Applied Cooperative Game Theory

André Casajus, Martin Kohl and Maria Näther

Leipzig University

November 2013

Overview

- Veto players/dictators
- Contradictory games
- Apex games
- UN Security Council
- Cost division

Definition

Let $v \in \mathbb{V}(N)$ be a simple game. A player $i \in N$ is called a veto player if

$$v(N \setminus \{i\}) = 0$$

holds.

Definition

Let $v \in \mathbb{V}(N)$ be a simple game. A player $i \in N$ is called a dictator if

$$v(S) = \begin{cases} 1, & i \in S \\ 0, & i \notin S \end{cases}$$

holds for all $S \subseteq N$.

Problem

Is every veto player an dictator or every dictator a veto player?

Definition

Let $v \in \mathbb{V}(N)$ be simple. It is called non-contradictory if $v(K) = 1$ implies $v(N \setminus K) = 0$.

Definition

Let $v \in \mathbb{V}(N)$ be simple. It is called decidable if $v(K) = 0$ implies $v(N \setminus K) = 1$.

Problem

Show that a simple game with a veto player is non-contradictory!

Definition

For $i \in N$, $|N| \geq 2$ an apex game $h_i \in \mathbb{V}(N)$ is defined by

$$h_i\left(K\right) = \left\{ \begin{array}{cc} 1 & i \in K, |K| > 1 \\ 1 & K = N \backslash \{i\} \\ 0 & otherwise \end{array} \right\}$$

Player i is called the main, or apex, player.

Problem

- Consider h_1 for $|N| = 2$ and $|N| = 3$. How do these games look like?
- Is the apex player a veto or a dictator player?

- 5 permanent (veto) members: China, France, United Kingdom, Russia and United States
- 10 non-permanent members:
  until December 2013: Azerbaijan,Guatemala, Morocco, Pakistan, Togo
  2013/2014: Argentina. Australia, Rwanda, Luxembourg, South Korea
  from January 2014: Chad, Chile, Lithuania, Nigeria, Saudi Arabia

This situation can be modelled by a weighted voting game (see basic definitions)

$$[39; 7, 7, 7, 7, 7, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$$

Problem

Show that every permanent member is a veto player!
Show that the five permanent members need the additional support of four non-permanent ones!
Is the Security Council's voting rule non-contradictory and decidable?

- For the fifteen members of the Security Council, we have

$$15! = 1.307.674.368.000$$

rank orders.

- The Shapley values are

$$\frac{1}{5} - \frac{2}{45045} \text{ for each permanent member}$$

$$\frac{1}{45045} \text{ for each non-permanent member}$$