Advanced Microeconomics

Midterm Winter 2015/2016

4th December 2015

You have to accomplish this test within **60 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Schreiben Sie einen finalen Lösungssatz!/
Write down a final sentence stating your answer!

1	2	3	4	5	6	7	$\sum$

Problem 1 (14 points)
Consider the utility function

$$U(x_1, x_2) = 9x_1^2 + x_2^2.$$

(a) Calculate the marginal rate of substitution! Does this utility function represent convex preferences?

(b) Derive the Hicksian demand!

(c) Assume $p_1^l > 3p_2$. Determine the compensating variation for a price increase of good 1 from p_1^l to $p_1^h > p_1^l$.

Solution:

(a) The marginal rate of substitution is given by

$$MRS = \frac{MU_1}{MU_2} = \frac{18x_1}{2x_2}.$$

By monotonicity, if x_1 goes up, x_2 goes down along an indifference curve. Therefore, if x_1 increases, the nominator of $\frac{18x_1}{2x_2}$ increases while the denominator decreases. Thus, if x_1 increases, the MRS increases and the above above utility function represents concave preferences.

(b) Therefore, the solution to the household's optimization problem is a corner solution. We compare the utilities:

$$\begin{aligned} U\left(\frac{m}{p_1}, 0\right) &= \frac{9m^2}{p_1^2} \\ U\left(0, \frac{m}{p_2}\right) &= \frac{m^2}{p_2^2} \end{aligned}$$

and find that the household consumes only good 1 if $p_1 < 3p_2$. Thus, the Hicksian demand is given by

$$\chi\left(\bar{U}, p_1, p_2\right) = \begin{cases} \left(\frac{\sqrt{\bar{U}}}{3}, 0\right), & \text{if } p_1 < 3p_2 \\ \left\{\left(\frac{\sqrt{\bar{U}}}{3}, 0\right), \left(0, \sqrt{\bar{U}}\right)\right\} & \text{if } p_1 = 3p_2 \\ \left(0, \sqrt{\bar{U}}\right), & \text{if } p_1 > 3p_2. \end{cases}$$

(c) Since $p_1^h > p_1^l > 3p_2$, the household consumes only good 2 for both prices p_1^l and p_1^h. The household optimum stays the same. Thus, the compensating variation is equal to zero.

Problem 2 (6 points)

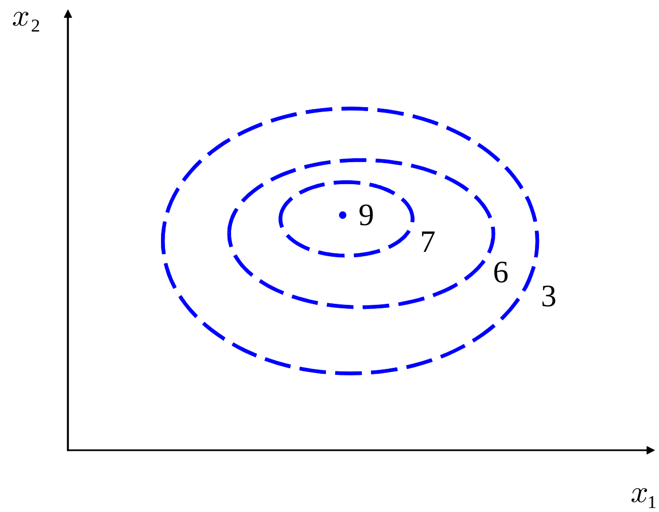
Consider a decision problem in strategic form with three strategies s_1, s_2 and s_3. Consider two mixed strategies $\sigma_1 = \left(0, \frac{1}{2}, \frac{1}{2}\right)$ and $\sigma_2 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. For a given state of the world $w \in W$, assume $\sigma_1 \in \sigma^{R,W}(w)$ and $\sigma_2 \notin \sigma^{R,W}(w)$. Find $s^{R,W}(w)$ and $\sigma^{R,W}(w)$!

Solution:

A best mixed strategy puts positive probabilities only on best pure strategies. Since σ_1 is a best mixed strategy, s_2 and s_3 are best pure strategies. Any mixed strategy that puts positive probabilities on best pure strategies, only, is a best mixed strategy. Thus, s_1 cannot be a best pure strategy–otherwise, σ_2 were a best mixed strategy. We obtain

$$\begin{aligned} s^{R,W}(w) &= \{s_2, s_3\}, \\ \sigma^{R,W}(w) &= \{(0, p, 1-p) : p \in [0,1]\}. \end{aligned}$$

Problem 3 (8 points)



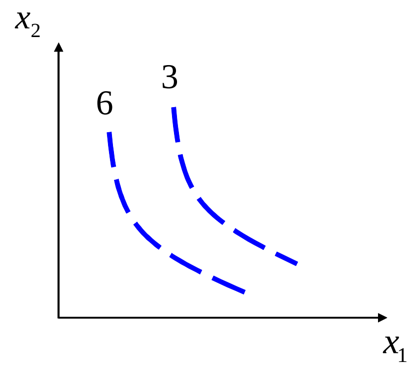
(a) Check whether the preferences illustrated above satisfy monotonicity, convexity or concavity.

(b) Sketch indifference curves that represent non-monotonic and concave preferences.

Solution:

(a) The preferences do not satisfy monotonicity because the utility of any point to the north east of the bliss point is lower than 9. The preferences are convex but not concave. Convexity: The better set for a given allocation consists of an ellipse (the indifference curve together with the points inside). These sets are obviously convex because the line connecting any two points of the ellipse always lies in the ellipse (for the bliss point, the convex combination of this point with itself is the bliss point, hence this is trivially true for the bliss point). Concavity: The worse set of a given allocation consists of all allocations excepting the open set within the ellipse. Consider any two different points A and B on the indifference curve marked with the number 6. The point $\frac{1}{2}A + \frac{1}{2}B$ is strictly preferred to A. Hence, the worse set is not convex and the preferences not concave.

(b) The following figure illustrates non-monotonic, concave preferences:



Problem 4 (10 Points)
Consider a firm facing the production function:

$$y = f(x_1, x_2) = \min\{x_1, 3x_2 + 1\}$$

(a) Does the production set fulfill the possibility of divine production?

(b) Show that input efficiency requires $x_2 = 0$ for $x_1 < 1$.

(c) Determine the cost function $C(y)$.

Solution:

(a) No divine production, since
$$f(0,0) = \min\{0,1\} = 0.$$

(b) First alternative: The input factors are used input-efficiently, if

$$\begin{aligned} x_1 &= 3x_2 + 1 \\ x_2 &= \frac{x_1 - 1}{3} \end{aligned}$$

However, if $x_1 < 1$ it follows that

$$x_2 = \frac{x_1 - 1}{3} < 0.$$

A negative input factor is not allowed. Therefore $x_2 = 0$. Second alternative: Consider the factor combination (x_1, x_2) with $x_1 < 1$. Then, $f(x_1, x_2) = x_1 = f(x_1, 0)$ for all $x_2 > 0$. This means that x_2 can be reduced down to zero without a decrease in output.

(c) The cost-minimization curve is thus given by:

$$x_2(x_1) = \begin{cases} \frac{x_1 - 1}{3} & \text{for } x_1 \geq 1 \\ 0 & \text{for } x_1 < 1 \end{cases}.$$

In terms of output y, this yields

$$\begin{aligned} x_1 &= y \\ x_2 &= \begin{cases} \frac{y-1}{3} & \text{for } y \geq 1 \\ 0 & \text{for } y < 1 \end{cases} \end{aligned}$$

Inserting this into $C(y) = w_1 x_1(y) + w_2 x_2(y)$, the cost function is given by

$$C(y) = \begin{cases} w_1 y & \text{for } y < 1 \\ w_1 y + w_2 \left(\frac{y-1}{3}\right) & \text{for } y \geq 1 \end{cases}.$$

Problem 5 (6 Points)
Consider an agent who maximizes his expected utility.

(a) Let $u(40) = 5$ and $u(20) = 4$. Assume that the agent is indifferent between $[20; 1]$ and $[40, 8; \frac{3}{4}, \frac{1}{4}]$. Determine $u(8)$.

(b) Is the agent risk loving?

Solution:

(a) By the indifference condition, we have

$$
\begin{aligned}
u(20) &= E_u\left([20,0;1,0]\right) \stackrel{!}{=} E_u\left(\left[40,10;\frac{3}{4},\frac{1}{4}\right]\right) = \frac{3}{4}u(40) + \frac{1}{4}u(8) \\
4 &= \frac{3}{4}\cdot 5 + \frac{1}{4}u(8) \\
u(8) &= 4\left(\frac{16}{4} - \frac{15}{4}\right) = 1
\end{aligned}
$$

(b) Risk loving agents fulfill

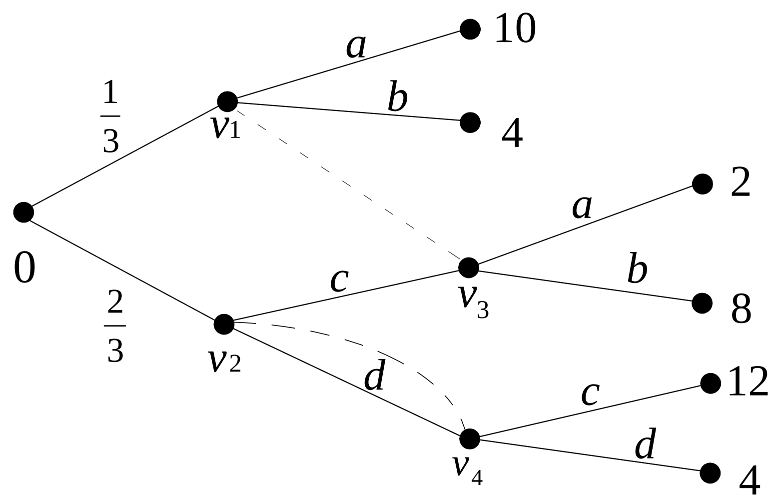
$$E_u(L) \geq u(E(L))$$

Assuming, as usual, monotonicity of the vNM function, we obtain

$$
\begin{aligned}
E_u\left(\left[40,10;\frac{3}{4},\frac{1}{4}\right]\right) &= E_u\left([20,0;1,0]\right) \\
&= u\left(20\right) \\
&< u(32) = u\left(E\left(\left[40,10;\frac{3}{4},\frac{1}{4}\right]\right)\right)
\end{aligned}
$$

Hence, the agent is not risk loving.

Problem 6 (12 Points)
Consider the following decision problem with move of nature at node 0:



(a) How many subtrees does this decision tree have? Name their initial nodes.

(b) Does this decision situation exhibit imperfect recall?

(c) Determine the best pure strategy.

Solution:

(a) There is just one subtree starting at 0.

(b) v_1 and v_3 lie in the same information set

$$I(v_1) = I(v_3),$$

but the experience is different:

$$X(v_1) = (I(v_1)) \neq X(v_3) = (I(v_2), c, I(v_3)).$$

Thus, the decision situation exhibits imperfect recall. (Similarly, consider nodes v_2 and v_4)

(c) In total, 4 pure strategies exist. Their payoffs are

$$\begin{aligned}
u(\lfloor a, c\rfloor) &= \frac{1}{3} \cdot 10 + \frac{2}{3} \cdot 2 = \frac{14}{3} \\
u(\lfloor a, d\rfloor) &= \frac{1}{3} \cdot 10 + \frac{2}{3} \cdot 4 = \frac{18}{3} \\
u(\lfloor b, c\rfloor) &= \frac{1}{3} \cdot 4 + \frac{2}{3} \cdot 8 = \frac{20}{3} \\
u(\lfloor b, d\rfloor) &= \frac{1}{3} \cdot 4 + \frac{2}{3} \cdot 4 = \frac{12}{3},
\end{aligned}$$

Therefore, the best pure strategy is

$$s = \lfloor b, c\rfloor$$

Problem 7 (4 points)
Do the following utility functions represent the same preferences? Explain!

$$\begin{aligned} u_1(x_1, x_2) &= \min\{x_1, x_2\} \\ u_2(x_1, x_2) &= \min\{x_1, 2x_2\} \end{aligned}$$

Solution:
Consider the consumption bundles $(2, 1)$ and $(2, 2)$. We have

$$u_1(2, 1) = 1 < 2 = u_1(2, 2)$$

and

$$u_2(2, 1) = 2 = u_2(2, 2).$$

Thus, bundle $(2, 1)$ is better than $(2, 2)$ under the preferences represented by u_1. However, a household having preferences represented by u_2 is indifferent between the two bundles. Therefore, the two utility functions do not represent the same preferences.