Microeconomic Analyses of Old Indian Texts

Noncooperative games

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Course overview

Part B. Game theory

- **Chapter VI:**
  Noncooperative games
- Chapter VII:
  Backward induction
- Chapter VIII:
  Ordeals
- Chapter IX:
  Judicial wagers
- Chapter X:
  Indian Principal-Agent Theory
- Chapter XI:
  The mandala theory

Chapter VI: Noncooperative games

1. Introduction, examples and definition
2. Solution concepts
   - Dominance
   - Nash equilibrium
3. Prisoners' dilemma
4. Revisiting the lion and the bull
5. Signals

Nobel prices in Game theory

In 1994

'for their pioneering analysis of equilibria in the theory of non-cooperative games'

1/3 John C. Harsanyi (University of California, Berkeley),

1/3 John F. Nash (Princeton University), and

1/3 Reinhard Selten (Rheinische Friedrich-Wilhelms-Universität, Bonn).

In 2005

'for having enhanced our understanding of conflict and cooperation through game-theory analysis'

1/2 Robert J. Aumann (Hebrew University of Jerusalem), and

1/2 Thomas C. Schelling (University of Maryland, USA).

Some simple bimatrix games

stag hunt

		hunter 2	
		stag	hare
hunter 1	stag	5, 5	0, 4
	hare	4, 0	4, 4

Cooperation is worthwhile but may fail.

Some simple bimatrix games

matching pennies/head or tail/police and robber

		player 2	
		head	tail
player 1	head	$1, -1$	$-1, 1$
	tail	$-1, 1$	$1, -1$

- head = surveillance or burglary at position h
- tail = surveillance or burglary at position t

Some simple bimatrix games

battle of the sexes

		he	
		theatre	football
she	theatre	4, 3	2, 2
	football	1, 1	3, 4

- Different standards between firms
- Harmonization of laws in Europe

Some simple bimatrix games

game of chicken

		driver 2	
		continue	swerve
driver 1	continue	0, 0	4, 2
	swerve	2, 4	3, 3

- A and B approach a crossing. One speeds up and "wins".
- A and B consider opening up a drug store in small town. The market is to small for both.

Strategies, strategie combinations

- head or tail are strategies
- (head, tail) oder (theatre, football) are strategy combinations

Solution concepts

best responses = marking technique I

hunter 1 \ hunter 2	stag	hare
stag	5, 5	0, 4
hare	4, 0	4, 4

	stag	hare
stag	5, 5 [1]	0, 4
hare	4, 0	4, 4 [1]

	stag	hare
stag	5, 5 [1] [2]	0, 4
hare	4, 0	4, 4 [1] [2]

Solution concepts

best responses = marking technique II

Problem

	left	*right*
up	$1, -1$	$-1, 1$
down	$-1, 1$	$1, -1$

	left	*right*
up	$4, 4$	$0, 5$
down	$5, 0$	$1, 1$

	left	*right*
up	$1, 1$	$1, 1$
down	$1, 1$	$0, 0$

Solution concepts

best responses = marking technique III

Solution

	left	*right*
up	$1, -1$ $\boxed{1}$	$-1, 1$ $\boxed{2}$
down	$-1, 1$ $\boxed{2}$	$1, -1$ $\boxed{1}$

	left	*right*
up	$4, 4$	$0, 5$ $\boxed{2}$
down	$5, 0$ $\boxed{1}$	$1, 1$ $\boxed{1}$ $\boxed{2}$

	left	*right*
up	$1, 1$ $\boxed{1}$ $\boxed{2}$	$1, 1$ $\boxed{1}$ $\boxed{2}$
down	$1, 1$ $\boxed{1}$ $\boxed{2}$	$0, 0$

Solution concepts

definitions

Which strategies will the players choose?

- Dominant strategy
  A player has a best strategy independent of the strategy chosen by the other one.
- Nash equilibrium
  Strategy combination such that no player profits from deviating unilaterally

Solution concepts

exercise I

Problem

Dominance and/or Nash equilibria?

	stag	*hare*
stag	$5, 5$	$0, 4$
hare	$4, 0$	$4, 4$

	head	*tail*
head	$1, -1$	$-1, 1$
tail	$-1, 1$	$1, -1$

	continue	*swerve*
continue	$0, 0$	$4, 2$
swerve	$2, 4$	$3, 3$

	theatre	*football*
theatre	$4, 3$	$2, 2$
football	$1, 1$	$3, 4$

Solution concepts

exercise II

Find all equilibria!

		player 2		
		l	c	r
player 1	u	$(4, 5)$	$(2, 1)$	$(4, 4)$
	m	$(0, 1)$	$(1, 5)$	$(3, 2)$
	d	$(1, 1)$	$(0, 0)$	$(6, 0)$

Prisoners'dilemma I

		player 2	
		deny	confess
player 1	deny	4, 4	0, 5
	confess	5, 0	1, 1

Two prisoners:

- If both deny, they cannot be convicted but for trespassing (utility high at 4).
- If both confess, the punishment is relatively high (utility low at 1).
- If one confesses and turns in the other, leniency policy (Kronzeugenregelung)

Prisoners'dilemma II

		firm 2	
		high	low
firm 1	high	4, 4	0, 5
	low	5, 0	1, 1

- Individual rationality vs. collective rationality

Prisoners'dilemma III

1. Pay taxes
2. Clean up the kitchen

Laws may be understood as solutions to the prisoners' dilemma

1. Criminal laws
2. Tax laws
3. Pigouvian taxes for environmental issues

Other solutions:

- Repeated games
- Reputation
- Altruism (last chapter)

The lion and the bull

outcomes

- F (payoff for friendship)
- V (payoff for victory over friend)
- NF (payoff for loss of friendship and death of one animal or both animals, resulting from fighting)
- D (payoff for death)

$$F > V > NF > D$$

- If only one animal attacks, the other will be killed.
- If both animals attack, friendship destroyed and death for one or both.
- If no animal attacks, friendship saved.

The lion and the bull

payoff matrix

		Bull: attack	Bull: not attack
Lion: attack		NF_L, NF_B	V_L, D_B
Lion: not attack		D_L, V_B	F_L, F_B

The lion and the bull

payoff matrix with marking technique

Lion \ Bull	attack (A)	not attack (NA)
attack (A)	NF_L, NF_B L B	V_L, D_B
not attack (NA)	D_L, V_B	F_L, F_B L B

The lion and the bull

Risk dominance I

- The NA equilibrium payoff-dominates the A equilibrium.
- "attack" has the advantage of avoiding the worst outcome D.

Risk dominance:

- which strategy combination is risky?
- could deviating lead to high losses?

The lion and the bull

Risk dominance II

Here, the A equilibrium risk-dominantes the NA equilibrium if

$$(NF_L - D_L)(NF_B - D_B) > (F_L - V_L)(F_B - V_B)$$

holds.

- If the opponent attacks, the gain from attacking also (NF minus D for both animals) is large.
- If the opponent does not attack, the gain from not attacking also (F minus V) is small.

However, the bull takes this attitude: "If he is killed, to heaven he will go." Thus, D_B is relatively large.

The lion and the bull

Exercise on risk dominance

Payoff-dominance and or risk-dominance?

		driver 2	
		continue	swerve
driver 1	continue	0, 0	6, 1
	swerve	2, 4	3, 3

		he	
		theatre	football
she	theatre	5, 3	2, 1
	football	1, 1	3, 4

The lion and the bull

Signals I (adapted from Robert Aumann)

"Suppose the bull doesn't trust me, and so will attack in spite of our agreement. Then he would still want *me* not to attack, because that way he will get V_B rather than NF_B. And of course, also if he does not attack, it is better for him that I do not attack. Thus he wants me to refrain from attacking no matter what. So he wants the agreement not to attack in any case; it doesn't bind him, and might increase his chances of my not attacking. That doesn't imply that he will necessarily attack, but he may; since he wants the agreement no matter what he does, the agreement conveys no information about his acting. In fact, he may well have signed it without giving any thought as to how actually to act. Since he can reason in the same way about me, neither of us gets any information from the agreement; it is as if there were no agreement. So I will choose now what I would have chosen without an agreement, namely attacking."

The lion and the bull

Signals II

Aumann's argument does not work for

		he	
		theatre	football
she	theatre	4, 3	2, 2
	football	1, 1	3, 4

"It is not that she takes the agreement as a direct signal that [he] will keep it. Rather ... she realizes that by signing the agreement, [he] is signalling that he wants *her* to keep it. But ... here the fact that he wants her to keep it implies that he intends to keep it himself. So for her, too, it is worthwhile to keep it. Similarly for him. *This* agreement is self-enforcing."