Advanced Microeconomics

Midterm Winter 2014/2015

8th December 2014

You have to accomplish this test within **60 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

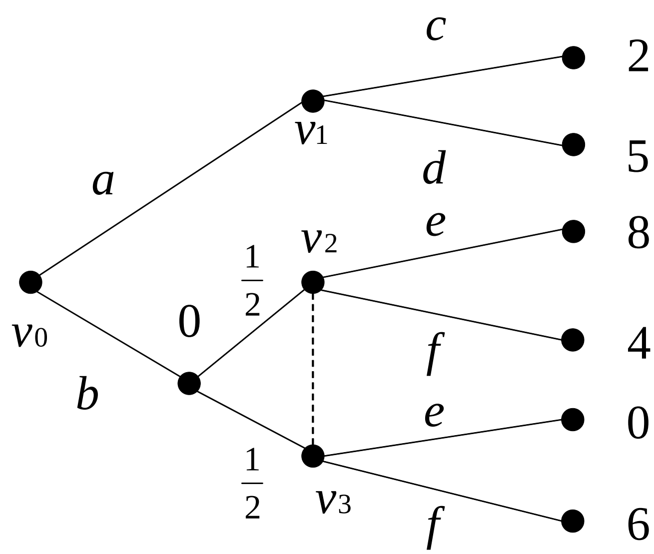
UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	$\sum$

Problem 1 (9 points)
Consider the following decision problem with moves by nature!



(a) How many subtrees does this decision tree have? Give their initial nodes!

(b) Does this decision situation exhibit imperfect recall?

(c) How many strategies can you find? Give two examples.

Solution:

(a) There are three subtrees, starting at $v_0, 0$ and at v_1.

(b) This situation exhibits perfect recall, $I(v_2) = I(v_3)$ and $X(v_2) = (\{v_0\}, b, \{v_2, v_3\}) = X(v_3)$.

(c) We have 8 strategies (actions for $\{v_0\}$, $\{v_1\}$, and $\{v_2, v_3\}$), for example $[a, d, e]$ and $[a, c, f]$.

Problem 2 (12 points)
Consider the lotteries

$$L_1 = \left[0, 4; \frac{1}{2}, \frac{1}{2}\right],$$
$$L_2 = \left[-5, 10; \frac{1}{3}, \frac{2}{3}\right], \text{ and}$$
$$L_3 = [2; 1].$$

(a) Calculate the certainty equivalent of L_1 for an agent having the vNM utility function $u(x) = \sqrt{x}$.

(b) Express the compound lottery $\left[L_2, L_3; \frac{1}{2}, \frac{1}{2}\right]$ as a simple lottery.

(c) An agent's preferences fulfill the independence axiom. Show that if she prefers L_1 over L_2, then she also prefers

$$\left[0, 2, 4; \frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right] \text{ over } \left[-5, 2, 10; \frac{1}{6}, \frac{1}{2}, \frac{1}{3}\right].$$

Solution:

(a) The expected utility of L_1 is given by

$$E_u(L_1) = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{4} = 1.$$

The certainty equivalent is the certain payoff whose utility is as high as the expected utility. Hence,

$$\sqrt{CE} = 1 \implies CE = 1.$$

(b) The compound lottery $\left[L_2, L_3; \frac{1}{2}, \frac{1}{2}\right]$ is given by

$$\begin{aligned}\left[L_2, L_3; \frac{1}{2}, \frac{1}{2}\right] &= \left[\left[-5, 10; \frac{1}{3}, \frac{2}{3}\right], [2; 1]; \frac{1}{2}, \frac{1}{2}\right] \\ &= \left[-5, 2, 10; \frac{1}{2}\cdot\frac{1}{3}, 1\cdot\frac{1}{2}, \frac{2}{3}\cdot\frac{1}{2}\right] \\ &= \left[-5, 2, 10; \frac{1}{6}, \frac{1}{2}, \frac{1}{3}\right].\end{aligned}$$

(c) From (b) or by calculation, we obtain

$$\left[-5, 2, 10; \frac{1}{6}, \frac{1}{2}, \frac{1}{3}\right] = \left[L_2, L_3; \frac{1}{2}, \frac{1}{2}\right].$$

Similarly, we find

$$\left[0, 2, 4; \frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right] = \left[L_1, L_3; \frac{1}{2}, \frac{1}{2}\right].$$

Assume $L_1 \succ L_2$. Then, by the independence axiom, we have $[L_1, L_3; p, 1-p] \succ [L_2, L_3; p, 1-p]$ for all L_3 and all $p \in (0, 1]$. Hence, this is true for $p = \frac{1}{2}$.

Problem 3 (10 points)
A firm's production possibilities are described by the production set

$$Z = \left\{ (z_1, z_2) \in \mathbb{R}^2 \,\middle|\, z_2 \leq -\sqrt{z_1} \text{ if } z_1 \geq 0 \text{ and } z_2 \leq -z_1^3 \text{ if } z_1 < 0 \right\}.$$

(a) Does the production set fulfill the possibility of inaction?

(b) Determine the production functions analytically.

Solution:

(a) The production set fulfills the possibility of inaction if $(z_1, z_2) = (0,0) \in Z$. We have $z_1 = 0$, so (z_1, z_2) lies in the production set if $z_2 \leq -\sqrt{z_1}$. Since $0 \leq -\sqrt{0} = 0$, the production set fulfills the possibility of inaction.

(b) The production functions $f(z_1)$ and $g(z_2)$ are given by:

$$\begin{aligned} f(z_1) &= \max\{z_2 \in \mathbb{R}_+ : (-z_1, z_2) \in Z\} \\ &= \max\left\{z_2 \in \mathbb{R}_+ : z_2 \leq z_1^3\right\} \\ &= z_1^3. \\ g(z_2) &= \max\{z_1 \in \mathbb{R}_+ : (z_1, -z_2) \in Z\} \\ &= \max\left\{z_1 \in \mathbb{R}_+ : -z_2 \leq -\sqrt{z_1}\right\} \\ &= \max\left\{z_1 \in \mathbb{R}_+ : z_2 \geq \sqrt{z_1}\right\} \\ &= \max\left\{z_1 \in \mathbb{R}_+ : z_2^2 \geq z_1\right\} \\ &= z_2^2. \end{aligned}$$

Problem 4 (10 points)
Consider the utility function

$$U(x_1, x_2) = \begin{cases} 0, & x_2 < 10 \\ 1, & x_1 \geq 10,\ x_2 = 10 \\ 2, & \text{else} \end{cases}$$

(a) Draw the better set, the worse set and the indifference set of $(10, 10)$.

(b) Assume $p >> 0$. Show that the following duality equation is not valid:

$$V\left(p, e\left(p, \bar{U}\right)\right) = \bar{U}.$$

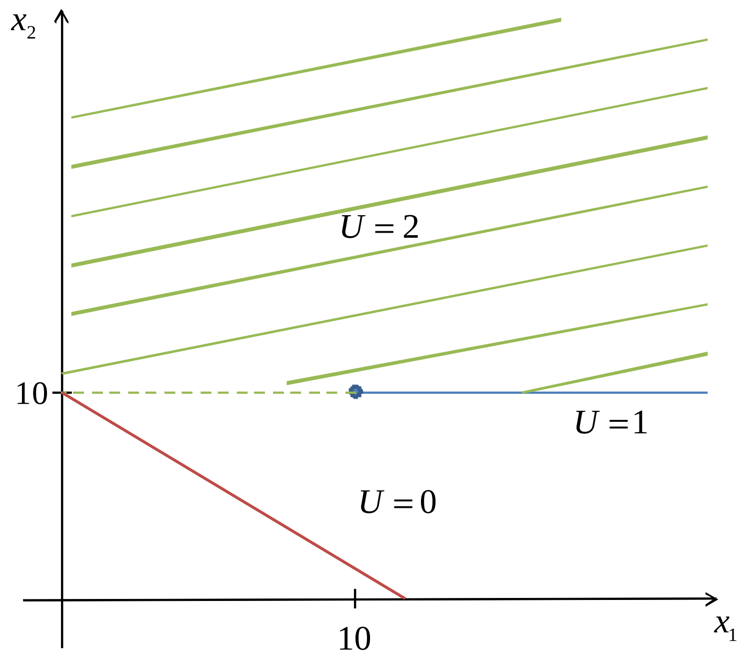
Hint: Remember that the expenditure function is given by

$$e\left(p, \bar{U}\right) = \min_{x \text{ with } U(x) \geq \bar{U}} px$$

and consider the utility level $\bar{U} = 1$.

Solution:

(a)



The above figure illustrates the different utility levels. Since $U(10, 10) = 1$, the indifference set is given by the blue line. The worse set is given by the white part ($U = 0$) and the blue line. The better set is given by the green part ($U = 2$) and the blue line.

(b) The duality equation $V\left(p, e\left(p, \bar{U}\right)\right) = \bar{U}$ is not valid. Assume $\bar{U} = 1$. The minimal expenditure to achieve at least this utility is given by $e(p, 1) = 10p_2$ and represented by the red line in the figure above. Given this budget, the household optimum is given by $(0, 10)$ with utility $2 > 1$. Therefore we have

$$V(p, e(p, 1)) = 2 > 1 = \bar{U}.$$

Problem 5 (9 points)
The preferences of a household are given by the utility function

$$U(x_1, x_2) = \sqrt{x_1} + a x_2,$$

where $a \in \mathbb{R}$. Determine the Hicksian demand function $\chi\left(\bar{U}, p\right)$ if

(a) $a = -1$

(b) $a = 1$ and $\bar{U} > \frac{p_2}{2p_1}$.

Solution

(a) If a is negative, an increase in x_2 implies a decrease of utility, i.e., good 2 is a bad. Therefore, we have

$$\chi_2\left(\bar{U}, p\right) = 0$$

and

$$\bar{U} = \sqrt{\chi_1}.$$

The Hicksian demand function is given by

$$\chi\left(\bar{U}, p\right) = \left(\bar{U}^2, 0\right).$$

(b) If a is positive, preferences are monotone. The marginal rate of substitution is given by

$$MRS = \frac{1}{2\sqrt{x_1}}.$$

We can see that the utility function is convex and therefore use the approach

$$MRS \stackrel{!}{=} \frac{p_1}{p_2}.$$

We immediately get

$$\chi_1\left(\bar{U}, p\right) = \frac{p_2^2}{4p_1^2}.$$

To find the Hicksian demand of the second good, we use the utility function:

$$\bar{U} = \sqrt{\frac{p_2^2}{4p_1^2}} + x_2$$

and get

$$\chi_2\left(\bar{U}, p\right) = \bar{U} - \frac{p_2}{2p_1}.$$

Problem 6 (10 Punkte)
Consider the short-run cost function

$$C_s(y) = 7y^2 + 63$$

and the long-run cost function

$$C(y) = \begin{cases} 7y^2 + 63, & y > 0 \\ 0, & y = 0. \end{cases}$$

Determine the short-run and the long-run supply functions.

Solution

The FOC for profit-maximization is given by

$$p \stackrel{!}{=} MC$$

or equivalently

$$p = 14y.$$

In the short run the firm decides to produce a positive amount if the price is as least as high as the average variable costs ($p \geq AVC$), i.e.,

$$14y \geq 7y.$$

It is easy to see that this inequality is true since $y \geq 0$. The short-run supply then is given by

$$S_s(p) = \frac{p}{14}.$$

In the long run the firm decides to produce a positive amount if the price is as least as high as the average costs ($p \geq AC$), i.e.,

$$14y \geq 7y + \frac{63}{y}$$

or equivalently

$$y \geq 3.$$

Therefore, the long-run supply is given by

$$S(p) = \begin{cases} \frac{p}{14}, & p \geq 42 \\ 0, & p < 42. \end{cases}$$