# Advanced Microeconomics

# Midterm Winter 2013/2014

9th December 2013

You have to accomplish this test within **60 minutes.**

**PRÜFUNGS-NR.:**

STUDIENGANG:

NAME, VORNAME:

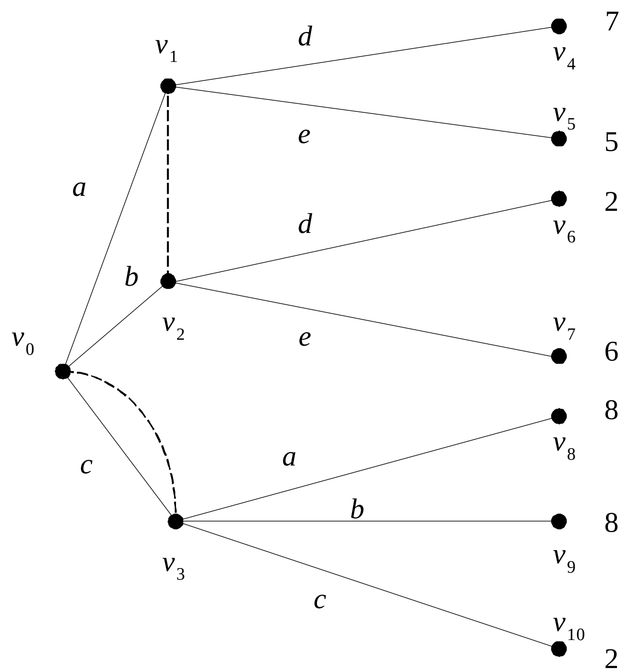
UNTERSCHRIFT DES STUDENTEN:

**ANFORDERUNGEN/REQUIREMENTS:**

**Lösen Sie die folgenden Aufgaben!/Solve all the exercises!**
**Schreiben Sie, bitte, leserlich!/Write legibly, please!**
**Sie können auf Deutsch schreiben!/You can write in English!**
**Begründen Sie Ihre Antworten!/Give reasons for your answers!**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | $\sum$ |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Problem 1 (12 points)**
Consider the following decision problem without moves by nature!




(a) How many subtrees does this decision tree have? Give their initial nodes!

(b) Show that this decision situation exhibits imperfect recall!

(c) How many strategies can you find? Give two examples.

(d) Find the optimal strategies!

**Solution:**

a) There is exactly one subtree, starting at $v_0$.

b) $I(v_1) = I(v_2)$, but $X(v_1) = (\{v_0, v_3\}, a, \{v_1, v_2\}) \neq (\{v_0, v_3\}, b, \{v_1, v_2\}) = X(v_2)$, which implies imperfect recall.

c) We have 6 strategies (actions for $\{v_0, v_3\}$ and $\{v_1, v_2\}$), for example $[a, d]$ and $[c, e]$.

(d) The payoff 8 cannot be reached by pure strategies. However the payoff for $[a, d]$ is equal to 7. Hence, $[a, d]$ is optimal.

**Problem 2 (6 points)**
An agent maximizes his expected utility. Let $u(0) = 0$ and $u(100) = 1$.

(a) Assume that the agent is indifferent between $[50; 1]$ and $[100, 0; 0.6, 0.4]$. Determine $u(50)$.

(b) Is this agent risk-loving?

**Solution**

(a) Indifference implies
$$u\,(50) = 0.6 \cdot u\,(100) + 0.4 \cdot u\,(0) = 0.6.$$

(b) By $u\,(E\,(L)) = u\,(0.6 \cdot 100) = u\,(60) > u\,(50) = 0.6 \cdot u\,(100) = 60 = E_u\,(L)$ we know that the agent prefers the expected value of the lottery over the lottery itself. Hence, he cannot be risk-loving!

**Problem 3 (8 points)**
For the following decision situation,

| | $w_1$ | $w_2$ |
|---|---|---|
| $s_1$ | 6 | 4 |
| $s_2$ | 2 | 3 |
| $s_3$ | 1 | 6 |

answer these four questions: Are strategies $s_1$ and $s_2$ rationalizable with respect to $W$ and/or with respect to $\Omega$?

**Solution**

- $s_1 \in s^{R,W}(w_1)$, because $6 > 2$ and $6 > 4$. We can infer that $s_1$ is rationalizable w.r.t. $W$. Therefore $s_1$ is rationalizable w.r.t. $\Omega$: $\{s_1\} = s^{R,\Omega}(1,0)$.

From

- $s_2 \notin s^{R,W}(w_1) = \{s_1\}$ because $6 > 2$ and $6 > 1$
- $s_2 \notin s^{R,W}(w_2) = \{s_3\}$ because $6 > 4$ and $6 > 3$

we can infer that $s_2$ is not rationalizable w.r.t. $W$.

Finally $s_2$ is not rationalizable w.r.t. $\Omega$, because for any probabilities $\sigma_1$ for $w_1$ and $1 - \sigma_1$ for $w_2$, we have

$$u(s_2, (\sigma_1, 1 - \sigma_1)) = 2\sigma_1 + 3(1 - \sigma_1) < 6\sigma_1 + 4(1 - \sigma_1) = u(s_1, (\sigma_1, 1 - \sigma_1)).$$

**Problem 4 (4 points)**
Do the following pairs of utility functions represent the same preferences?

(a) $U(x_1, x_2) = x_1 \cdot x_2$ and $V(x_1, x_2) = (x_1 + 3)\, x_2$,

(b) $U(x_1, x_2) = x_1 + x_2$ and $V(x_1, x_2) = e^{x_1} \cdot e^{x_2}$.

Solution

(a) Consider the comsumption bundles $(0,0)$ and $(0,1)$. On the one hand we have $U(0,0) = 0 = U(0,1)$ and thus $(0,0) \sim_U (0,1)$. On the other hand we have $V(0,0) = 0 < 3 = V(0,1)$ and thus $(0,0) \prec_V (0,1)$. The utility functions do not represent the same preferences.

(b) Consider the function $f(x) = e^x$,which is monotone increasing. Furthermore, we have $V(x_1, x_2) = e^{x_1} e^{x_2} = e^{x_1 + x_2} = e^{U(x_1, x_2)} = f \circ U(x_1, x_2)$. Thus, the utility functions $U$ and $V$ represent the same preferences.

**Problem 5 (6 points)**
The preferences of a houshold are given by the utility function

$$U(x_1, x_2) = \ln x_1 + 2x_2.$$

Assume $\frac{m}{p_2} - \frac{1}{2} \geq 0$. Determine the Hicksian demand function $\chi\left(\overline{U}, p\right)$.

**Solution**
The marginal rate of substitution is given by (1 point)

$$MRS = \frac{1}{2x_1}.$$

Since the utility function is convex we can use the approach (1 point)

$$MRS = \frac{p_1}{p_2}$$

Now, we immediately know (1 point)

$$\chi_1\left(\overline{U}, p\right) = \frac{p_2}{2p_1}.$$

To find the Hicksian demand of the second good we use the utility function: (1 point)

$$\overline{U} = \ln\left(\frac{p_2}{2p_1}\right) + 2x_2$$

and get (2 points)

$$\chi_2\left(\overline{U}, p\right) = \frac{1}{2}\left(\overline{U} - \ln\left(\frac{p_2}{2p_1}\right)\right).$$

**Problem 6 (14 Punkte)**
Consider a firm facing the production function

$$y = f(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}.$$

(a) Examine the returns to scale characterizing this production function!

(b) In the short run factor 1 is fixed at level $\overline{x_1} > 0$. Determine the short-run cost function $C_s(y, \overline{x_1})$.

(c) Determine the demand function of both input factors.

**Solution**

**(a)** If $t \geq 1$, we have

$$\begin{aligned} f(tx_1, tx_2) &= \sqrt{tx_1} + \sqrt{tx_2} = \sqrt{t}\left(\sqrt{x_1} + \sqrt{x_2}\right) \\ &\leq t\left(\sqrt{x_1} + \sqrt{x_2}\right) = tf(x_1, x_2). \end{aligned}$$

Thus, the production function is characterized by decreasing returns to scale. (3 Points)

**(b)** The short-run cost function is given by (1 point)

$$C_s(y, \overline{x_1}) := \min_{x_2} \left\{ w_1 \overline{x_1} + w_2 x_2 : \sqrt{\overline{x_1}} + \sqrt{x_2} \geq y \right\}$$

We have to distinguish the cases $\sqrt{\overline{x_1}} \geq y$ and $\sqrt{\overline{x_1}} < y$. The firm cannot use less than $\overline{x_1}$ of input factor 1. Thus, the output cannot be below $\sqrt{\overline{x_1}}$, even if the firm aims to sell less. The short-run costs cannot decrease below $w_1 \overline{x_1}$ (3 points). We obtain (3 points)

$$C_s(y, \overline{x_1}) := \begin{cases} w_1 \overline{x_1}, & \sqrt{\overline{x_1}} \geq y \\ w_1 \overline{x_1} + w_2 \left(y - \sqrt{\overline{x_1}}\right)^2, & \sqrt{\overline{x_1}} < y. \end{cases}$$

**(c)** The firms profit is given by

$$\Pi(x_1, x_2) = p\left(\sqrt{x_1} + \sqrt{x_2}\right) - w_1 x_1 - w_2 x_2.$$

(2 points). In order to maximize profit the partial derivatives with respect to $x_1$ and $x_2$ have to equal 0 (1 point):

$$\begin{aligned} \frac{\partial \Pi}{\partial x_1} &= \frac{p}{2\sqrt{x_1}} - w_1 \overset{!}{=} 0 \\ \frac{\partial \Pi}{\partial x_2} &= \frac{p}{2\sqrt{x_2}} - w_2 \overset{!}{=} 0. \end{aligned}$$

We receive the demand functions (1 points):

$$x_1 = \frac{p^2}{4w_1^2};\ x_2 = \frac{p^2}{4w_2^2}.$$

**Problem 7 (10 points)**

Consider the following strategic form game, where the payoffs/utilities of player $A$ are the left numbers in the matrix entries.

| Player $A$ \ Player $B$ | $b1$ | $b2$ | $b3$ |
|---|---|---|---|
| $a1$ | 1,2 | 4,5 | 2,2 |
| $a2$ | 2,6 | 1,4 | 2,3 |
| $a3$ | 4,2 | 3,3 | 5,4 |

a) Successively delete strictly dominated strategies as long as this is possible (i.e., apply iterative strict dominance)! Provide **all necessary inequalities**!

b) Determine the Nash equilibria in **pure** strategies (if any) of the **original** game!

**Solution**

(a) We know that the order of eliminating strictly dominated strategies does not affect the outcome.

Strategy $a2$ is strictly dominated by $a3$ , because

$$4 > 2, 3 > 1, 5 > 2.$$

After deletion of $a2$ we have

| | | Player $B$ | | |
|---|---|---|---|---|
| | | $b1$ | $b2$ | $b3$ |
| Player $A$ | $a1$ | 1,2 | 4,5 | 2,2 |
| | $a3$ | 4,2 | 3,3 | 5,4 |

(2 points). Here $b1$ is strictly dominated by $b3$ (1 point), because

$$3 > 2, 5 > 2.$$

After deletion of $b1$ we have

| | | Player $B$ | |
|---|---|---|---|
| | | $b2$ | $b3$ |
| Player $A$ | $a1$ | 4,5 | 2,2 |
| | $a3$ | 3,3 | 5,4 |

(2 points). In this game, none of $A$'s strategies is dominated (2 points) because

$$4 > 3, \text{ but } 2 < 5,$$

and similarly for $B$,

$$5 > 2, \text{ but } 3 < 4.$$

(b) The game

| | | Player $B$ | |
|---|---|---|---|
| | | $b2$ | $b3$ |
| Player $A$ | $a1$ | 4,5 | 2,2 |
| | $a3$ | 3,3 | 5,4 |

has two pure-strategy equilibria (3 points): (a1,b1), because

$$5 > 2, 4 > 3$$

and (a2,b2), because

$$4 > 3, 5 > 2$$

With respect to part (a) this already shows that the original game has the same and no further pure-strategy equilibria (1 point).