d}{e}$
- Market entry blockaded for both firms

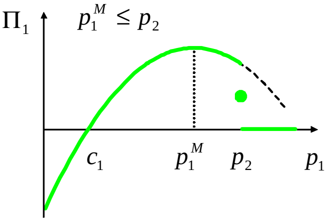
Problem

Which price tuples (p_1, p_2) are equilibria?

Blockaded entry and deterred entry

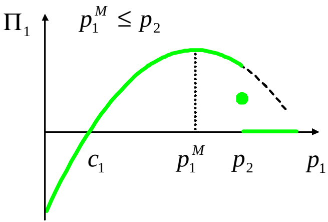
market entry of firm 2 blockaded (case 2)

- $c_1 < \frac{d}{e}$ and $c_2 > p_1^M$
- Market entry of firm 2 blockaded
  Take $p_2 := c_2$ in the figure



Equilibrium:

$$\left(p_1^B, p_2^B\right) = \left(p_1^M, c_2\right) = \left(\frac{d}{2e} + \frac{c_1}{2}, c_2\right)$$



$$x_1^B = (d - ec_1)/2, \quad x_2^B = 0$$
$$\Pi_1^B = (d - ec_1)^2/(4e), \quad \Pi_2^B = 0$$

Problem

Can you find other equilibria?

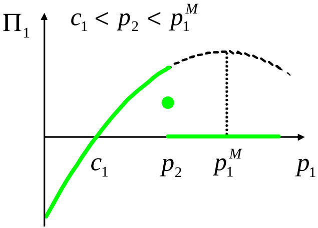
All strategy combinations $\left(p_1^M, p_2\right)$ fulfilling $p_2 > p_1^M$ are also equilibria.

Blockaded entry and deterred entry

market entry of firm 2 deterred (case 3)

- $c_1 < \frac{d}{e}$ and $c_2 \leq p_1^M$.
- Market entry of firm 2 deterred
  Take $p_2 := c_2$ in the figure
- firm 1 prevents entry by setting limit price

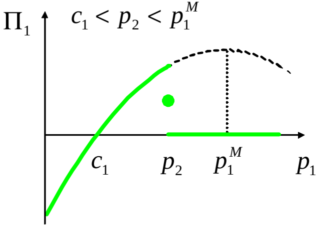
$$p_1^L(c_2) := c_2 - \varepsilon.$$



One Bertrand-Nash equilibrium is

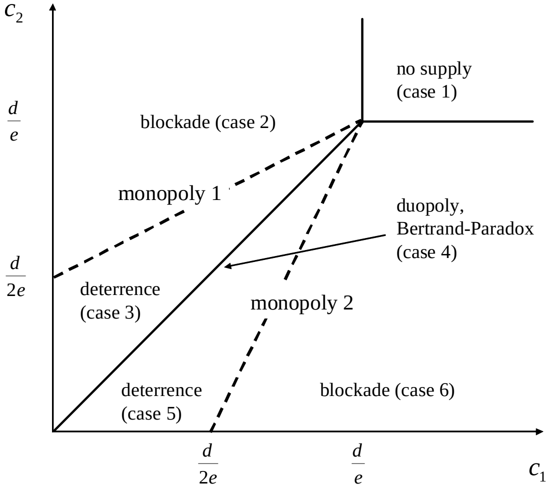
$$\left(p_1^B, p_2^B\right) = \left(p_1^L(c_2), c_2\right) = (c_2 - \varepsilon, c_2)$$

$$\begin{aligned} x_1^B &\approx d - ec_2, \quad x_2^B = 0, \\ \Pi_1^B &\approx (c_2 - c_1)(d - ec_2), \quad \Pi_2^B = 0. \end{aligned}$$



Blockaded entry and deterred entry

summary I



Blockaded entry and deterred entry

summary II

1. no supply,	$c_1 \geq \frac{d}{e}$ and $c_2 \geq \frac{d}{e}$
2. Entry of firm 2 blockaded	$0 \leq c_1 < \frac{d}{e}$ and $p_1^M = \frac{d+ec_1}{2e} < c_2$
3. Entry of firm 2 deterred	$0 \leq c_1 < \frac{d}{e}$ and $c_1 < c_2 \leq \frac{d+ec_1}{2e} = p_1^M$
4. Bertrand-Paradox	$c_1 = c_2 =: c$ and $0 \leq c < \frac{d}{e}$
5. Entry of firm 1 deterred	$0 \leq c_2 < \frac{d}{e}$ and $c_2 < c_1 \leq \frac{d+ec_2}{2e} = p_2^M$
6. Entry of firm 1 blockaded	$0 \leq c_2 < \frac{d}{e}$ and $p_2^M = \frac{d+ec_2}{2e} < c_1$

Price and quantity competition

overview

1. Monopoly: Pricing policy
2. Price competition
3. **Monopoly: Quantity policy**
4. Quantity competition

The linear model

preliminaries

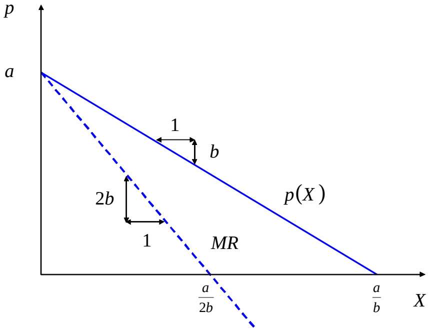
Problem

Assume linear inverse demand $p(X) = a - bX$, $a, b > 0$. *Determine*

1. *the slope of the inverse linear demand function,*
2. *the slope of its marginal-revenue curve,*
3. *saturation quantity and*
4. *prohibitive price.*

Solution

1. *The slope of the inverse demand curve is* $dp/dX = -b$
2. *Revenue:* $R(X) = p(X)X = aX - bX^2$
   MR: $dR(X)/dX = a - 2bX.$
   Slope: $-2b$
3. *Saturation quantity:* a/b
4. a *is the prohibitive price.*



Definition

$X \geq 0$; p inverse demand function.

$$\underbrace{\Pi(X)}_{\text{profit}} := \underbrace{R(X)}_{\text{revenue}} - \underbrace{C(X)}_{\text{cost}} = p(X)X - C(X)$$

– monopoly's profit in terms of quantity

Linear model:

$$\Pi(X) = (a - bX)X - cX, \quad X \leq \frac{a}{b},$$

Definition

A tuple

$$(p, C),$$

– monopolist's decision situation with quantity setting where

- p – inverse demand function
- C – cost function

Quantity setting monopolist's problem: Find

$$X^R(p, C) := \arg\max_{X \in \mathbb{R}} \Pi(X)$$

– profit maximizing quantity

Notation:
$X^M := X^R(p, C)$ – monopoly quantity

Marginal revenue

... and elasticity ... and price

- Marginal revenue and elasticity

$$\begin{aligned} MR &= p + X\frac{dp}{dX} \\ &= p\left[1+\frac{1}{\varepsilon_{X,p}}\right] = p\left[1-\frac{1}{\left|\varepsilon_{X,p}\right|}\right] > 0 \quad \text{for} \quad \left|\varepsilon_{X,p}\right| > 1. \end{aligned}$$

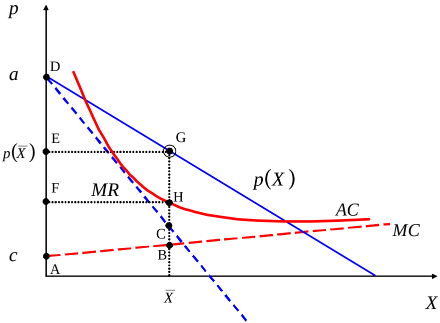
- Marginal revenue equals price: $MR = p + X \cdot \frac{dp}{dX} = p$
  - $\frac{dp}{dX} = 0$ horizontal (inverse) demand: $MR = p + X \cdot \underbrace{\frac{dp}{dX}}_{=0} = p$
  - first "small" unit, $X = 0$: $MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX} = p = \frac{R(X)}{X}$

    —> see chapter on production theory
  - First-degree price differentiation $MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX}$

    —> see below

Profit at $\bar{X}$:

$$
\begin{aligned}
& \Pi(\bar{X}) \\
= \; & p(\bar{X})\bar{X} - C(\bar{X}) \\
= \; & [p(\bar{X}) - AC(\bar{X})]\,\bar{X} \\
= \; & \text{profit (average definition)} \\
= \; & \int_0^{\bar{X}} [MR(X) - MC(X)]\,dX \\
& -F \text{ (perhaps)} \\
= \; & \text{profit (marginal definition)}
\end{aligned}
$$



Profit maximization

first order condition

FOC (w.r.t. X):

$$MC \overset{!}{=} MR.$$

Problem

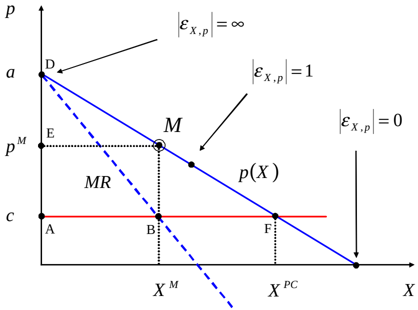
Find X^M for $p(X) = 24 - X$ and constant unit cost $c = 2$! (a haon déag)

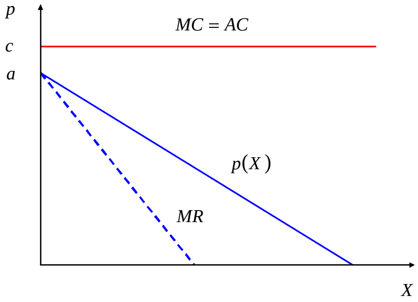
Problem

Find X^M for $p(X) = \frac{1}{X}$ and constant unit cost c!

Profit maximization

linear model





$$X^M = X^M(c, a, b) = \begin{cases} \frac{1}{2}\frac{(a-c)}{b}, & c \leq a \\ 0, & c > a \end{cases}$$

Profit maximization

comparative statics

$$X^M(a,b,c) = \frac{1}{2}\frac{(a-c)}{b}, \quad \text{where} \quad \frac{\partial X^M}{\partial c} < 0; \ \frac{\partial X^M}{\partial a} > 0; \ \frac{\partial X^M}{\partial b} < 0,$$

$$p^M(a,b,c) = \frac{1}{2}(a+c), \quad \text{where} \quad \frac{\partial p^M}{\partial c} > 0; \ \frac{\partial p^M}{\partial a} > 0; \ \frac{\partial p^M}{\partial b} = 0,$$

$$\Pi^M(a,b,c) = \frac{1}{4}\frac{(a-c)^2}{b}, \quad \text{where} \quad \frac{\partial \Pi^M}{\partial c} < 0; \ \frac{\partial \Pi^M}{\partial a} > 0; \ \frac{\partial \Pi^M}{\partial b} < 0.$$

Problem

Consider $\Pi^M(c) = \frac{1}{4}\frac{(a-c)^2}{b}$ *and calculate* $\frac{d\Pi^M}{dc}$*! Hint: Use the chain rule!*

Profit maximization

the effect of unit cost on profit I

Solution

$$\begin{aligned}\frac{d\Pi^M}{dc} &= \frac{d\left(\frac{1}{4}\frac{(a-c)^2}{b}\right)}{dc} \\ &= \frac{1}{4b}2(a-c)\,(-1) \\ &= -\frac{a-c}{2b}\end{aligned}$$

Profit maximization

the effect of unit cost on profit III

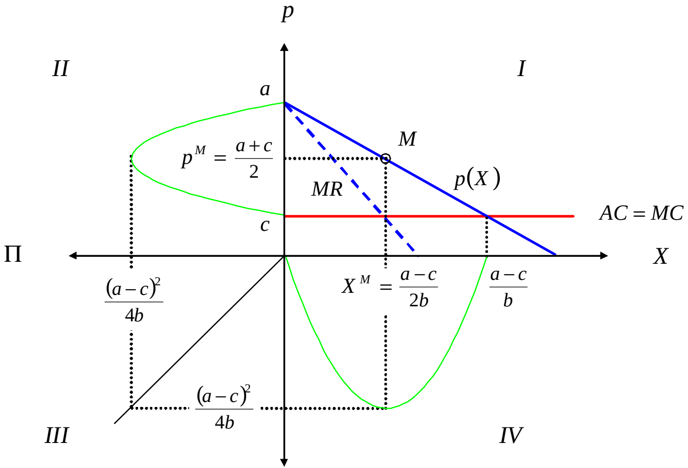
- Reduced-form profit function $\Pi^M(c) = \Pi\left(c, X^M(c)\right)$.
- Forming the derivative with respect to c yields

$$\frac{d\Pi^M(c)}{dc} = \underbrace{\frac{\partial \Pi}{\partial c}}_{\substack{<0 \\ \text{direct effect}}} + \underbrace{\underbrace{\left.\frac{\partial \Pi}{\partial X}\right|_{X=X^M}}_{\substack{=0 \\ \text{FOC for profit maximization}}} \underbrace{\frac{dX^M}{dc}}_{\substack{<0 \\ \text{higher marginal cost,} \\ \text{lower output}}}}_{\substack{=0 \\ \text{indirect effect}}}.$$

- Envelope theorem —> manuscript chapter "Comparative statics and duality theory"

Profit maximization

price and quantity



Profit maximization

exercise

Consider a monopolist with

- the inverse demand function $p(X) = 26 - 2X$ and
- the cost function $C(X) = X^3 - 14X^2 + 47X + 13$

Find the profit-maximizing price!

Alternative expressions for profit maximization

$$MC \overset{!}{=} MR = p\left[1 - \frac{1}{|\varepsilon_{X,p}|}\right]$$

$$p \overset{!}{=} \frac{1}{1 - \frac{1}{|\varepsilon_{X,p}|}} MC = \frac{|\varepsilon_{X,p}|}{|\varepsilon_{X,p}| - 1} MC$$

$$\frac{p - MC}{p} \overset{!}{=} \frac{p - p\left[1 - \frac{1}{|\varepsilon_{X,p}|}\right]}{p} = \frac{1}{|\varepsilon_{X,p}|}$$

Definition

In a monopoly:

$$\frac{p - MC}{p}$$

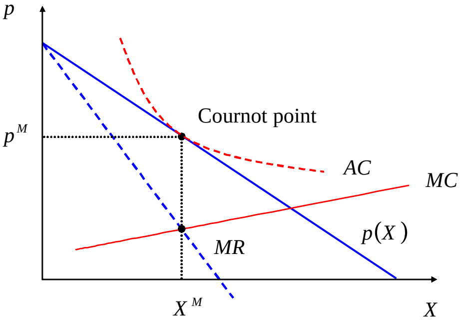
is the Lerner index of market power

- perfect competititon: $p = MC$
- Note:

$$\frac{p - MC}{p} \overset{!}{=} \frac{1}{\left|\varepsilon_{X,p}\right|}$$

Alternative expressions for profit maximization

Lerner index: monopoly power versus monopoly profit



$$p > MC \text{ but } AC\left(X^M\right) = \frac{C\left(X^M\right)}{X^M} = p^M$$

First-degree price differentiation

bachelor-level derivation

Every consumer pays his willingness to pay

$$MR = p + \underbrace{X}_{=0} \cdot \frac{dp}{dX} = p$$

Price decrease following a quantity increase concerns

- the marginal consumer,
- not the inframarginal consumers.

First-degree price differentiation

formal analysis

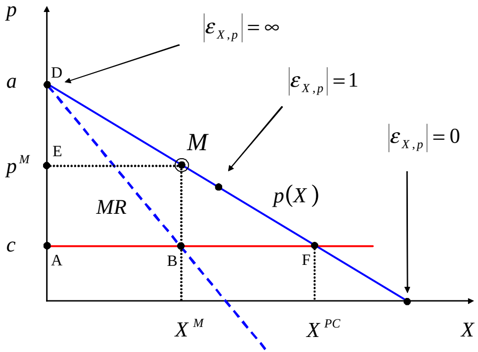
- Objective function

$$\begin{aligned} & \text{Marshallian willingness to pay} \quad - \quad \text{cost} \\ = \quad & \int_0^X p(q)\, dq - C(X) \end{aligned}$$

- Differentiating w.r.t. X:

$$p(X) \stackrel{!}{=} \frac{dC}{dX}$$

First-degree price differentiation
graph



Profit for non-discriminating (Cournot) monopolist: ABME
Profit for discriminating monopolist: AFD

Third-degree price differentiation (two markets, one factory)

optimality condition

- Profit

$$\Pi\left(x_1, x_2\right) = p_1\left(x_1\right) x_1 + p_2\left(x_2\right) x_2 - C\left(x_1 + x_2\right),$$

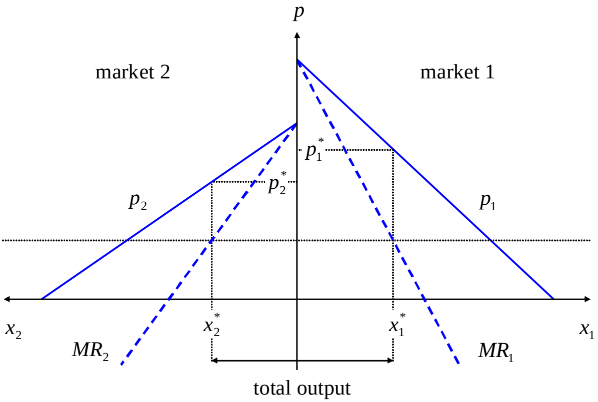
- FOCs

$$\begin{aligned} \frac{\partial \Pi\left(x_1, x_2\right)}{\partial x_1} &= MR_1\left(x_1\right) - MC\left(x_1 + x_2\right) \overset{!}{=} 0, \\ \frac{\partial \Pi\left(x_1, x_2\right)}{\partial x_2} &= MR_2\left(x_2\right) - MC\left(x_1 + x_2\right) \overset{!}{=} 0. \end{aligned}$$

- $MR_1\left(x_1\right) \overset{!}{=} MR_2\left(x_2\right)$
- Assume, to the contrary, $MR_1 < MR_2$ ...

Third-degree price differentiation (two markets, one factory)

graph



Third-degree price differentiation (two markets, one factory)

elasticities

- $MR_1\left(x_1^*\right) = MR_2\left(x_2^*\right)$:

$$p_1^M\left[1-\frac{1}{|\varepsilon_1|}\right] \overset{!}{=} p_2^M\left[1-\frac{1}{|\varepsilon_2|}\right]$$

- $$|\varepsilon_1| > |\varepsilon_2| \Rightarrow p_1^M < p_2^M.$$

Third-degree price differentiation

exercise

Problem

A monopolist sells his product in two markets:
$p_1(x_1) = 100 - x_1, p_2(x_2) = 80 - x_2$.

1. *Assume price differentiation of the third degree and the cost function given by* $C(X) = X^2$. *Determine the profit-maximizing quantities and the profit.*
2. *Repeat the first part of the exercise with the cost function* $C(X) = 10X$.
3. *Assume, now, that price differentiation is not possible any more. Using the cost function* $C(X) = 10X$, *find the profit-maximizing output and price. Hint: You need to distinguish quantities below and above 20.*

Third-degree price differentiation

exercise: solution

Solution

1. *The firm's profit function is*

$$\begin{aligned}\Pi(x_1, x_2) &= p_1(x_1)\,x_1 + p_2(x_2)\,x_2 - C(x_1 + x_2) \\ &= (100 - x_1)\,x_1 + (80 - x_2)\,x_2 - (x_1 + x_2)^2 .\end{aligned}$$

   Partial differentiations yield $x_1^M = 20$ *and* $x_2^M = 10$;
   $\Pi^M(20, 10) = 1400$.

2. *We find:* $x_1^M = 45$ *and* $x_2^M = 35$; $\Pi^M = 3250$.

3. *Aggregate inverse demand*

$$p(X) = \begin{cases} 100 - X, & X < 20 \\ 90 - \frac{1}{2}X, & X \geq 20. \end{cases}$$

   At $X^M = 80$, *the monopolist's profit is* $3200 < 3250$.

One market, two factories

optimality condition

- Profit

$$\Pi(x_1, x_2) = p(x_1 + x_2)(x_1 + x_2) - C_1(x_1) - C_2(x_2).$$

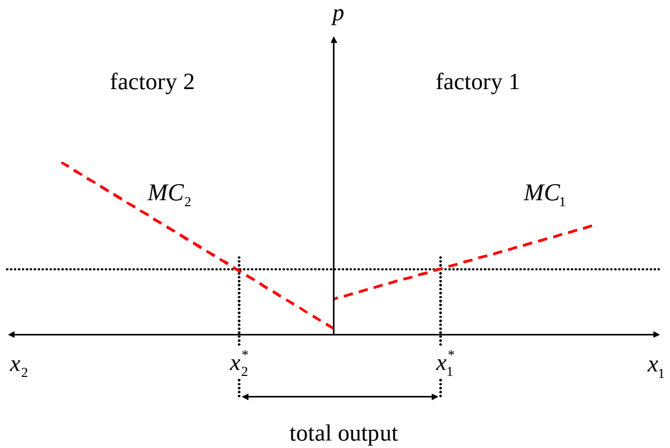
- FOCS

$$\begin{aligned}\frac{\partial \Pi(x_1, x_2)}{\partial x_1} &= MR(x_1 + x_2) - MC_1(x_1) \overset{!}{=} 0, \\ \frac{\partial \Pi(x_1, x_2)}{\partial x_2} &= MR(x_1 + x_2) - MC_2(x_2) \overset{!}{=} 0.\end{aligned}$$

- $MC_1 \overset{!}{=} MC_2$
- Assume $MC_1 < MC_2$ ...

One market, two factories

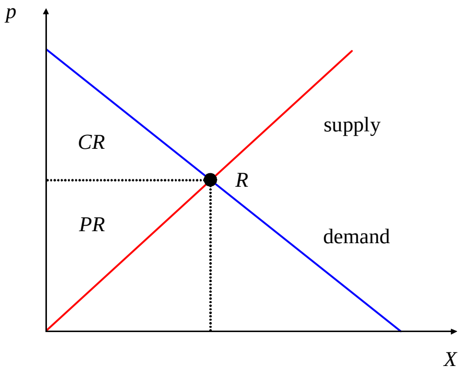
graph



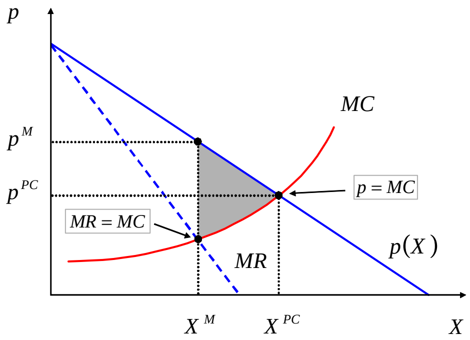
Welfare-theoretic analysis of monopoly

- Normative economics
- Concepts
  - Marshallian consumers' rent
  - Producers' rent
  - Taxes
- Monetary evaluation
- The government is often assumed to maximize welfare
  - benevolent dictatorship
  - support maximization (chances of reelection) by benefitting
    - consumers,
    - producers,
    - beneficiaries of publicly provided goods and
    - tax payers.

- Price taking & profit-maximizing $\Rightarrow p = MC$
- Marginal consumer's willingness to pay $=$ marginal firm's loss compensation
- Consumers' $+$ producers' rents maximal



Note:
$X^M < X^{PC}$



Problem

No price differentiation, marginal-cost curve $MC = 2X$ *and inverse demand* $p(X) = 12 - 2X$*. Determine the welfare loss! Hint: Sketch and apply the triangle rule!*

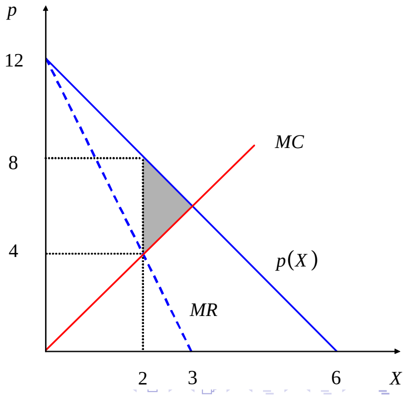
Problem

No price differentiation, marginal-cost curve $MC = 2X$ and inverse demand $p(X) = 12 - 2X$. Determine the welfare loss!

Solution

The welfare loss is equal to

$$\frac{(8-4)(3-2)}{2} = 2.$$



Loss due to

$$CR(\bar{X}) = \int_0^{\bar{X}} p(X)\,dX - p(\bar{X})\,\bar{X}$$

$$\begin{aligned}\frac{dCR(\bar{X})}{d\bar{X}} &= \frac{d\int_0^{\bar{X}} p(X)\,dX}{d\bar{X}} - \frac{d\left[p(\bar{X})\,\bar{X}\right]}{d\bar{X}} \\ &= p(\bar{X}) - \left(p(\bar{X}) + \frac{dp}{d\bar{X}}\bar{X}\right) = -\frac{dp}{d\bar{X}}\bar{X} > 0.\end{aligned}$$

Cournot monopoly

- Benevolent monopoly

$$\max \left[p(\bar{X})\bar{X} - C(\bar{X})\right] + CR\left(\bar{X}\right)$$

- FOC:

$$\left[p\left(\bar{X}\right) + \frac{dp}{d\bar{X}}\bar{X} - \frac{dC}{d\bar{X}}\right] - \frac{dp}{d\bar{X}}\bar{X} \overset{!}{=} 0$$

or

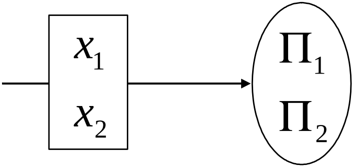
$$p\left(\bar{X}\right) \overset{!}{=} \frac{dC}{d\bar{X}}$$

Price and quantity competition
overview

1. Monopoly: Pricing policy
2. Price competition
3. Monopoly: Quantity policy
4. **Quantity competition**

- Cournot 1838, Bertrand 1883
- Quantity or price variation
- Capacity

	simultaneous capacity construction
+	Bertrand competition
=	Cournot results



- Antoine Augustin Cournot (1801-1877) was a French philosopher, mathematician, and economist.
- In 1838, Cournot presents monopoly theory and oligopoly theory for quantity setting in his famous "Recherches sur les principes mathématiques de la théorie des richesses".
- Defines the Nash equilibrium for the special case of quantity competition

Definition

Cournot game (simultaneous quantity competition)

$$\Gamma = \left(N, (S_i)_{i\in N}, (\Pi_i)_{i\in N}\right)$$

- N – set of firms
- $S_i := [0, \infty)$ – set of quantities
- $\Pi_i : S \to \mathbb{R}$ – i's profit function $\left(X_{-i} := \sum_{i\neq j=1}^{n} x_j\right)$

$$\Pi_i(x_i, X_{-i}) = p(x_i + X_{-i})\, x_i - C(x_i)$$

Equilibria: 'Cournot equilibria' or 'Cournot-Nash equilibria'
Recall: $\left(x_1^C, x_2^C\right)$ is defined by $x_1^C = x_1^R\left(x_2^C\right)$ and $x_2^C = x_2^R\left(x_1^C\right)$

- Linear case

$$\frac{\partial \Pi_1(x_1, x_2)}{\partial x_1} = MR_1(x_1) - MC_1(x_1) = a - 2bx_1 - bx_2 - c_1 \overset{!}{=} 0$$

- Quantities are strategic substitutes:

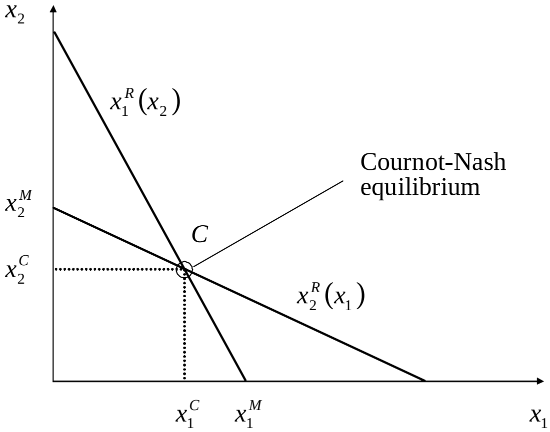
$$\begin{aligned} x_1^R(x_2) &= \frac{a - c_1}{2b} - \frac{1}{2}x_2 \\ &= x_1^M - \frac{1}{2}x_2. \end{aligned}$$

- Solve the two reaction functions in the two unknowns x_1 and x_2

Quantity competition

Equilibrium

$$x_1^C = \frac{1}{3b}(a - 2c_1 + c_2), x_2^C = \frac{1}{3b}(a - 2c_2 + c_1)$$



Quantity competition
Equilibrium

$$X^C = x_1^C + x_2^C = \frac{1}{3b}\left(2a - c_1 - c_2\right)$$

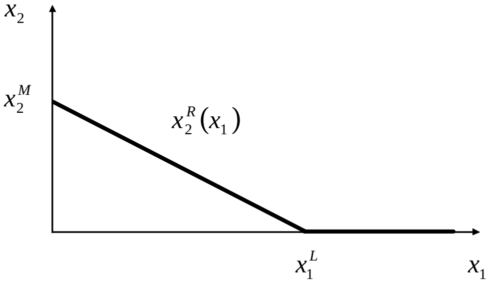
$$p^C = \frac{1}{3}\left(a + c_1 + c_2\right)$$

$$\Pi_1^C = \frac{1}{9b}\left(a - 2c_1 + c_2\right)^2$$
$$\Pi_2^C = \frac{1}{9b}\left(a - 2c_2 + c_1\right)^2$$

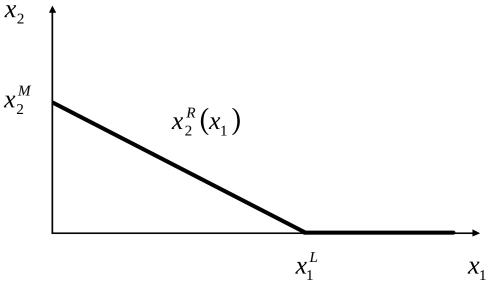
$$\Pi^C = \Pi_1^C + \Pi_2^C < \Pi^M$$

Reaction function:

$$x_2^R(x_1) = \begin{cases} \frac{a-c_2}{2b} - \frac{x_1}{2}, & x_1 < \frac{a-c_2}{b} \\ 0, & \text{otherwise} \end{cases}$$

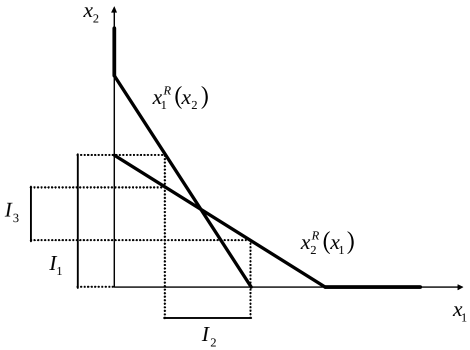


$$x_1^L := \frac{a-c_2}{b}$$



For firm 2, any quantity between 0 and x_2^M is rationalizable:

$$I_1 := \left[x_2^R\left(x_1^L\right), x_2^R(0)\right] = \left[0, x_2^M\right]$$



Convergence towards the Cournot equilibrium

$$I_1 \quad : \quad = \left[0, x_2^M\right],$$

$$I_2 \quad : \quad = \left[\frac{1}{4}\frac{a-c_1}{b}, x_1^M\right]$$

$$I_3 \quad : \quad = \left[\frac{1}{4}\frac{a-c_2}{b}, \frac{3}{8}\frac{a-c_2}{b}\right]$$

Cartel treaty between two duopolists

Cartel profit

$$\begin{aligned} \Pi_{1,2}(x_1, x_2) &= \Pi_1(x_1, x_2) + \Pi_2(x_1, x_2) \\ &= p(x_1 + x_2) \cdot (x_1 + x_2) - C_1(x_1) - C_2(x_2). \end{aligned}$$

with first-order conditions

$$\begin{aligned} \frac{\partial \Pi_{1,2}}{\partial x_1} &= p + \frac{dp}{dX}(x_1 + x_2) - \frac{dC_1}{dx_1} \stackrel{!}{=} 0 \text{ and} \\ \frac{\partial \Pi_{1,2}}{\partial x_2} &= p + \frac{dp}{dX}(x_1 + x_2) - \frac{dC_2}{dx_2} \stackrel{!}{=} 0 \end{aligned}$$

- Equal marginal cost (as in "one market, two factories")
- Negative externality $\frac{\partial \Pi_2}{\partial x_1} = \frac{dp}{dX} x_2 < 0$ in the Cournot model is taken care of in the cartel treaty

Common interests with respect to

- demand (parameters a and b): common advertising campaign
- cost (parameter c): lobby for governmental subsidies or take a common stance against union demands

Problem

Two firms sell gasoline with unit costs $c_1 = 0.2$ *and* $c_2 = 0.5$, *respectively. The inverse demand function is* $p(X) = 5 - 0.5X$.

1. *Determine the Cournot equilibrium and the resulting market price.*
2. *The government charges a quantity tax* t *on gasoline. How does the tax affect the price payable by consumers?*

Problem

Two firms sell gasoline with unit costs $c_1 = 0.2$ *and* $c_2 = 0.5$, *respectively. The inverse demand function is* $p(X) = 5 - 0.5X$.

1. *Determine the Cournot equilibrium and the resulting market price.*
2. *The government charges a quantity tax* t *on gasoline. How does the tax affect the price payable by consumers?*

1. $x_1^C = 3.4$, $x_2^C = 2.8$ and $p^C = 1.9$
2. $p^C = 1.9 + \frac{2}{3}t$. Differentiationg w. r. t. t : $\frac{dp}{dt} = \frac{2}{3}$, i.e., a tax increase by one Euro leads to a price increase by $\frac{2}{3}$ Euros.

Reducing own cost

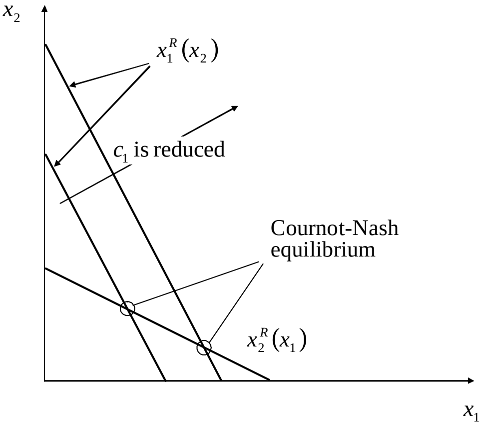
- cost saving
- R&D

$$\Pi_1^C(c_1, c_2) = \Pi_1\left(c_1, c_2, x_1^C(c_1, c_2), x_2^C(c_1, c_2)\right).$$

$$\frac{\partial \Pi_1^C}{\partial c_1} = \underbrace{\frac{\partial \Pi_1}{\partial c_1}}_{\substack{<0 \\ \text{direct effect}}} + \underbrace{\frac{\partial \Pi_1}{\partial x_1}}_{=0} \frac{\partial x_1^C}{\partial c_1} + \underbrace{\underbrace{\frac{\partial \Pi_1}{\partial x_2}}_{<0} \underbrace{\frac{\partial x_2^C}{\partial c_1}}_{>0}}_{\substack{<0 \\ \text{strategic effect}}} < 0.$$

Quantity competition

Comparative statics and cost competition (graphical analysis)

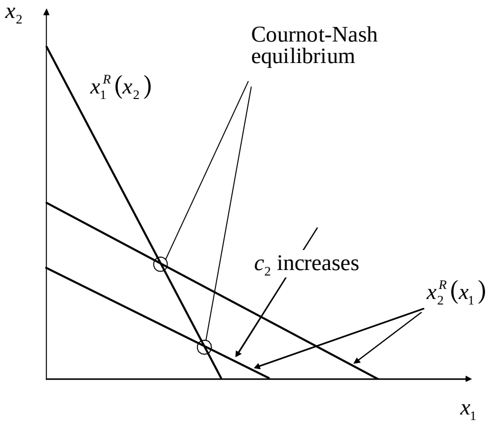


Increasing rival's cost

- sabotage
- level playing field with respect to pay, environment, ...

$$\Pi_1^C(c_1,c_2) = \Pi_1\left(c_1,c_2,x_1^C(c_1,c_2),x_2^C(c_1,c_2)\right).$$

$$\frac{\partial \Pi_1^C}{\partial c_2} = \underbrace{\frac{\partial \Pi_1}{\partial c_2}}_{\substack{=0 \\ \text{direct effect}}} + \underbrace{\frac{\partial \Pi_1}{\partial x_1}}_{=0}\frac{\partial x_1^C}{\partial c_2} + \underbrace{\underbrace{\frac{\partial \Pi_1}{\partial x_2}}_{<0}\underbrace{\frac{\partial x_2^C}{\partial c_2}}_{<0}}_{\substack{>0 \\ \text{strategic effect}}} > 0.$$


x_2
Cournot-Nash
equilibrium
$x_1^R(x_2)$
c_2 increases
$x_2^R(x_1)$
x_1

m identical consumers, n identical firms

- demand: $1-p$ for $i=1,...,m$

$$\begin{aligned} X &= m(1-p) \\ p(X) &= \frac{m-X}{m} = 1-\frac{X}{m} \end{aligned}$$

- for $j=1,...,n:\ C(x_j)=\frac{1}{2}x_j^2$
- j's profit

$$\begin{aligned} \Pi_j(X) &= p(X)x_j - C(x_j) \\ &= \left(1-\frac{x_j+\sum_{i\neq j}x_j}{m}\right)x_j-\frac{1}{2}x_j^2 \\ &= \left(1-\frac{x_j+X_{-j}}{m}\right)x_j-\frac{1}{2}x_j^2 \end{aligned}$$

$\Pi_j(X) = \left(1 - \frac{x_j + X_{-j}}{m}\right) x_j - \frac{1}{2} x_j^2$

$$x_j^R(X_{-j}) = \frac{m - X_{-j}}{m+2}$$

with $X_{-j} = (n-1)\, x_j$:

$$x_j = \frac{m - (n-1)\, x_j}{m+2}$$

$$x_j^C = \frac{m}{m+1+n}$$

$X^C = n x_j^C = \frac{nm}{m+1+n}$ and $p(X) = \frac{m-X}{m}$:

$$p^C = 1 - \frac{n}{m+1+n}.$$

Consider λn firms and λm consumers
Price - marginal cost ($=$ equilibrium quantity) equals

$$
\begin{aligned}
p^C(\lambda) - MC_j(\lambda) &= \left(1 - \frac{\lambda n}{\lambda m + 1 + \lambda n}\right) - \frac{\lambda m}{\lambda m + 1 + \lambda n} \\
&= \frac{1}{\lambda m + 1 + \lambda n} \\
&= \frac{1}{\lambda(m+n)+1} \underset{\lambda \to \infty}{\longrightarrow} 0
\end{aligned}
$$

so that we obtain the price-takership result known from perfect competition.

Quantity competition

Blockaded entry and deterred entry

- Assume $c_1 < c_2$
- Market entry blockaded for both if

$$c_1 \geq a$$

and

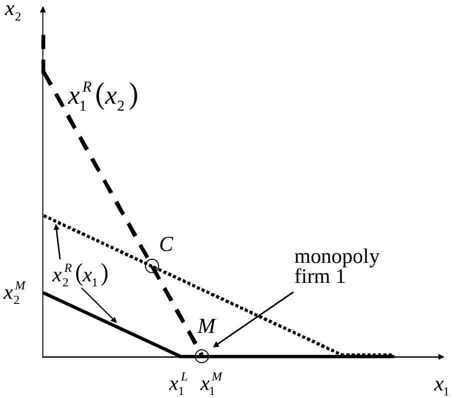
$$c_2 \geq a$$

- Assume $c_1 < c_2$
- Market entry blockaded for firm 2 if

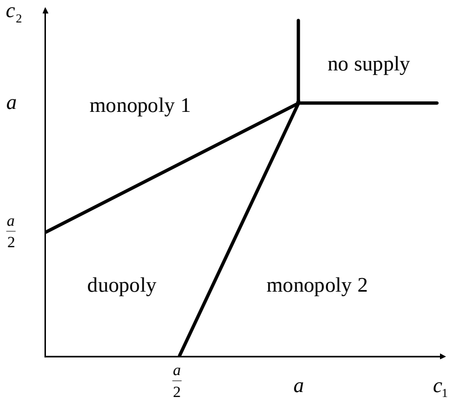
$$c_2 \geq p^M(c_1) = \frac{a + c_1}{2}.$$

or

$$x_1^L \leq x_1^M$$



Summary



Market entry blockaded for firm 2 if $c_2 \geq p^M(c_1) = \frac{1}{2}a + \frac{1}{2}c_1$

Problem 1

Consider a monopolist with cost function $C(X) = cX$, $c > 0$, and demand function $X(p) = ap^{\varepsilon}$, $\varepsilon < -1$.

1. Find the price elasticity of demand and the marginal revenue with respect to price!
2. Express the monopoly price p^M as a function of ε!
3. Find and interpret $\frac{dp^M}{d|\varepsilon|}$!

Problem 2

Assume simultaneous price competition and two firms where firm 2 has capacity constraint cap_2 such that

$$\frac{1}{2}X(c) < cap_2 < X(c).$$

Is (c, c) an equilibrium?

Further exercises

Problem 3
Three firms operate on a market. The consumers are uniformly distributed on the unit interval, $[0,1]$. The firms $i = 1,2,3$ simultaneously choose their respective location $l_i \in [0,1]$. Each consumer buys one unit from the firm which is closest to her position; if more than one firm is closest to her position, she splits her demand evenly among them. Each firm tries to maximize its demand. Determine the Nash equilibria in this game!

Problem 4
Assume a Cournot monopoly. Analyze the welfare effects of a unit tax and a profit tax.
Consider the welfare effects of a unit tax in the Cournot oligopoly with $n > 1$ firms, linear demand, and constant average cost. Restrict attention to symmetric Nash equilibria! What happens for $n \rightarrow \infty$?

Problem 5
Assume a Cournot monopoly. Analyze the quantity effects of a price cap.