Advanced Microeconomics

Midterm Winter 2016/2017

25th November 2017

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	$\sum$
PUNKTE:							

Problem 1 (16 points)

The preferences of a household are represented by the utility function

$$U(x_1, x_2) = (x_1 + x_2)^4.$$

Assume that good one is twice as expensive as good two.

(a) Derive the indirect utility function.

(b) Determine Hicksian demand.

(c) Now assume that both goods are equally expensive. Determine Hicksian demand.

Solution:

(a) Since U is equivalent to $U_2(x_1, x_2) = x_1 + x_2$, the preferences are linear and we have $MRS = 1$. Additionally,

$$MOC = \frac{p_1}{p_2} = 2 > 1 = MRS$$

holds. Therefore the houshold only consumes good two. Marshallian demand is given by

$$x_1^*(p, m) = 0;\ x_2^*(p, m) = \frac{m}{p_2}.$$

We derive the indirect utility function

$$V(p, m) = \left(\frac{m}{p_2}\right)^4.$$

(b) Given $p_1 = 2p_2$, the houshold consumes only good two, i.e., $\chi_1(p, \bar{U}) = 0$. Inserting this in the utility function yields

$$\bar{U} = (0 + \chi_2)^4$$

or equivalently $\chi_2(p, \bar{U}) = \sqrt[4]{\bar{U}}$.

(c) If $p_1 = p_2$, we have

$$MRS = 1 = MOC.$$

The houshold then has the same expenditure for all bundles on a given indifference curve, i.e., for a given utility level $\bar{U}$ all bundles on the indifference curve with utility level $\bar{U}$ are optimal. The Hicksian demand satisfies

$$\bar{U} = (\chi_1 + \chi_2)^4$$

or equivalently

$$\chi_1 + \chi_2 = \sqrt[4]{\bar{U}}.$$

More precisely, Hicksian demand $\chi(p, \bar{U})$ equals

$$\left\{(\chi_1, \chi_2) :\ \chi_1 \in \left[0, \sqrt[4]{\bar{U}}\right],\ \chi_2 = \sqrt[4]{\bar{U}} - \chi_1\right\}.$$

Problem 2 (5 points)

Consider the preferences represented by the utility function

$$U(x_1, x_2) = -x_1 + x_2.$$

Do the preferences satisfy

	yes	no
monotonicity		
local non-satiation		
convexity		

Hint: A false answer offsets a correct answer! An explanation is not required.

Solution:

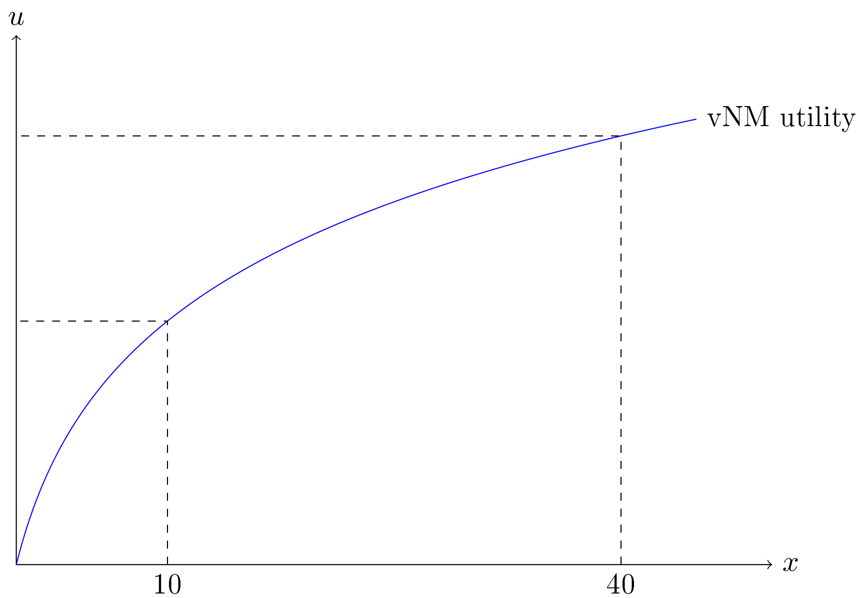
	yes	no
monotonicity		x
local non-satiation	x	
convexity	x	

Problem 3 (10 Punkte)

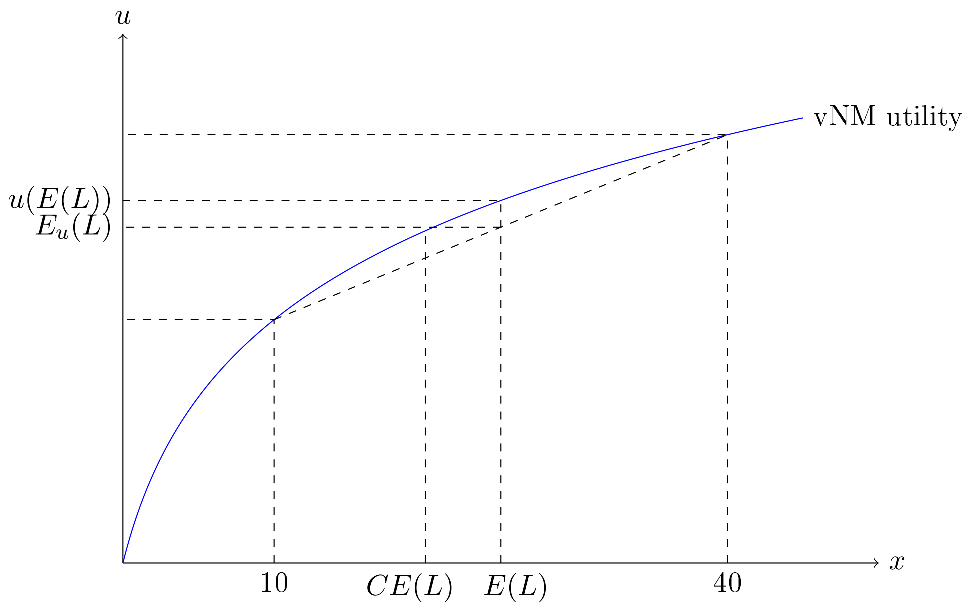
Consider the lottery $L = \left[10, 40; \frac{1}{2}, \frac{1}{2}\right]$ and the vNM-utility function depicted below. Derive *graphically*

- the expected value of the lottery $E(L)$,
- the utility of the expected value $u(E(L))$,
- the expected utility of the lottery $E_u(L)$, and
- the certainty equivalent $CE(L)$

Explain whether the utility function exhibits risk-averse, risk-neutral or risk-loving preferences.



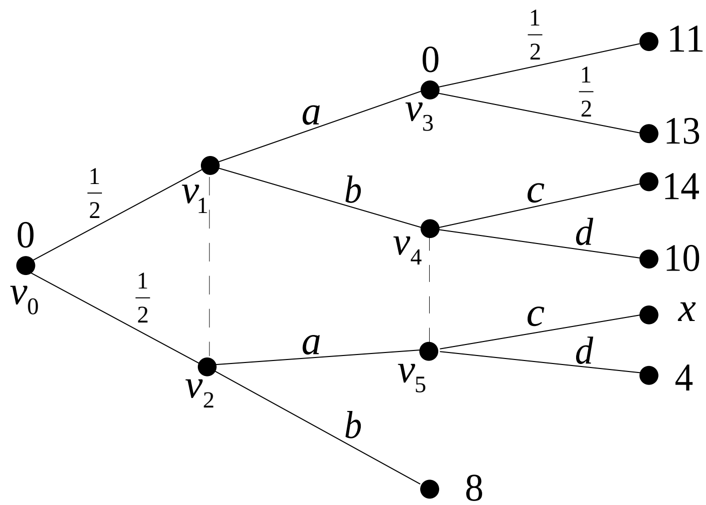
Solution:



The vNM-utility function is concave and therefore exhibits risk-averse preferences.

Problem 4 (14 Punkte)

Consider the following decision problem with $x \in (0, \infty)$ and moves by nature at v_0 and v_3:



(a) Does this decision situation exhibit imperfect recall?

(b) How many strategies do you find? State their payoffs!

(c) Determine the best pure strategy.

Solution:

(a) v_4 and v_5 lie in the same information set

$$I(v_4) = I(v_5)$$

but the expericence is different:

$$X(v_4) = (I(v_1), b, I(v_4)) \neq (I(v_1), a, I(v_4)) = (I(v_2), a, I(v_5)) = X(v_5).$$

Thus, the decision situation exhibits imperfect recall.

(b) 4. The payoffs are given by

$$\begin{aligned}
u(\lfloor a, c \rfloor) &= \frac{1}{2} \cdot 12 + \frac{1}{2} x = 6 + \frac{x}{2} \\
u(\lfloor a, d \rfloor) &= \frac{1}{2} \cdot 12 + \frac{1}{2} \cdot 4 = 8 \\
u(\lfloor b, c \rfloor) &= \frac{1}{2} \cdot 14 + \frac{1}{2} \cdot 8 = 11 \\
u(\lfloor b, d \rfloor) &= \frac{1}{2} \cdot 10 + \frac{1}{2} \cdot 8 = 9.
\end{aligned}$$

(c) The best pure strategies are $\lfloor b, c \rfloor$ for $x \leq 10$ and $\lfloor a, c \rfloor$ for $x \geq 10$.

Problem 5 (7 punkte)

Consider the production set

$$Z = \left\{(z_1, z_2) \in \mathbb{R}^2 : \ z_2 \le -\sqrt{z_1} \text{ if } z_1 \ge 0 \text{ and } z_2 \le -z_1 \text{ if } z_1 < 0\right\}.$$

Determine the production functions analytically.

Solution:

For $z_1 \ge 0$, z_2 is the input factor. We set $x_2 = -z_2$. The input-efficient production function is given by

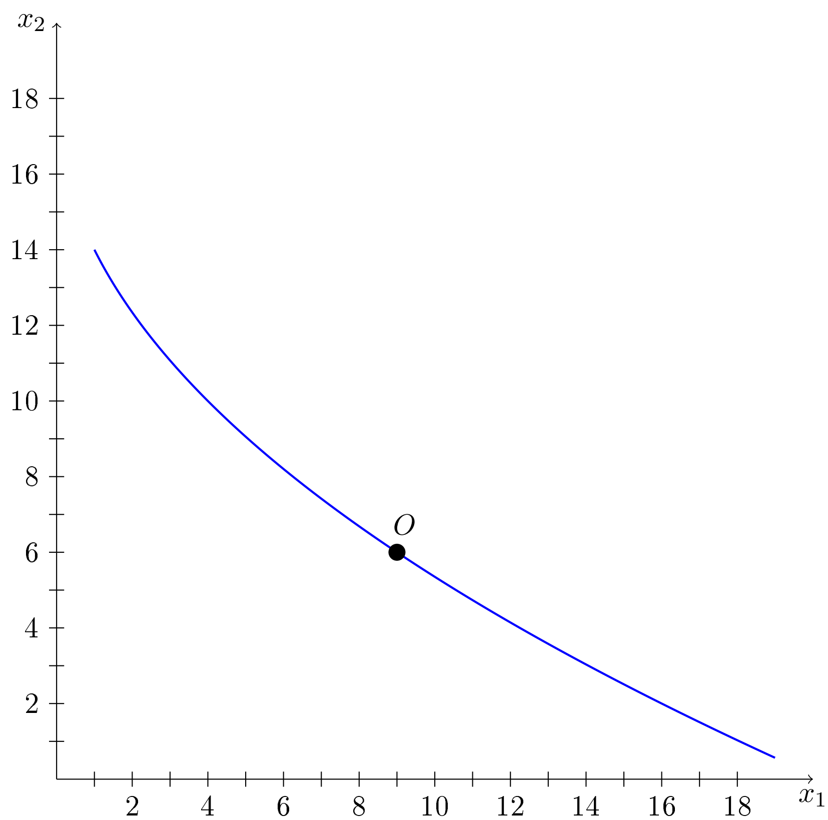
$$\begin{aligned} f_1(x_2) &= \max\{z_1 \in \mathbb{R}_+ : (z_1, -x_2) \in Z\} \\ &= \max\{z_1 \in \mathbb{R}_+ : \ -x_2 \le -\sqrt{z_1}\} \\ &= \max\{z_1 \in \mathbb{R}_+ : x_2 \ge \sqrt{z_1}\} \\ &= \max\left\{z_1 \in \mathbb{R}_+ : x_2^2 \ge z_1\right\} \\ &= x_2^2. \end{aligned}$$

For $z_1 < 0$, z_1 is the input factor. We set $x_1 = -z_1$. The input-efficient production function is given by

$$\begin{aligned} f_2(x_1) &= \max\{z_2 \in \mathbb{R}_+ : (-x_1, z_2) \in Z\} \\ &= \max\{z_2 \in \mathbb{R}_+ : z_2 \le x_1\} \\ &= x_1. \end{aligned}$$

Problem 6 (8 punkte)

Consider a household with strictly monotonic and strictly convex preferences living in a world of two goods. His income is given by $m = 72$. For the prices $p_1 = 4$ and $p_2 = 6$, his optimal consumption bundle is given by $O = (9, 6)$. The corresponding indifference curve is drawn in the figure below.

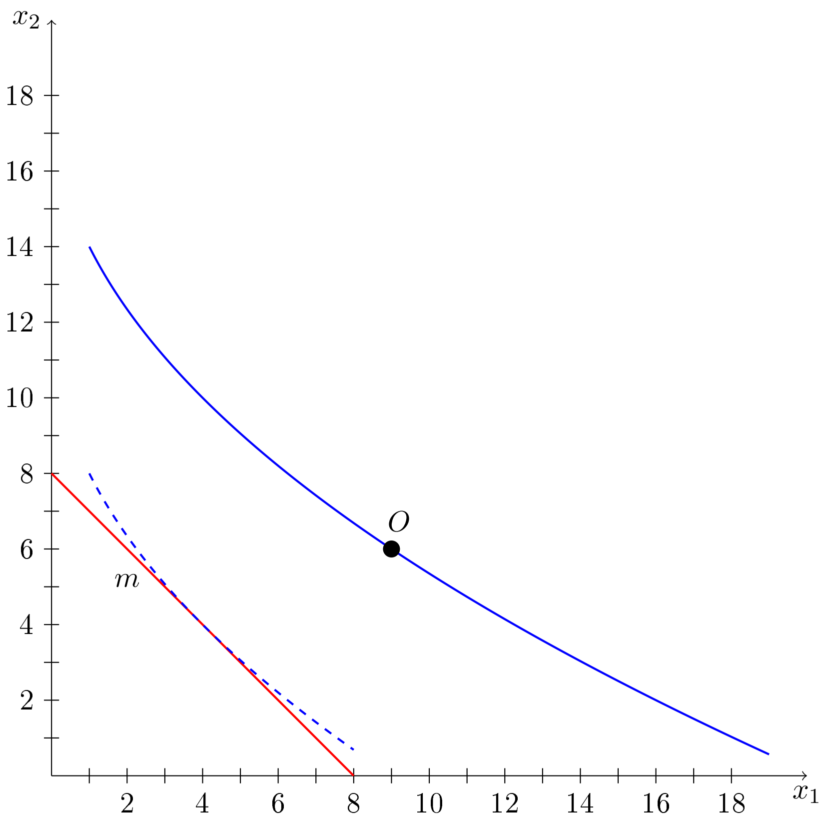


(a) Consider a price change towards $p_1' = 9$, $p_2' = 9$. Draw the new budget line and sketch the indifference curve with the maximum utility level within this budget.

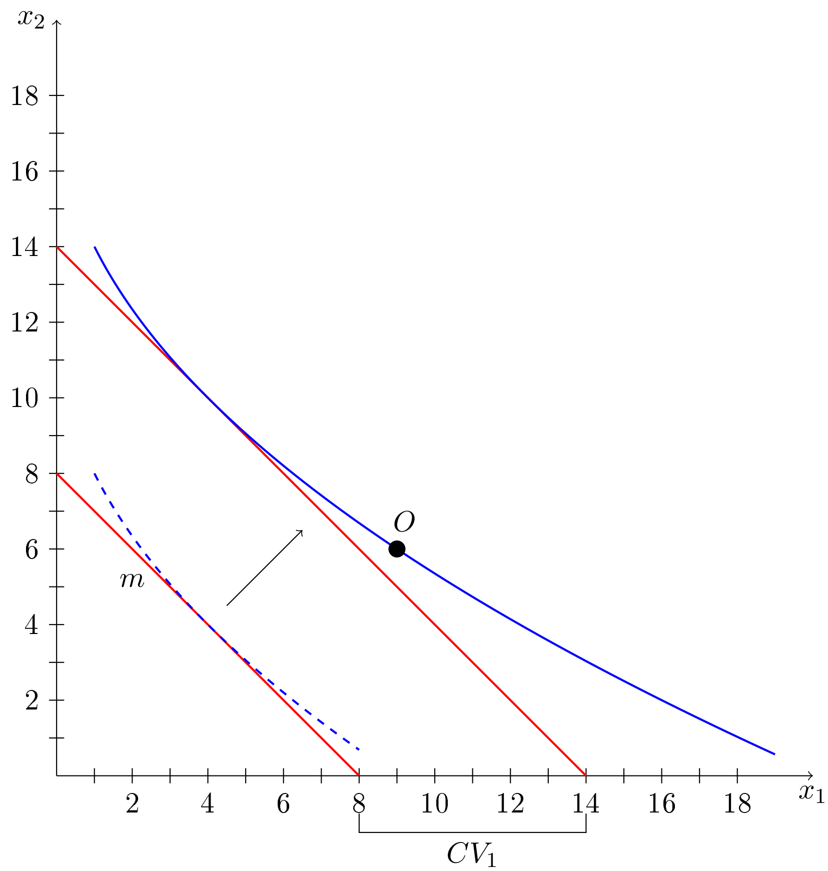
(b) Derive the compensating variation in units of good 1 graphically. State the compensating variation in monetary units.

Solution:

(a) The budget line is drawn in red. Axes intercepts are determined by $\left(\frac{m}{p_1'}, 0\right) = (8, 0)$ and $\left(0, \frac{m}{p_2'}\right) = (0, 8)$. The sketched indifference curve (drawn in blue and dashed) is strictly convex and tangent to the budget line in one point due to strict monotonicity and strict convexity.



(b) The new budget line has a slope of $-\frac{p_1'}{p_2'} = -1$. It is shifted until it is tangent to the initianlly drawn indifference curve. The intercept of the new budget line with the x_1-axis is given by $(14, 0)$. Hence the compensating variaton in units of good one is given by $CV_1 = 14 - 8 = 6$. And the compensating variation in monetary units by $CV = p_1' \, CV_1 = 9 \cdot 6 = 54$.



x_2
18
16
14
12
10
8
6
4
2
O
m
2
4
6
8
10
12
14
16
18
x_1
CV_1