Advanced Microeconomics

Midterm Winter 2017/2018

1st December 2017

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	7	$\sum$
PUNKTE:								

Problem 1 (12 points)
Consider the following two-person game with mixed strategies. Calculate both reaction functions and illustrate them graphically. Determine all equilibria in pure and properly mixed strategies.

		player 2	
		l	r
player 1	o	$(3,3)$	$(4,5)$
	u	$(0,2)$	$(4,1)$

Solution
Let σ_1 denote the probability of player 1 to play o and let σ_2 denote the probility of player 2 to play l. Player 1 prefers o over u if

$$\begin{gathered} 3\sigma_2 + 4(1-\sigma_2) \geq 4(1-\sigma_2) \\ \sigma_2 \geq 0 \end{gathered}$$

holds. We get

$$\sigma_1^R(\sigma_2) = \begin{cases} [0,1], & \sigma_2 = 0 \\ 1, & \sigma_2 > 0. \end{cases}$$

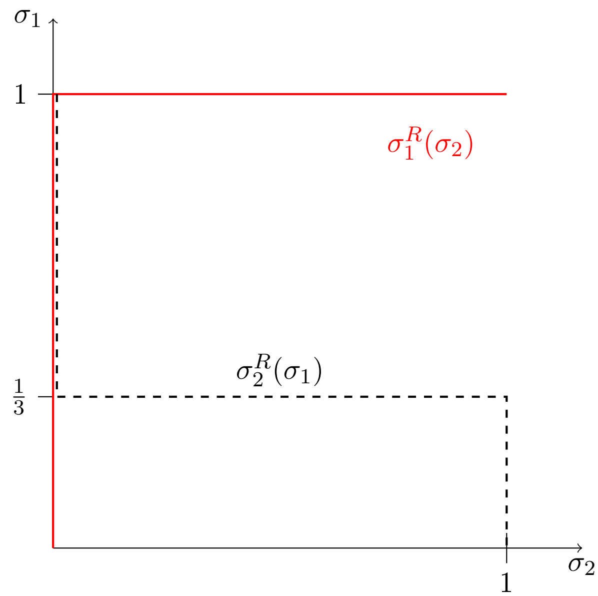
Player 2 prefers l over r if

$$\begin{gathered} 3\sigma_1 + 2(1-\sigma_1) \geq 5\sigma_1 + (1-\sigma_1) \\ \frac{1}{3} \geq \sigma_1 \end{gathered}$$

holds. We get

$$\sigma_2^R(\sigma_1) = \begin{cases} 0, & \sigma_1 > \frac{1}{3} \\ [0,1], & \sigma_1 = \frac{1}{3} \\ 1, & \sigma_1 < \frac{1}{3}. \end{cases}$$

Hence, there are infinitely many equilibria in mixed strategies given by $\sigma_1 \in \left[\frac{1}{3}, 1\right]$ and $\sigma_2 = 0$. Especially, the equilibrium given by $\sigma_1 = 1$ and $\sigma_2 = 0$ is the only equilibrium in pure strategies. A graphical illustration is given below.


σ_1
1
$\sigma_1^R(\sigma_2)$
$\sigma_2^R(\sigma_1)$
$\frac{1}{3}$
1
σ_2

Problem 2 (13 points)
Consider the utility funciton

$$U(x_1, x_2) = x_1^2 x_2.$$

Determine (i) the indirect utility function, (ii) the expenditure function, and (iii) the Hicksian demand function for both goods!

Solution
Marginal utility $\frac{\partial U}{\partial x_i} \geq 0$, $(i = 1, 2)$ is non-negative. Preferences are thus monotonic. The marginal rate of subsitution

$$MRS = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{2x_2}{x_1}$$

is decreasing in x_1. Preferences are thus convex. Applying

$$MRS = \frac{2x_2}{x_1} \stackrel{!}{=} \frac{p_1}{p_2} = MOC$$

leads to

$$x_2 = \frac{p_1}{2p_2} x_1.$$

Inserting the latter into the budget constraint

$$m = p_1 x_1 + p_2 x_2 = p_1 x_1 + p_2 \frac{p_1}{2p_2} x_1 = \frac{3}{2} p_1 x_1$$

leads to

$$x_1^*(p, m) = \frac{2m}{3p_1}$$
$$x_2^*(p, m) = \frac{p_1}{2p_2} x_1^*(p, m) = \frac{m}{3p_2}.$$

The indirect utility function reads

$$V(p, m) = U(x_1^*(p, m), x_2^*(p, m)) = \left(\frac{2m}{3p_1}\right)^2 \frac{m}{3p_2} = \frac{4}{27} \frac{m^3}{p_1^2 p_2}.$$

By duality,

$$\bar{U} = V(p, E(p, \bar{U})) = \frac{4}{27} \frac{E(p, \bar{U})^3}{p_1^2 p_2}$$

holds. This leads to the expenditure function

$$E(p, \bar{U}) = 3 \sqrt[3]{\frac{\bar{U} p_1^2 p_2}{4}}.$$

By duality, $\chi_i(p, \bar{U}) = x_i^*(p, E(p, \bar{U}))$ holds. This leads to the Hicksian demand functions

$$\chi_1(p, \bar{U}) = \frac{2E(p, \bar{U})}{3p_1} = \sqrt[3]{\frac{2\bar{U} p_2}{p_1}}$$
$$\chi_2(p, \bar{U}) = \frac{E(p, \bar{U})}{3p_2} = \sqrt[3]{\frac{\bar{U} p_1^2}{4p_2^2}}.$$

Problem 3 (7 points)
An agent maximizes his expected utility. Let $u(0) = 0$ and $u(20) = 1$. Assume that the agent is indifferent between $L_1 = [10; 1]$ and $L_2 = [20, 0; 0.3, 0.7]$.

a) Determine $u(10)$.

b) Is this agent risk-averse?

Solution

a) First, we have
$$E_u(L_1) = u(10).$$
Second, for L_2
$$E_u(L_2) = 0.3 \cdot u(20) + 0.7 \cdot u(0) = 0.3$$
holds. Finally, $L_1 \sim L_2$ implies $E_u(L_1) = E_u(L_2)$. This yields
$$u\,(10) = 0.3.$$

b) Risk aversion implies $E_u(L) \leq u(E(L))$ for all lotteries L. For L_2 we have $E_u(L_2) = u(10)$ and
$$E(L_2) = 0.3 \cdot 20 = 6.$$
Assuming, as usual, monotonicity of the vNM function, we obtain
$$E_u(L_2) = u(10) > u(6) = u(E(L_2)).$$
Hence, the agent is not risk averse.

Problem 4 (8 points)
For the following decision situation,

	w_1	w_2
s_1	4	4
s_2	1	8
s_3	5	1

answer the following four questions: Are strategies s_1 and s_2 rationalizable with respect to W and/or with respect to Ω?

Solution
We have $s_2 \in s^{R,W}(w_2) = \{s_2\}$ because $8 > 1$ and $8 > 4$. Consequently, s_2 is rationalizable w.r.t. W. Due to $s_2 \in s^{R,W}(w_2)$, s_2 is rationalizable w.r.t. Ω: $s_2 \in s^{R,\Omega}(0,1) = \{s_2\}$.
From

- $s_1 \notin s^{R,W}(w_1) = \{s_3\}$ because $5 > 4$ and $5 > 1$
- $s_1 \notin s^{R,W}(w_2) = \{s_2\}$ because $8 > 4$ and $8 > 1$

we can infer that s_1 is not rationalizable w.r.t. W.
Finally, s_1 is rationializable w.r.t. Ω because $s_1 \in s^{R,\Omega}(\frac{2}{3}, \frac{1}{3}) = \{s_1\}$ due to $4 > \frac{2}{3} + \frac{1}{3} \cdot 8 = \frac{10}{3}$ and $4 > \frac{2}{3} \cdot 5 + \frac{1}{3} = \frac{11}{3}$.

Problem 5 (8 points)

Consider the production function $f(K,L) = 2\sqrt{K} + \sqrt{L}$ and factor prices w_K and w_L. The output price is given by p.

a) Determine the production elasticity of factor K!

b) Determine the demand function for the production factor L!

Solution

a) The the production elasticity of factor K is given by

$$\varepsilon_{f,K} = \frac{\partial f}{\partial K}\frac{K}{f(K,L)} = \frac{1}{\sqrt{K}}\frac{K}{2\sqrt{K}+\sqrt{L}} = \frac{\sqrt{K}}{2\sqrt{K}+\sqrt{L}}.$$

b) Profit is given by

$$\Pi(K,L) = pf(K,L) - w_K K - w_L L$$

whose first order maximization constraint w.r.t. L

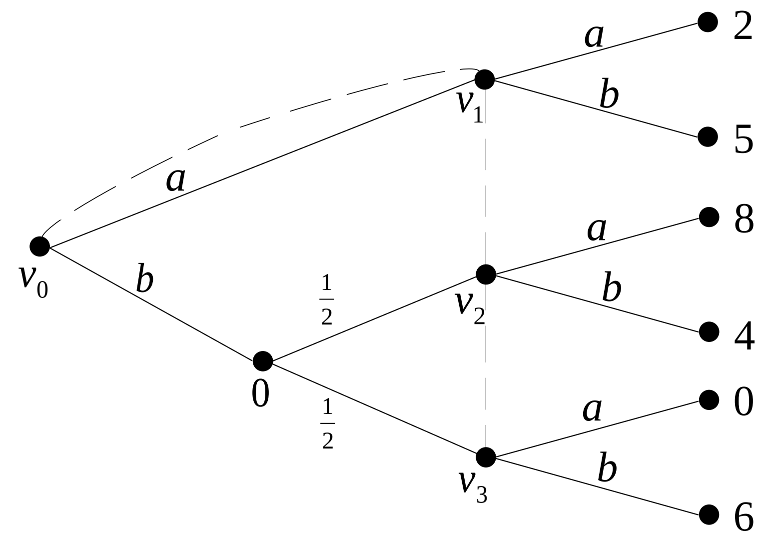
$$p\frac{\partial f}{\partial L} - w_L \stackrel{!}{=} 0$$

$$\frac{p}{2\sqrt{L}} \stackrel{!}{=} w_L$$

yields the demand function for the production factor L

$$L(w_L) = \frac{p^2}{4w_L^2}.$$

Problem 6 (8 points)
Consider the following decision problem with moves by nature!



a) Does this decision situation exhibit imperfect recall?

b) How many pure strategies do you find? Give two examples and state their payoffs!

Solution

a) The nodes v_0 and v_1 lie in the same information set

$$I(v_0) = I(v_1) = \{v_0, v_1, v_2, v_3\}$$

but the experience is different:

$$X(v_0) = (I(v_0)) \neq (I(v_0), a, I(v_0)) = (I(v_0), a, I(v_1)) = X(v_1).$$

Thus, the decison situation exhibits imperfect recall.

b) There are two pure strategies, a and b. Their payoffs are

$$\begin{aligned} u(\lfloor a \rfloor) &= 2 \\ u(\lfloor b \rfloor) &= \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 6 = 5. \end{aligned}$$

Problem 7 (4 points)
Arina's indirect utility function is given by

$$V(p, m) = \frac{3m}{p_1 + p_2 + 3p_3}.$$

How much is Arina willing to pay in order to avoid a price increase from $p_1 = 2$ to $p_1' = 4$ if she currently faces $p = (2, 4, 4)$ and $m = 120$?

Solution
If the price increase occurs, Arina's utility is given by

$$V(p', m) = \frac{3 \cdot 120}{4 + 4 + 3 \cdot 4} = 18.$$

She is willing to pay EV to avoid the price increase if

$$V(p, m - EV) = \frac{3 \cdot 120 - 3 \cdot EV}{2 + 4 + 3 \cdot 4} \geq 18 = V(p', m)$$

holds. This leads to $EV \leq 12$. Thus, Arina is willing to pay 12.