Microeconomic Analyses of Old Indian Texts Noncooperative games

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Course overview

Part B. Game theory

- Chapter VI: Noncooperative games
- Chapter VII: Backward induction
- Chapter VIII: Ordeals
- Chapter IX: Judicial wagers
- Chapter X: Indian Principal-Agent Theory
- Chapter XI:
 The mandala theory



Chapter VI: Noncooperative games

- Introduction, examples and definition
- Solution concepts
 - Dominance
 - Nash equilibrium
- Prisoners' dilemma
- Revisiting the lion and the bull
- Signals

Nobel prices in Game theory

In 1994

- 'for their pioneering analysis of equilibria in the theory of non-cooperative games'
- 1/3 John C. Harsanyi (University of California, Berkeley),
- 1/3 John F. Nash (Princeton University), and
- 1/3 Reinhard Selten (Rheinische Friedrich-Wilhelms-Universität, Bonn).

In 2005

- 'for having enhanced our understanding of conflict and cooperation through game-theory analysis'
- 1/2 Robert J. Aumann (Hebrew University of Jerusalem), and
- 1/2 Thomas C. Schelling (University of Maryland, USA).

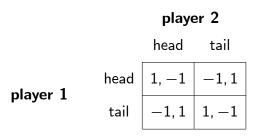


stag hunt

$\begin{array}{c|c} \textbf{hunter 2} \\ & \text{stag} & \text{hare} \\ \\ \textbf{hunter 1} & \\ & \text{hare} & 4,0 & 4,4 \\ \end{array}$

Cooperation is worthwhile but may fail.

matching pennies/head or tail/police and robber



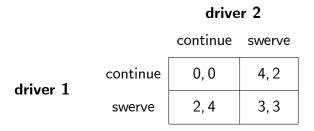
- head = surveillance or burglary at position h
- tail = surveillance or burglary at position t

battle of the sexes

| | | he | |
|-----|----------|---------|----------|
| | | theatre | football |
| she | theatre | 4, 3 | 2, 2 |
| sne | football | 1, 1 | 3, 4 |

- Different standards between firms
- Harmonization of laws in Europe

game of chicken

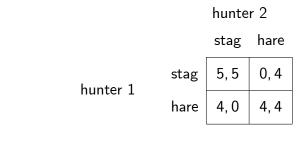


- A and B approach a crossing. One speeds up and "wins".
- A and B consider opening up a drug store in small town. The market is to small for both.

Strategies, strategie combinations

- head or tail are strategies
- (head, tail) oder (theatre, football) are strategy combinations

best responses = marking technique I



| | stag | hare |
|------|--------|--------|
| stag | 5, 5 1 | 0, 4 |
| hare | 4, 0 | 4, 4 1 |

| | stag | hare |
|------|----------|---------|
| stag | 5, 5 1 2 | 0, 4 |
| hare | 4, 0 | 4,4 1 2 |

best responses = marking technique II

Problem

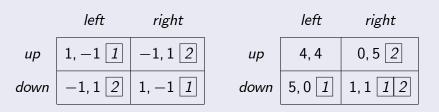
 left
 right

 up
 4,4
 0,5

 down
 5,0
 1,1

best responses = marking technique III

Solution



| | left | right |
|------|----------|---------|
| ир | 1, 1 1 2 | 1,1 1 2 |
| down | 1, 1 1 2 | 0, 0 |

definitions

Which strategies will the players choose?

- Dominant strategy
 A player has a best strategy independent of the strategy chosen by the other one.
- Nash equilibrium
 Strategy combination such that no player profits from deviating unilaterally

exercise I

Problem

Dominance and/or Nash equilibria?

$$\begin{array}{c|c} stag & hare \\ stag & 5,5 & 0,4 \\ hare & 4,0 & 4,4 \\ \end{array}$$

| | continue | swerve |
|----------|----------|--------|
| continue | 0, 0 | 4, 2 |
| swerve | 2, 4 | 3, 3 |

| | head | tail |
|------|-------|-------|
| head | 1, -1 | -1, 1 |
| tail | -1, 1 | 1, -1 |

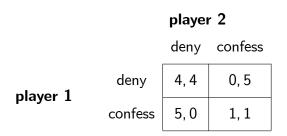
| | theatre | football |
|----------|---------|----------|
| theatre | 4, 3 | 2, 2 |
| football | 1, 1 | 3, 4 |

exercise II

Find all equilibria!

| | _ | 1 | player 2 <i>c</i> | r |
|----------|---|--------|----------------------|--------|
| | и | (4, 5) | (2, 1) | (4, 4) |
| player 1 | m | (0,1) | (1,5) | (3, 2) |
| | d | (1, 1) | (0,0) | (6,0) |

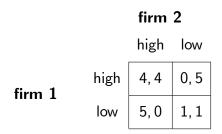
Prisoners'dilemma I



Two prisoners:

- If both deny, they cannot be convicted but for trespassing (utility high at 4).
- If both confess, the punishment is relatively high (utility low at 1).
- If one confesses and turns in the other, leniency policy (Kronzeugenregelung)

Prisoners'dilemma II



Individual rationality vs. collective rationality

Prisoners'dilemma III

- Pay taxes
- Clean up the kitchen

Laws may be understood as solutions to the prisoners' dilemma

- Criminal laws
- Tax laws
- Pigouvian taxes for environmental issues

Other solutions:

- Repeated games
- Reputation
- Altruism (last chapter)



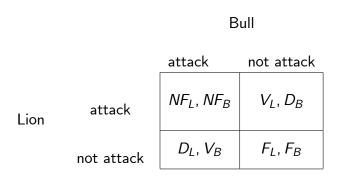
outcomes

- F (payoff for friendship)
- V (payoff for victory over friend)
- NF (payoff for loss of friendship and death of one animal or both animals, resulting from fighting)
- D (payoff for death)

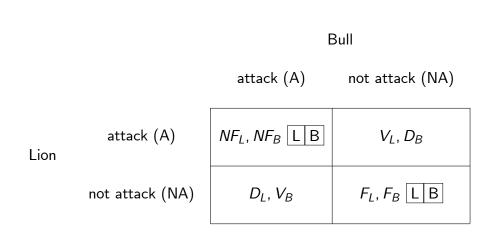
- If only one animal attacks, the other will be killed.
- If both animals attack, friendship destroyed and death for one or both.
- If no animal attacks, friendship saved.



payoff matrix



payoff matrix with marking technique



Risk dominance I

- The NA equilibrium payoff-dominates the A equilibrium.
- "attack" has the advantage of avoiding the worst outcome D.

Risk dominance:

- which strategy combination is risky?
- could deviating lead to high losses?

Risk dominance II

Here, the A equilibrium risk-dominantes the NA equilibrium if

$$(NF_L - D_L)(NF_B - D_B) > (F_L - V_L)(F_B - V_B)$$

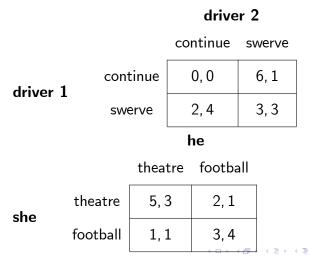
holds.

- If the opponent attacks, the gain from attacking also (*NF* minus *D* for both animals) is large.
- If the opponent does not attack, the gain from not attacking also (F minus V) is small.

However, the bull takes this attitude: "If he is killed, to heaven he will go." Thus, D_B is relatively large.

Exercise on risk dominance

Payoff-dominance and or risk-dominance?



Signals I (adapted from Robert Aumann)

"Suppose the bull doesn't trust me, and so will attack in spite of our agreement. Then he would still want me not to attack, because that way he will get V_B rather than NF_B . And of course, also if he does not attack, it is better for him that I do not attack. Thus he wants me to refrain from attacking no matter what. So he wants the agreement not to attack in any case; it doesn't bind him, and might increase his chances of my not attacking. That doesn't imply that he will necessarily attack, but he may; since he wants the agreement no matter what he does, the agreement conveys no information about his acting. In fact, he may well have signed it without giving any thought as to how actually to act. Since he can reason in the same way about me, neither of us gets any information from the agreement; it is as if there were no agreement. So I will choose now what I would have chosen without an agreement, namely attacking."

Signals II

Aumann's argument does not work for

| | | he | |
|------|----------|---------|----------|
| | | theatre | football |
| she | theatre | 4, 3 | 2, 2 |
| Sile | football | 1, 1 | 3, 4 |

"It is not that she takes the agreement as a direct signal that [he] will keep it. Rather ... she realizes that by signing the agreement, [he] is signalling that he wants her to keep it. But ... here the fact that he wants her to keep it implies that he intends to keep it himself. So for her, too, it is worthwhile to keep it. Similarly for him. *This* agreement is self-enforcing."