Advanced Microeconomics

Midterm Winter 2019/2020

4th December 2019

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

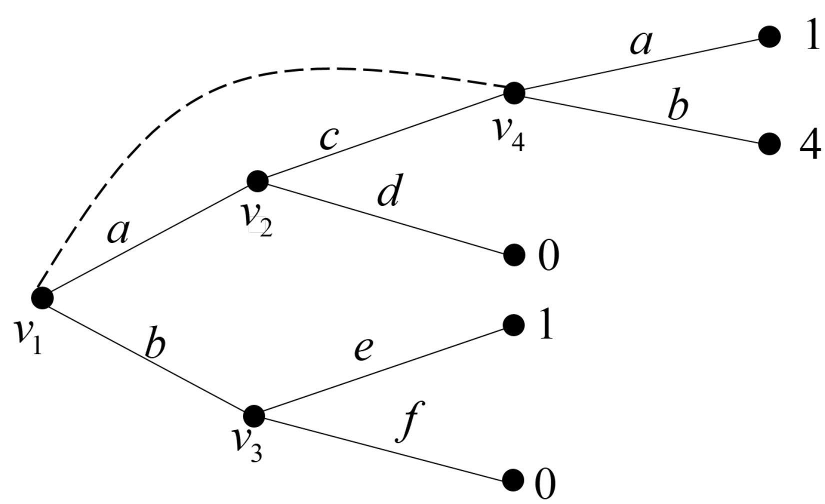
UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	7	8	$\sum$
PUNKTE:									

Problem 1 (12 points)
Consider the following decision problem without moves by nature!



a) How many subtrees do you find? State their initial nodes.

b) Determine the best pure strategies.

c) Determine the best behavioral strategies.

Solution:
a) There are 2 subtrees starting at v_1 and v_3.
b) The payoff of 4 cannot be reached by a pure strategy. The second highest payoff of 1 is achieved by the strategies $s_1 = \lfloor b, c, e \rfloor$, $s_2 = \lfloor b, d, e \rfloor$, $s_3 = \lfloor a, c, e \rfloor$ and $s_4 = \lfloor a, c, f \rfloor$.
c) Let α, β, γ denote the probability weights put on action a, c, and e, respectively. A behavioral strategy is defined by $s = (\alpha, \beta, \gamma)$. If node v_3 is reached with positive probability, i.e., if $\alpha < 1$, $\gamma = 1$ is optimal because $1 > 0$. If node v_2 is reached with positive probability, i.e., if $\alpha > 0$, $\beta = 1$ is optimal because $4, 1 > 0$. The payoff of 4 is reached with positive probability if and only if $0 < \alpha < 1$. In this case, we find the best behavioral strategy by maximizing the expected payoff

$$\begin{aligned} u(\alpha, 1, 1) &= (1-\alpha) \cdot 1 + \alpha \cdot \alpha \cdot 1 + \alpha \cdot (1-\alpha) \cdot 4 \\ &= 1 - \alpha + \alpha^2 + 4\alpha - 4\alpha^2 \\ &= 1 + 3\alpha - 3\alpha^2 \end{aligned}$$

over α. Maximization leads to

$$\begin{aligned} \frac{\partial u(\alpha, 1, 1)}{\partial \alpha} &= 3 - 6\alpha \overset{!}{=} 0 \\ \Rightarrow \alpha^* &= \frac{1}{2}. \end{aligned}$$

The expected payoff of $s^* = (\alpha^*, 1, 1)$ is found to be

$$u(s^*) = 1 + 3 \cdot \frac{1}{2}(1 - \frac{1}{2}) = 1 + \frac{3}{4} = \frac{7}{4} > 1.$$

Since a payoff of $1 < 7/4$ is reached at best if α=1 or $\alpha = 0$, $s^* = (\frac{1}{2}, 1, 1)$ is the best behavioral strategy.

Problem 2 (5 points)
A firm's production possibilities are described by the production set

$$Z = \{(z_1, z_2) \in \mathbb{R}^2 : z_1 \leq 0,\ z_1 + z_2^3 \leq 0)\}.$$

Determine the production function analytically.
Solution:
Since $z_1 \leq 0$, z_1 must be the input factor. We set $x_1 = -z_1$ and obtain

$$\begin{aligned}
y_2 = f(x_1) &= \max\{z_2 \in \mathbb{R}_+ :\ (-x_1, z_2) \in Z\} \\
&= \max\{z_2 \in \mathbb{R}_+ :\ -x_1 + z_2^3 \leq 0\} \\
&= \max\{z_2 \in \mathbb{R}_+ :\ +z_2^3 \leq x_1\} \\
&= \max\{z_2 \in \mathbb{R}_+ :\ z_2 \leq \sqrt[3]{x_1}\} \\
&= \sqrt[3]{x_1}.
\end{aligned}$$

Problem 3 (6 points)
Consider a household with money budget $m > 0$ and utility function $U(x_1, x_2) = x_1 x_2$. The household optimum is given by

$$x_1(p_1, p_2, m) = \frac{m}{2p_1}, \qquad x_2(p_1, p_2, m) = \frac{m}{2p_2}.$$

Show that the old-household-optimum substitution effect of good 1 is strictly negative.
Solution:
Let $x^* = (x_1^*, x_2^*)$ denote the old household optimum. If the household is compensated for a price increase in p_1 such that the household can effort the old household optimum x^*, the household consumes

$$\begin{aligned} x_1^S(p_1^n, p_2, x_1^*, x_2^*) &= x_1(p_1^n, p_2, x_1^* p_1^n + x_2^* p_2), \\ &= \frac{x_1^* p_1^n + x_2^* p_2}{2p_1^n} \\ &= \frac{1}{2} x_1^* + \frac{p_2}{2p_1^n} x_2^*, \end{aligned}$$

where p_1^n is the new price of good 1. The substitution effect of good 1 is given by

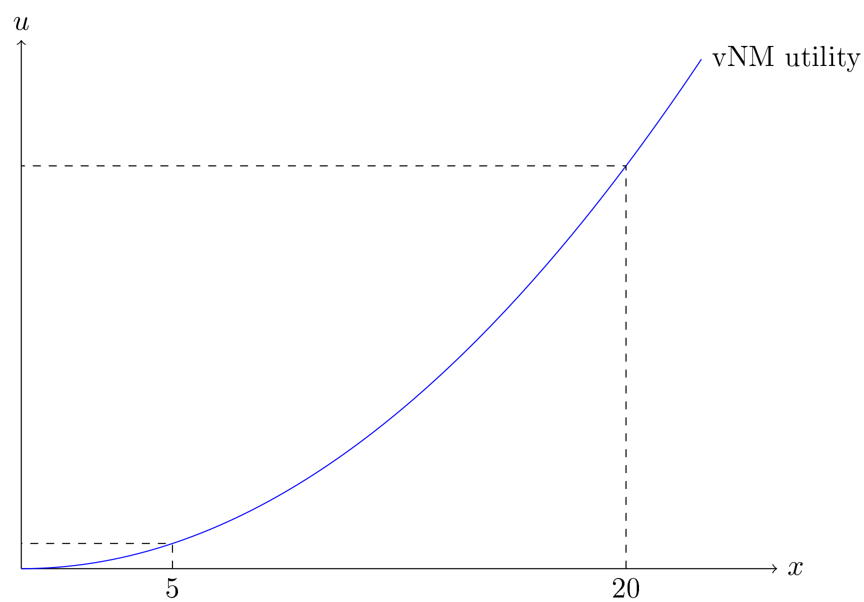
$$\frac{\partial x_1^S}{\partial p_1^n} = -\frac{p_2}{2(p_1^n)^2} x_2^* < 0$$

because $x_2^* > 0$ due to $m > 0$. Hence, the substitution effect is strictly negative.

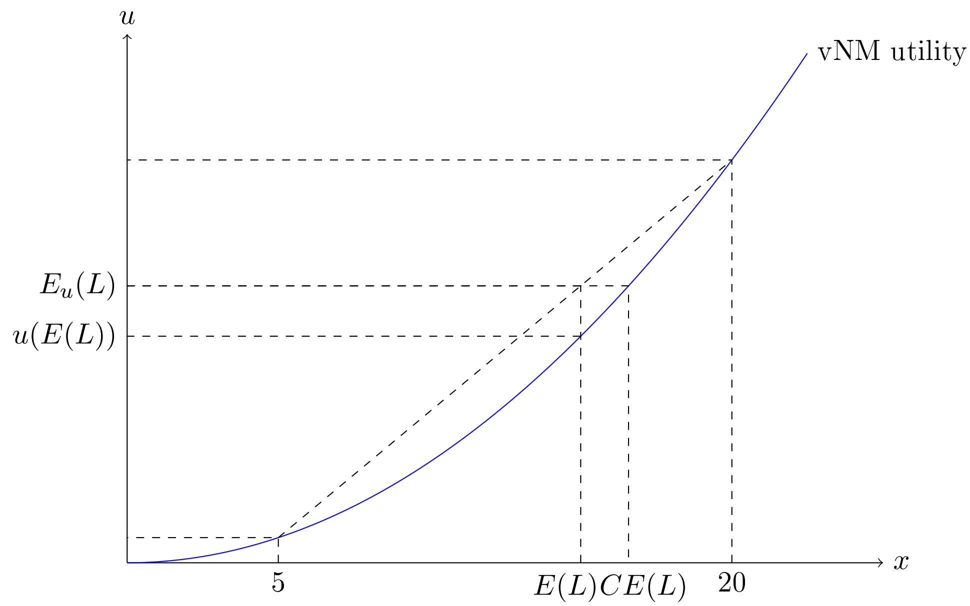
Problem 4 (10 points)
Consider the lottery $L = \left[5, 20; \frac{1}{3}, \frac{2}{3}\right]$ and the vNM-utility function depicted below. Derive *graphically*

- the expected value of the lottery $E(L)$,
- the utility of the expected value $u(E(L))$,
- the expected utility of the lottery $E_u(L)$, and
- the certainty equivalent $CE(L)$

Explain whether the utility function exhibits risk-averse, risk-neutral or risk-loving preferences.



Solution:



The vNM-utility function is convex and therefore exhibits risk-loving preferences.

Problem 5 (12 points)

Consider a game with players 1 and 2 who choose $s_1 \in [0, \infty)$ and $s_2 \in [0, \infty)$, respectively, and who have the following utility functions:

$$u_1(s_1, s_2) = \begin{cases} -s_1, & s_1 < s_2 \\ 2 - s_1, & s_1 = s_2 \\ 4 - s_2, & s_1 > s_2 \end{cases}$$

and

$$u_2(s_1, s_2) = \begin{cases} 2 - s_1, & s_1 < s_2 \\ 1 - s_2, & s_1 = s_2 \\ -s_2, & s_1 > s_2 \end{cases}$$

a) What is player 1's best response if player 2 chooses $s_2 = 3$, what is her best response if he chooses $s_2 = 5$?

b) Determine the reaction function of player 1.

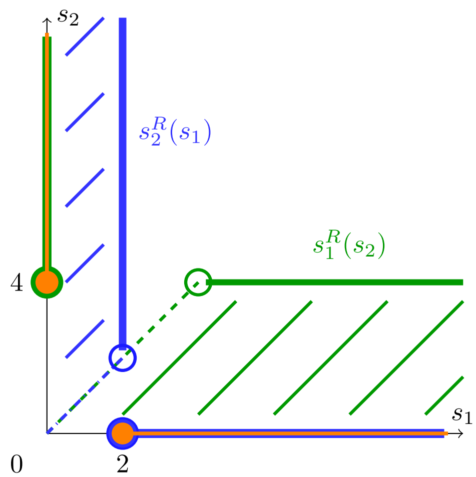
c) Is the strategy combination $(s_1, s_2) = (3, 0)$ a Nash equilibrium?

Solution:

a) The best response of player 1 to $s_2 = 5$ is $s_1^R(s_2 = 5) = 0$. If player 1 chooses $s_1 = s_2 = 5$, her payoff is $u_1(5, 5) = 2-5 = -3$. If she chooses $s_1 > s_2 = 5$, her payoff is $u_1(s_1, 5) = 4-5 = -1$. By choosing $s_1 = 0 < s_2 = 5$, she can realize a payoff of $u_1(0, 5) = 0$.
The best response of player 1 to $s_2 = 3$ is $s_1^R(s_2 = 5) = s_1 > 3$. If player 1 chooses $s_1 = s_2 = 3$, her payoff is $u_1(3, 3) = 2 - 3 = -1$. If she chooses $s_1 < s_2 = 3$, her maximal payoff is $u_1(0, 5) = 0$. By choosing $s_1 > s_2 = 3$, she can realize a payoff of $u_1(s_1, 3) = 4 - 3 = 1$.

b) The best response function of player 1 is given by: $s_1^R(s_2) = \begin{cases} (s_2, \infty), & s_2 < 4 \\ (s_2, \infty) \cup \{0\}, & s_2 = 4 \\ \{0\}, & s_2 > 4 \end{cases}$.

c) The answer can be solved graphically. The best response function of player 2 is given by $s_2^R(s_1) = \begin{cases} (s_1, \infty) & , s_1 < 2 \\ (s_1, \infty) \cup \{0\} & , s_1 = 2 \\ \{0\} & , s_1 > 2 \end{cases}$. Illustrating the reaction functions gives:



The green function is the reaction function $s_1^R(s_2)$ of player 1, the blue function the reaction function of player 2 $s_2^R(s_1)$.
The intersection of the reactions functions is depicting the reciprocal best responses of the two players (marked in orange). We see that the Nash equilibria of this game are given by $(s_1^*, s_2^*) = (x, 0)$, with $x \geq 2$, and $(0, y)$, with $y \geq 4$.
$(3, 0)$ is thus a Nash equilibrium.

Alternative: $(s_1, s_2) = (3, 0)$ is a Nash equilibrium if no player has an incentive to deviate unilaterally. For the given strategy combination, the payoffs are given by $(u_1, u_2) = (4, 0)$.
Given that player 1 plays $s_1 = 3$, player 2 cannot increase his payoff above $u_2 = 0$ by deviating unilaterally. Choosing $s_2 = s_1 = 3$ results in a payoff of $u_2(3, 3) = -2$. Choosing $s_2 > s_1 = 3$ results in a payoff of $u_2(3, s_2) = 2 - 3 = -1$. By choosing $s_2 < s_1$, the payoff of player 2 is ≤ 0. Thus by choosing $s_2 = 0$, player 2 can achieve the maximal payoff of 0 given that player 1 chooses $s_1 = 3$.
Given that player 2 plays $s_2 = 0$, player 1 cannot increase her payoff above $u_1 = 4$ by deviating unilaterally. Choosing $s_1 < s_2 = 0$ is not possible. Choosing $s_1 = s_2 = 0$ results in a payoff of $u_1(0, 0) = 0$. By choosing $s_2 > 0$, player 1 can achieve the payoff of $u_1(s_1 > 0, 0) = 4$. Thus by deviating, player 1 cannot increase her payoff above 4.
Since no player has an incentive to deviate unilaterally, the strategy combination $(s_1, s_2) = (3, 0)$ is a Nash equilibrium.

Problem 6 (4 points)
Consider the following production function: $y = f(x_1, x_2) = \min\left\{x_1^2, x_2\right\}$. Determine the cost function.
Solution:
Efficient factor inputs require $x_1^2 = x_2$. Expressing input factors in terms of production, we obtain: $x_1 = \sqrt{y}$, $x_2 = y$. The cost function is given as:

$$C(y) := w_1 x_1 + w_2 x_2 = w_1 \sqrt{y} + w_2 y.$$

Problem 7 (3 points)
Consider the expenditure function

$$e(\bar{U}, p) = p_2 \left(\bar{U} + 1 - \ln \frac{p_2}{p_1} \right).$$

Assume $\bar{U} > \ln \frac{p_2}{p_1}$. Derive Hicksian demand for good 2.

Solution:

The demand can be determined using Shephard's lemma:

$$\chi_2(\bar{U}, p) = \frac{\partial e(\bar{U}, p)}{\partial p_2} = 1 \cdot \left[\bar{U} + 1 - \ln \frac{p_2}{p_1} \right] + p_2 \cdot \left[-\frac{1}{p_2} \right] = \bar{U} - \ln \frac{p_2}{p_1}$$

Problem 8 (8 points)
Consider the second-price auction with two bidders, $1, 2$, whose reservation prices are given by $r_1 = 10$ and $r_2 = 20$, respectively, and whose bids are $s_1, s_2 \in [0, \infty) = W$, respectively. A fair coin determines who gets the object if $s_1 = s_2$.

Apply iterative rationalizabilty with respect to W.

Solution:

The utility function of bidder 1 is given by

$$u_1(s_1, s_2) = \begin{cases} r_1 - s_2, & s_1 > s_2 \\ \frac{r_1 - s_2}{2} & s_1 = s_2 \\ 0 & s_1 < s_2 \end{cases}.$$

A strategy $s_1 \in [0, \infty)$ is rationalizable with respect to W if there exists an $s_2 \in W$ such that $s_1 \in s_1^{R,W}(s_2)$, i.e., s_1 is a best response to strategy s_2. For any $s_1 \in [0, \infty)$, we define the strategy $\bar{s}_2 := \max(s_1, r_1) + 1$. Then, bidder 1 does not get the object. He has $u_1(s_1, \bar{s}_2) = 0$. If he wants to get the object, he must bid $s_1' \geq \bar{s}_2$, which would lead to $u_1(s_1', \bar{s}_2) < u_1(s_1, \bar{s}_2) = 0$. Hence, $s_1 \in s_1^{R,W}(\bar{s}_2)$. Hence, every strategy $s_1 \in [0, \infty)$ is rationalizable. Similarly, for any $s_2 \in [0, \infty)$, we define the strategy $\bar{s}_1 := \max(s_2, r_2) + 1$ that ensures $s_2 \in s_2^{R,W}(\bar{s}_1)$. Hence, iterative rationalizability does not restrict the agents' sets of strategies. The two sets of rationalizable strategies are given by $[0, \infty)$ and $[0, \infty)$.