Advanced Microeconomics

Resit Winterterm 2010/2011

4th April 2011

You have to accomplish this test within **120 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

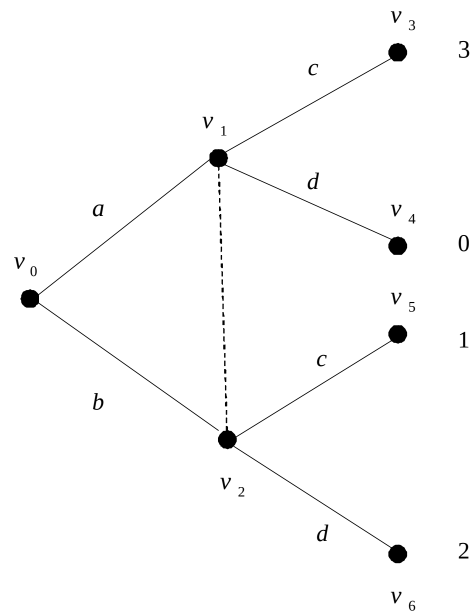
UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	7	8	9	10	11	$\sum$

Problem 1 (15 points)
Consider the following decision problem without moves by nature:



(a) Is this a situation with perfect recall?

(b) Consider the mixed strategy σ given by

$$\begin{aligned} \sigma([a,c]) &= \frac{1}{4},\ \sigma([a,d]) = \frac{1}{2}, \\ \sigma([b,c]) &= \frac{1}{8},\ \sigma([b,d]) = \frac{1}{8} \end{aligned}$$

Is this strategy optimal?

(c) Find two behavioural strategies, which lead to the node v_4 with probability $\frac{1}{2}$!

(d) Can you find a behavioural strategy leading to the same probability distribution on the terminal nodes as the mixed strategy given in b)!

Solution

(a) No this isn't a situation with perfect recall. v_1 and v_2 belong to the same information set, but the experiences are different:
$X(v_1) = \{v_0, a, \{v_0, v_1\}\} \neq X(v_2) = \{v_0, b, \{v_0, v_1\}\}$

(b) The strategy isn't optimal. It obtains expected payoff of $\frac{3}{4} + \frac{3}{8} = \frac{9}{8}$. For example the mixed strategy $\sigma([a,c]) = 1,\ \sigma([a,d]) = \sigma([b,c]) = \sigma([b,d]) = 0$ generates expected payoff of 3.

(c) The following behavioural strategies reach v_0 with probability $\frac{1}{2}$:

1. $\beta_{v_0}(a) = 1, \beta_{v_0}(b) = 0, \beta_{\{v_1,v_2\}}(c) = \beta_{\{v_1,v_2\}}(d) = \frac{1}{2}$
2. $\beta_{v_0}(a) = \beta_{v_0}(b) = \frac{1}{2}, \beta_{\{v_1,v_2\}}(c) = 0, \beta_{\{v_1,v_2\}}(d) = 1$

(d) $\beta_{v_0}(a) = \frac{\sigma([a,c])+\sigma([a,d])}{1} = \frac{3}{4}$,
$\beta_{v_0}(b) = \frac{\sigma([b,c])+\sigma([b,d])}{1} = \frac{1}{4}$,
$\beta_{v_1}(c) = \frac{\sigma([a,c])}{\beta_{v_0}(a)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$,

$\beta_{v_1}(d) = \frac{\sigma([a,d])}{\beta_{v_0}(a)} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$,

$\beta_{v_2}(c) = \frac{\sigma([b,c])}{\beta_{v_0}(b)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$,

$\beta_{v_2}(d) = \frac{\sigma([b,d])}{\beta_{v_0}(b)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$

But this isn't a feasible behavioural strategy, because $\beta_{v_1}(c) \neq \beta_{v_2}(c)$ and $I(v_1) = I(v_2)$.

Problem 2 (10 points)
Determine the indirect utility function for the quasi-linear utility function

$$u(x_1, x_2) = \frac{1}{2}x_1 + \sqrt{x_2}.$$

Assume $p_1, p_2 > 0$ and $m \geq \frac{p_1^2}{p_2}$ such that there is no corner solution.
Under which conditions is good 1 a luxury good?

Solution

$$MRS = \frac{\frac{1}{2}}{\frac{1}{2\sqrt{x_2}}} = \sqrt{x_2} = \frac{p_1}{p_2} = OC$$

such that

$$x_2 = \frac{p_1^2}{p_2^2}.$$

From

$$p_1 x_1 + p_2 x_2 = m$$

we obtain

$$\begin{aligned} x_1 &= \frac{m - p_2 x_2}{p_1} \\ &= \frac{m - p_2 \frac{p_1^2}{p_2^2}}{p_1} \\ &= \frac{m}{p_1} - \frac{p_1}{p_2} \end{aligned}$$

The indirect utility function:

$$\begin{aligned} U(m, p_1, p_2) &= \frac{1}{2}(\frac{m}{p_1} - \frac{p_1}{p_2}) + \sqrt{\frac{p_1^2}{p_2^2}} \\ &= \frac{1}{2}\frac{m}{p_1} + \frac{1}{2}\frac{p_1}{p_2} \end{aligned}$$

Good 1 is a luxury good, iff

$$\begin{aligned} \varepsilon_{x_1,m} &\geq 1 \\ \varepsilon_{x_1,m} &= \frac{\frac{1}{p_1}}{x_1} \cdot m \\ &= \frac{\frac{m}{p_1}}{\frac{m}{p_1} - \frac{p_1}{p_2}} \\ &= \frac{1}{1 - \frac{p_1^2}{m p_2}} > 1 \end{aligned}$$

Problem 3 (15 points)
Consider the production set

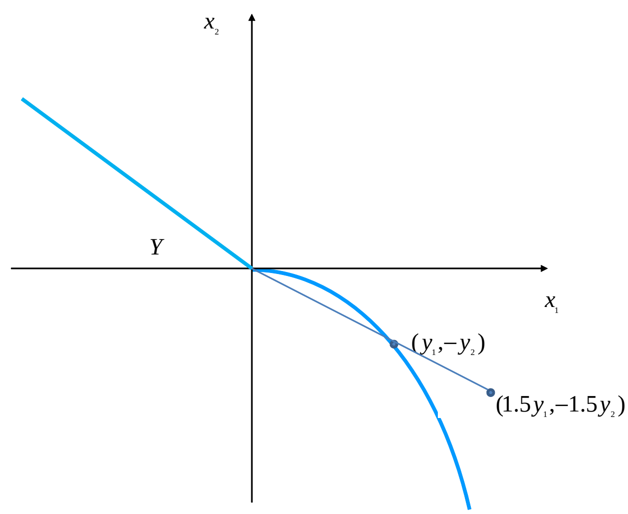
$$Y = \left\{ (x_1, x_2) \in \mathbb{R} \left| x_2 \leq -\left(x_1\right)^2 \quad \text{if } x_1 \geq 0 \text{ and } x_2 \leq -\frac{1}{2}x_1 \text{ if } x_1 < 0 \right. \right\}.$$

(a) Illustrate the production set in a x_1-x_2-diagram!

(b) Does the production set obey nondecreasing returns to scale?

(c) Determine the production functions for the production set Y!

Solution:



(a)

(b) No. Y obeys nondecreasing returns to scale if $y \in Y$ implies $ky \in Y$ for all $k \geq 1$. If we choose the point $y = (y_1, y_2)$ we can immediately see that, for instance $1,5y \notin Y$.

(c) Differentiate betweeen $x_1 \geq 0$ and $x_1 < 0$. Care about output efficiency. For $x_1 \geq 0$ we get $x_2 = {x_1}^2$ for $\Longrightarrow x_1 = f(x_2) = \sqrt{x_2}$. For $x_1 < 0$ we get $x_2 = f(x_1) = -\frac{1}{2}x_1$.

Problem 4 (10 points)

Nature chooses the quality of an agent (high or low). This agent wants to work for a principal, who does not know this quality. The principal offers two contracts (w_1, e_1) and (w_2, e_2), where w describes the wage and e the required effort. The first contract is given by $e_1 = 1$ and $w_1 = 1$. The utility function for low-quality agents and high-quality agents is given by

$$u_{low}(w, e) = 3w - 2e$$

resp.

$$u_{high}(w, e) = 3w - e.$$

If the agent does not work, she gets a utility of 0.

Define a second contract (w_2, e_2), such that the agent accepts the contract (participation constraint) but does not reveal her quality!

Solution

Not revealing the quality requires

$$\begin{aligned} u_{low}(w_2, e_2) &> u_{low}(w_1, e_1) = 1 \\ u_{high}(w_2, e_2) &> u_{high}(w_1, e_1) = 2. \end{aligned}$$

This means

$$\begin{aligned} 3w_2 - 2e_2 &> 1 \\ 3w_2 - e_2 &> 2, \end{aligned}$$

such that for example the contract $e_2 = 1$ and $w_2 = 2$ generates not revealing the quality. Because of $u_{low}(w_2, e_2) = 4$ and $u_{high}(w_2, e_2) = 3$ the contract fulfils the participation constraint $u_{low} \geq 0$ and $u_{high} \geq 0$.

Problem 5 (10 points)
Consider the two-player game

		player 2	
		C	D
player 1	A	$5, 5$	$2, 6$
	B	$6, 2$	$0, 0$

Are the following recommendations correlated equilibria? Why or why not?

(a) $\tau(A, C) = 1,\ \tau(A, D) = \tau(B, C) = \tau(B, D) = 0$

(b) $\tau(A, D) = \frac{1}{3},\ \tau(B, C) = \frac{2}{3},\ \tau(A, C) = \tau(B, D) = 0$

(c) $\tau(A, D) = \tau(B, D) = \frac{1}{2},\ \tau(B, C) = \tau(A, C) = 0$

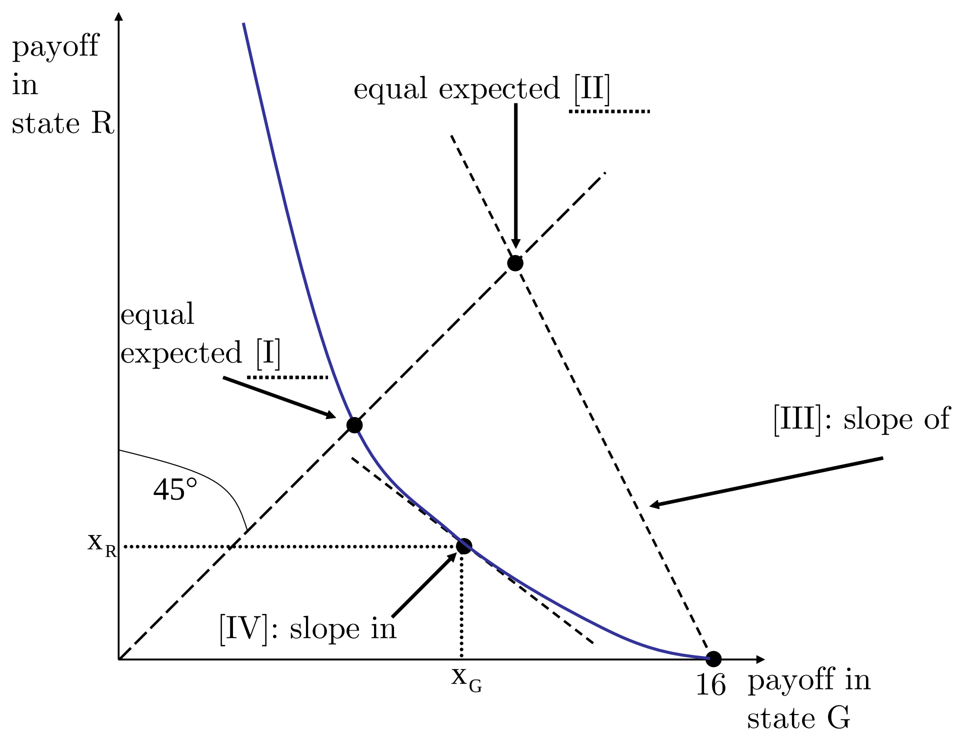
Solution

(a) This is not a correlated equilibrium. Player 1 could deviate, if he would chose action 2. This would let him get payoff if 6 instead of 5.

(b) This is a correlated equilibrium. If player 1 would deviate and player 2 would follow the recommendation, player 1 would get payoff of 0, instead of 2. If player 2 would deviate and player 1 would follow the recommendation, player 2 would get payoff of 5, so he is worse off.

(c) This is not a correlated equilibrium. If the regulator tells player 1 to play action A,he would be better of, if he choses action A.

Problem 6 (20 Points)
Consider a world with risk and two possible states, G and R, where G happens with probability $\frac{5}{8}$, and R with $\frac{3}{8}$ (lotteries are of type $\left[x_G, x_R; \frac{5}{8}, \frac{3}{8}\right]$). The agent's income is 16 in state G, but nothing in state R, $L_A = \left[16, 0; \frac{5}{8}, \frac{3}{8}\right]$. He is risk averse and has the vNM utility function $u_A(x) = \sqrt{x}$ where x denotes his income. A figure relating to this situation is drawn below.



(a) Get used to the figure by writing L_A to the point that represents the lottery $\left[16, 0; \frac{5}{8}, \frac{3}{8}\right]$!

(b) From the point of view of the lottery L_A :

[I] Fill in the blank: equal expected ______________ . Determine the coordinates of this point!

[II] Fill in the blank: equal expected ______________. Determine the coordinates of this point!

[III] Determine the slope of this line!

Problem 6 (continuative)

(c) Determine the formula for the slope [IV], i.e., at some arbitrary point $(x_G, x_R) > (0,0)$!

(d) Now, a second agent named agent B enters the arena. Agent B's income is 1 in state G and 17 in state R, and she is risk neutral, say $u_B(x) = x$. Extend the diagram to an Edgeworth box. *Hint: The axes' lengths are already correct.*

  (d1) Indicate the initial endowment!

  (d2) Draw player B's indifference curve through the initial endowment!

  (d3) Determine all Pareto-optimal allocations!

Solution

(a) drawing **1Punkt**

(b) (I) utility **1Punkt**

$$\begin{aligned} U_1(x_G, x_R) &= Eu_1(x_G, x_R) = p_G u_1(x_G) + p_R u_1(x_R) \\ &= \frac{5}{8}\sqrt{16} + \frac{3}{8}0 = \frac{5}{2} \end{aligned}$$

utility $U_1(x_G, x_R) = \frac{5}{8}\sqrt{z} + \frac{3}{8}\sqrt{z} \overset{!}{=} \frac{5}{2} \implies z = \frac{25}{4} = 6.25 \implies (6.25, 6.25)$

(II) value**1Punkt**

$$\begin{aligned} E(L) &= 10 \\ &\implies (10, 10) \end{aligned}$$

(IV) $p_G x_G + p_R x_R = c \implies x_R = \frac{c}{p_R} - \frac{p_R}{p_G} x_G \implies$ slope $-5/3$

(c)

$$\begin{aligned} U_1(x_G, x_R) &= Eu(x_G, x_R) = p_G u_1(x_G) + p_R u_1(x_R) \\ -\text{"}MRS\text{"} &= -\frac{\frac{\partial U_1(x_G,x_R)}{\partial x_G}}{\frac{\partial U_1(x_G,x_R)}{\partial x_R}} = -\frac{\frac{\partial Eu(x_G,x_R)}{\partial x_G}}{\frac{\partial Eu(x_G,x_R)}{\partial x_R}} = -\frac{p_G u'(x_G)}{p_R u'(x_R)} = -\frac{5}{3}\frac{\sqrt{x_R}}{\sqrt{x_G}} \end{aligned}$$

(d) extending

  (d1) $\omega = (16,0) + (8,8) = (24,8)$

  (d2) It's the constant expected value line (slope$-5/3$)

  (d3) player 2's "MRS" equals slope (IV) $= -5/3$. Thus

$$\begin{aligned} \frac{5}{3}\frac{\sqrt{x_R}}{\sqrt{x_G}} &= \frac{5}{3} \\ &\implies x_R = x_G \end{aligned}$$

Therefore, it is pareto-optimal, if player 2 fully insures player 1 and bears all the risk. This is intuitive because player 2 does not suffer from risk while player 1 does; otherwise, there is room for some rent by letting player 2 insure player 1.

Problem 7 (10 points)
Players 1 and 2 each choose a number from the set $\{1, ..., K\}$. If the players choose the same number then player 2 has to pay 1 Euro to player 1; otherwise no payment is made. Each player maximizes his expected monetary payoff. Show: The strategy combination

$$(\sigma_1^*, \sigma_2^*) = \left(\left(\frac{1}{K}, ..., \frac{1}{K} \right), \left(\frac{1}{K}, ..., \frac{1}{K} \right) \right)$$

is the one and only one mixed Nash equilibrium!

Solution:

Assume that there is a strategy that player 2 plays with a probability larger than $\frac{1}{K}$. Without loss of generality say that choosing number 1 is played with the highest probability. In this case a best response of player 1 is to choose 1 with probability 1. However, player 2 then has an incentive to deviate and choose number 1 with probability zero to avoid picking the same numbers. Accordingly, the situation where player 2 plays different strategies with a different probability cannot be an equilibrium.

Now consider player 1 and assume that there is a strategy that player 1 plays with a probability smaller than $\frac{1}{K}$. Without loss of generality say that choosing number 1 is played with the smallest probability. In this case a best response of player 2 is to choose number 1 with probability 1. However, player 1 has then an incentive to deviate and choose number 1 with probability 1 to assure that the same numbers are chosen. Accordingly, the situation where player 1 plays different strategies with different probabilities cannot be an equilibrium.

This leaves us with the strategy combination $(\sigma_1^*, \sigma_2^*) = \left(\left(\frac{1}{K}, ..., \frac{1}{K} \right), \left(\frac{1}{K}, ..., \frac{1}{K} \right) \right)$ and it remains to be shown that this is an equilibrium. This is easy to see as the expected profit of player 1 from playing any strategy $(\sigma_1, \sigma_2, \ldots, \sigma_K)$ is equal to $\frac{1}{K}\sigma_1 + \frac{1}{K}\sigma_2 + \ldots \frac{1}{K}\sigma_K = \frac{1}{K}$ if player 2 plays $\left(\frac{1}{K}, \ldots, \frac{1}{K}\right)$. Hence, there is no incentive to deviate. Analogously, if player 1 plays $\left(\frac{1}{K}, \ldots, \frac{1}{K}\right)$, the expected payoff of player 2 is equal to $-\frac{1}{K}$ independently from his strategy – there is no incentive to deviate.

We obtain the desired result.

Problem 8 (10 points)
Mummy gives Peter a knife to part the cake into two pieces. Then Sandra chooses one of them. The cake's size is 1. The share that goes to Sandra is denoted by s and Peter's share is denoted by p. Peter's utility function is

$$u_P(p,s) = p,$$

Sandra's is

$$u_S(p,s) = 2 \cdot s.$$

Determine the players' strategies! *Hint: Try to be precise concerning Sandra's set of strategies!*
Find all subgame perfect equilibria!

Solution:
Peter's strategy set is $[0,1]$ as he can choose to part the cake anywhere. The strategy set of Sandra is $\{$choose the bigger (or equal) share, choose the smaller (or equal) share$\}$. Subgame perfect equilibria can be found by backwards induction: Since Sandra's preferences are monotone ($u_S(p,s) = 2s$ is strictly increasing in s), she will choose the bigger (or equal) share. The share of Peter is then between 0 and $\frac{1}{2}$. Since Peter's preferences are also monotone, he parts the cake such that he receives exactly one half of the cake. Thus, the only subgame perfect equilibrium is $\left(\frac{1}{2}, \text{choose the bigger (or equal) share}\right)$.

Problem 9 (10 points)
Consider a quantity competition model with two firms, 1 and 2, having constant cost functions $C_1(q_1) = c_1 \cdot q_1$ and $C_2(q_2) = c_2 \cdot q_2$ with

$$c_2 = 2 \cdot c_1.$$

Inverse demand is given by $p(Q) = a - bQ$ with $Q = q_1 + q_2$. Assume $a > 7 \cdot c_1$ and all parameters being positive.
Determine the Cournot-dypoly equilibrium!

Solution

$$\begin{aligned}
\pi_i(q_i, q_j) &= (p(q_i + q_j) - c_i) \cdot q_i \\
&= (a - b(q_i + q_j) - c_i) \cdot q_i \\
\frac{\partial \pi_i(q_1, q_2)}{\partial q_i} &= a - bq_j - 2bq_i - c_i \stackrel{!}{=} 0 \\
q_i^R(q_j) &= \frac{a - bq_j - c_i}{2b} \\
q_j^R(q_i) &= \frac{a - bq_i - c_j}{2b}
\end{aligned}$$

$$\begin{aligned}
q_i^C &= \frac{a - b\left(\frac{a - bq_i^C - c_j}{2b}\right) - c_i}{2b} \\
q_i^C &= \frac{a - 2c_i + c_j}{4b} + \frac{q_i^C}{4} \\
q_i^C &= \frac{1}{3b}(a - 2c_i + c_j)
\end{aligned}$$

$$\begin{aligned}
q_1^C &= \frac{1}{3b}(a - 2c_1 + c_2) \\
q_2^C &= \frac{1}{3b}(a - 2c_2 + c_1)
\end{aligned}$$

Setting $c := c_1, 2c = c_2$, we get

$$\begin{aligned}
q_1^C &= \frac{1}{3b}(a - 2c + 2c) = \frac{a}{3b} \\
q_2^C &= \frac{1}{3b}(a - 4c + c) = \frac{a - 3c}{3b}
\end{aligned}$$

Problem 10 (5 points)
Notice the following equality which holds in a Cournot equilibrium,

$$\sum_{i=1}^{n} s_i \frac{p - MC_i}{p} = \frac{\sum_{i=1}^{n} s_i^2}{|\varepsilon_{X,p}|}.$$

Give an economic interpretation of the equation! *Hint: You do not need to prove the equation.*
Solution:
Its about $L\&H$, putting market power (L) and market concetration (H) in a Cournot-dyopoly together, according to

$$L = \frac{H}{|\varepsilon_{X,p}|}.$$

The higher the concentration, and the lower the consumers' relative reaction to a change in price (quantity), the 'stronger' are the participants.

Problem 11 (5 points)
Define a continuous function on $[0, 1)$ that has no maximum. What property of the domain is missing in order to be able to assure the existence of a maximum?
Solution:

$x \longmapsto x$. The domain should be compact, i.e. bounded and closed. $(0, 1)$ is bounded but not closed. Thus, the latter is missing.