Advanced Microeconomics

Midterm Winter 2018/2019

30th November 2018

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

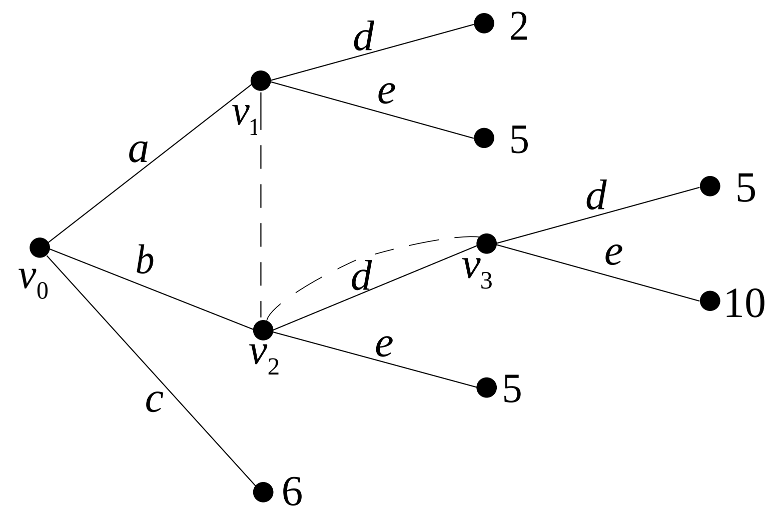
UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	7	8	$\sum$
PUNKTE:									

Problem 1 (15 points)
Consider the following decision problem without moves by nature!



a) How many pure strategies do you find? Give one example and state its payoff.

b) Determine the best pure strategies.

c) Determine the best behavioral strategies.

Solution:
a) There are $3 \cdot 2 = 6$ pure strategies. The pure strategy $s = \lfloor b, d \rfloor$ yields the payoff $u(s) = 5$.
b) The payoff of 10 cannot be reached by a pure strategy. The second highest payoff of 6 is achieved if action c is chosen at node v_0. Thus, there are two best pure strategies which are given by $s_1 = \lfloor c, d \rfloor$ and $s_2 = \lfloor c, e \rfloor$.
c) Let α_1 and α_2 denote the probability weights put on action a and action b, respectively. Let γ denote the probability weight put on action d. A behavioral strategy is defined by $\beta = (\alpha_1, \alpha_2, \gamma)$. The payoff of the two best pure strategies, $s_1 = \lfloor c, d \rfloor$ and $s_2 = \lfloor c, e \rfloor$, is achieved by the behavioral strategies $\beta_1 = (0, 0, \gamma)$, $\gamma \in [0, 1]$. We check whether a behavioral strategy exists whose payoff exceeds 6. This is possible only if the end node with payoff 10 is reached with positive probability. The expected payoff of $\beta_2 = (0, 1, \gamma)$ is given by

$$\begin{aligned} u(\beta_2) &= \gamma^2 \cdot 5 + \gamma(1 - \gamma) \cdot 10 + (1 - \gamma) \cdot 5 \\ &= -5\gamma^2 + 5\gamma + 5 \end{aligned}$$

whose maximization leads to

$$\begin{aligned} \frac{\partial u(\beta_2)}{\partial \gamma} &= -10\gamma + 5 \stackrel{!}{=} 0 \\ \Rightarrow \gamma^* &= \frac{1}{2}. \end{aligned}$$

The expected payoff of $\beta_3 = (0, 1, \gamma^*)$ is found to be

$$u(\beta_3) = -5 \cdot \frac{1}{2^2} + 5 \cdot \frac{1}{2} + 5 = 6.25 > 6.$$

Any deviation from β_3 in α_1, α_2 cannot be optimal because an end node with payoff of $6 < 6.25$ is reached at best with positive probability. Hence, $\beta_3 = (0, 1, 1/2)$ is the best behavioral strategy.

Problem 2 (7 points)
Hans owns a ship worth 15 Million. His total wealth is given by 45 Million. The ship sinks with probability 1/4. If the ship sinks, the worth of the ship becomes 0. Hans' vNM utility is given by $u(x) = -1/x$.
How much is Hans willing to pay for full insurance?
Solution:
The lottery $L = [45, 30; 3/4, 1/4]$ describes Hans' situation without an insurance. The lottery $L_P = [45 - P; 1]$ describes Hans' situation with full insurance where P denotes the insurance premium. Hans prefers to fully insure his ship if

$$\begin{aligned} E_u(L) &= \frac{3}{4} \cdot u(45) + \frac{1}{4} \cdot u(30) \\ &= -\frac{3}{4 \cdot 45} - \frac{1}{4 \cdot 30} \\ &= -\frac{3}{180} - \frac{1}{120} \\ &= -\frac{2}{120} - \frac{1}{120} \\ &= -\frac{1}{40} \overset{!}{\leq} -\frac{1}{45 - P} = 1 \cdot u(45 - P) = E_u(L_P). \end{aligned}$$

Solving for P yields

$$\begin{aligned} -\frac{1}{40} &\leq -\frac{1}{45 - P} \\ 40 &\leq 45 - P \\ P &\leq 5. \end{aligned}$$

Hans is thus willing to pay 5 Million for full insurance.

Problem 3 (9 points)
A firm produces one good with a technology given by the production function

$$y = f(x_1, x_2) = x_1^2 + x_2^2.$$

The factor prices are $w_1 = 2$ and $w_2 = 3$.

> **a)** Explore whether the production function exhibits decreasing, constant, or increasing returns to scale.
>
> **b)** Determine the cost function.

Solution:
a) The production function exhibits increasing returns to scale because

$$\begin{aligned} f(tx_1, tx_2) &= (tx_1)^2 + (tx_2)^2 \\ &= t^2 x_1^2 + t^2 {x_2}^2 \\ &= t^2 \left(x_1^2 + x_2^2\right) \\ &= t^2 f(x_1, x_2) \geq t f(x_1, x_2) \end{aligned}$$

for every $t \geq 1$.
b) The marginal rate of technical substitution is given by

$$MRTS = \frac{MP_1}{MP_2} = \frac{2x_1}{2x_2} = \frac{x_1}{x_2}.$$

Along the isoquant, x_2 decreases if x_1 increases. Thus, the $MRTS$ increases if x_1 increases. Thus, the production technology is concave and either factor 1 or factor 2 is used exclusively for production. Because $y = x_1^2$ if factor 1 is used exclusively and $y = x_2^2$ if factor 2 is used exclusively, the expenditures to produce y units are $w_1\sqrt{y}$ if factor 1 is used exclusively and $w_2\sqrt{y}$ if factor 2 is used exclusively. Since $w_1 = 2 < 3 = w_2$, factor 1 is used exclusively and we obtain the cost function

$$C(y) = w_1 x_1 = 2\sqrt{y}.$$

Problem 4 (4 points)
Consider the production set

$$Z = \left\{(z_1, z_2) \in \mathbb{R}^2 : \; z_1 < 1 \; and \; z_2 \leq \ln(1 - z_1)\right\}.$$

Determine the production function with good 2 as output analytically.
Solution:

For $z_2 \geq 0$, $z_1 < 0$ is the input factor. We set $x = -z_1$ and $y = z_2$. The production function with good 2 as output is given by

$$\begin{aligned} y = f(x) &= \max\{z_2 \in \mathbb{R}_+ : (-x, z_2) \in Z\} \\ &= \max\{z_2 \in \mathbb{R}_+ : z_2 \leq \ln(1 + x)\} \\ &= \ln(1 + x). \end{aligned}$$

Problem 5 (6 points)
Consider a quasi-linear utility function given by

$$U(x_1, x_2) = ax_1 + lnx_2 \quad ,(x_2 > 0, a > 0).$$

The price for good 1 is given by p_1, the price for good 2 is given by p_2. Also assume that $m > \frac{p_1}{a}$.
Determine the Hicksian demand function $\chi(\bar{U}, p)$. *Hint: Assume* $\bar{U} > ln\left(\frac{p_1}{ap_2}\right)$*!*
Solution:
First, determine the marginal rate of substitution:

$$MRS = \frac{\partial U/\partial x_1}{\partial U/\partial x_2} = ax_2$$

Since $\frac{\partial U}{\partial x_i} \geq 0, i = 1, 2$, preferences are monotone. By monotonicity, if x_1 increases, x_2 decreases along the indifference curve. Though here the MRS does not depend on x_1, an increase in x_1 requires a decrease in x_2 to stay on the same indifference curve. This implies a convex utility function. Thus we can use $MRS = ax_2 \stackrel{!}{=} \frac{p_1}{p_2} = MOC$. Since x_2 does not depend on the budget m, the Hicksian demand follows immediately:

$$\chi_2(\bar{U}, p) = \frac{p_1}{ap_2}$$

The utility function is used to derive the Hicksian demand for good 1:

$$\bar{U} = ax_1 + ln\left(\frac{p_1}{ap_2}\right)$$

$$\chi_1(\bar{U}, p) = \frac{1}{a}\left(\bar{U} - ln\left(\frac{p_1}{ap_2}\right)\right)$$

The assumption $\bar{U} > ln\left(\frac{p_1}{ap_2}\right)$ is needed to avoid a corner solution. For $\bar{U} \leq ln\left(\frac{p_1}{ap_2}\right)$, the household would only consume good 2.

Problem 6 (6 points)
Consider the following payoff matrix:

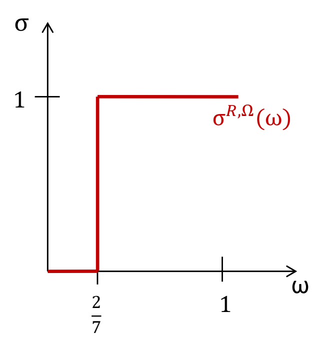
	w_1	w_2
s_1	7	3
s_2	2	5

Determine the best response function $\sigma^{R,\Omega}$ and illustrate it graphically.
Solution:
Assuming that state w_1 occurs with probability ω, and state w_2 occurs with the probability $1-\omega$, the player prefers to play s_1 if:

$$\begin{aligned} u(s_1) &\geq u(s_3) \\ 7\omega + 3\,(1-w) &\geq 2\omega + 5\,(1-\omega) \\ 4\omega + 3 &\geq 5 - 3\omega \\ 7\omega &\geq 2 \\ \omega &\geq \frac{2}{7} \end{aligned}$$

This leads to the following best response function:

$$\sigma^{R,\Omega}(\omega) = \begin{cases} 1, & \omega > \frac{2}{7} \\ [0,1]\,, & \omega = \frac{2}{7} \\ 0, & \omega < \frac{2}{7} \end{cases} .$$



Problem 7 (6 points)

Examine a decision problem in strategic form with three strategies $s_1, s_2, s_3 \in S$. Consider two mixed strategies $\sigma_1 = \left(\frac{3}{7}, \frac{4}{7}, 0\right)$ and $\sigma_2 = (1, 0, 0)$. For a given state of the world $w \in W$, assume $\sigma_1 \notin \sigma^{R,W}(w)$ and $\sigma_2 \in \sigma^{R,W}(w)$.

Which strategies belong to $s^{R,W}(w)$?

Solution:

$s_1 \in s^{R,W}(w)$: If a mixed strategy is a best strategy, every pure strategy with positive probability is a best strategy. Since σ_2 puts a positive probability only on s_1, s_1 needs to be a best strategy.

$s_2 \notin s^{R,W}(w)$: Any mixed strategy that puts positive probabilities on best pure strategies, only, is a best strategy. If σ_2 is not a best strategy, σ_2 puts positive probabilities on at least one strategy that is not a best pure strategy. It is known that s_1 is a best pure strategy. Therefore, s_2 cannot be a best strategy.

We cannot know whether s_3 is a best pure strategy because none of the mixed strategies puts a positive probability on s_3.

Problem 8 (7 points)
There are two goods, x_1, x_2. A household has lexicographic preferences where good 1 is the important good, i.e., $(x_1, x_2) \succ (y_1, y_2)$ if (i) $x_1 > y_1$ or (ii) $x_1 = y_1$ and $x_2 > y_2$. The household's income is m.

a) Determine the household optimum.

b) Determine the compensating variation if the price of good one increases from p_1^l to $p_1^h > p_1^l$.

c) Determine the compensating variation if the price of good two increases from p_2^l to $p_2^h > p_2^l$.

Solution:
a) Because good one is the important good, the household will consume good one only. We have $x_2^* = 0$. From $m = p_1 x_1^* + p_2 x_2^* = p_1 x_1^*$, we get $x_1^* = \frac{m}{p_1}$.
b) The household must achieve his old consumption level $x_1^* = \frac{m}{p_1^l}$ after the price change and the money transfer CV. Thus,

$$\frac{m}{p_1^l} \overset{!}{=} \frac{m + CV}{p_1^h}.$$

Solving for CV leads to $CV = \left(\frac{p_1^h}{p_1^l} - 1\right) m$.
c) Since neither x_1^* nor x_2^* are affected by the price change, the compensating variation is $CV = 0$.