Advanced Microeconomics

Final Winter 2013/2014

28th February 2014

You have to accomplish this test within **60 minutes.**

PRÜFUNGS-NR/ REGISTRATION NUMBER.:

STUDIENGANG/ DEGREE PROGRAM:

NAME, VORNAME/ NAME, FIRST NAME:

UNTERSCHRIFT DES STUDENTEN/ SIGNATURE:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	$\sum$	$\sum_{\text{mid}}$	Grade

Problem 1 (8 points)
Consider the cooperative game $(\{1,2,3\}, v)$ with the coalition function

$$v(K) = \begin{cases} 0, & K \in \{\emptyset, \{1\}, \{2\}, \{3\}\} \\ 1, & \text{else.} \end{cases} .$$

Examine, whether the Shapley payoff vector lies in the core.

Solution

Obviously all players are symmetric, which yields together with efficiency

$$Sh_1 = Sh_2 = Sh_3 = \frac{1}{3}.$$

The payoff vector $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ does not lie in the core - for example we have

$$v(\{1,2\}) = 1 > \frac{2}{3} = Sh_1 + Sh_2.$$

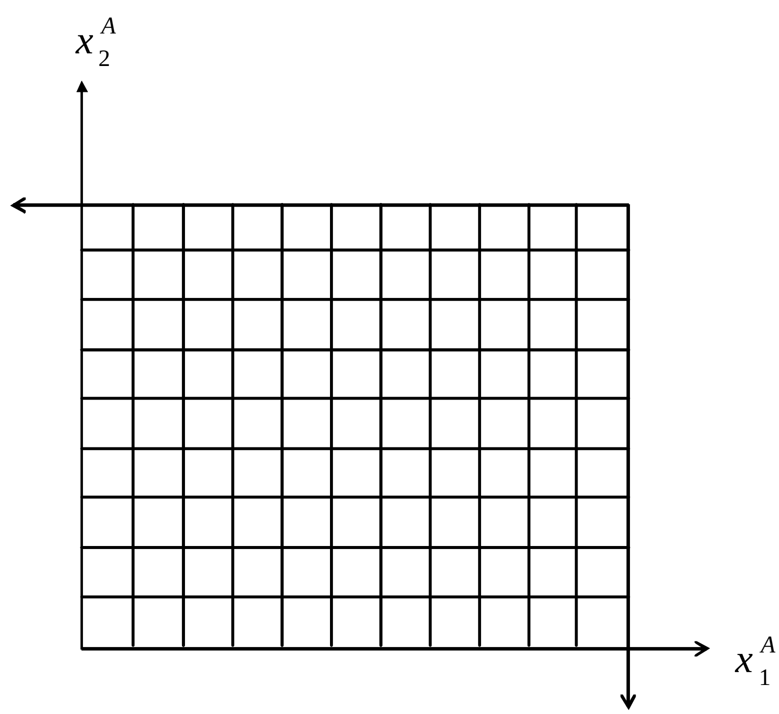
Problem 2 (12 points)
Consider an exchange economy with two agents A and B. Agent A has lexicographic preferences, where good 1 is the important good. The preferences of agent B are represented by the utility function

$$u^B\left(x_1^B, x_2^B\right) = x_1^B + x_2^B.$$

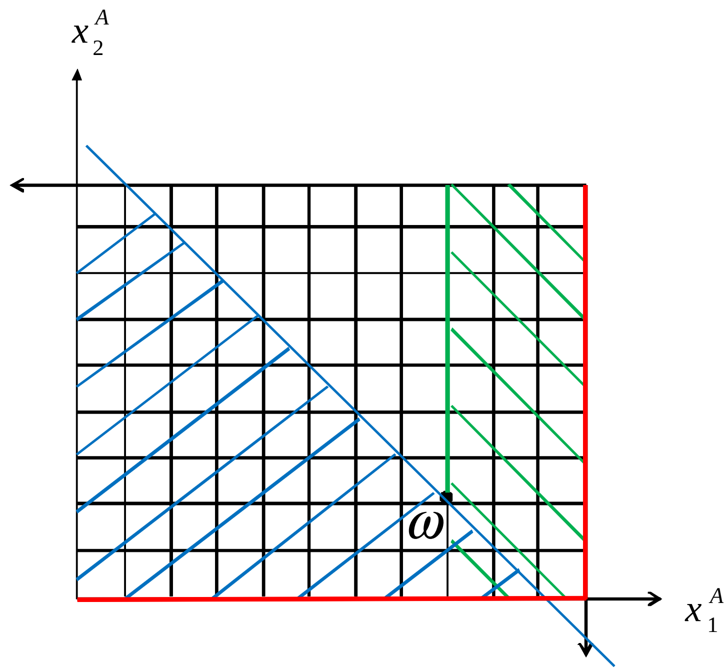
The endowment is given by

$$\omega^A = (8, 2) \text{ und } \omega^B = (3, 7).$$

Use the graphic below to illustrate endowments, the better sets of agent A and B with respect to ω and the exchange lense. Illustrate and determine extensively all Pareto-efficient allocations.



Solution



The endowment is denoted by ω. The green part, respectively the blue part illustrates the better set of agent A, respectively agent B. The exchange lense is that part of the box where both better sets intersect and thus the triangle which is blue and green not including the left border. Contract curve:

- assume A has a positive quantity of good 2 and B has a positive quantity of good 1. If the agent exchange one unit of good 1 against one unit of good 2, A is better of while B stays at the same utility level. Thus, there is a Pareto-improvement, these points cannot be Pareto-optimal
- assume A has nothing of good 2. If A gives something of good 1 to B, he is worse of. If A gives nothing to B, B will be worse of if he gives something to A. There is no Pareto-improvement and thus these points are Pareto-optimal.
- assume B has nothing of good 1. If A gives something of good 1 to B, he is worse of. If A gives something of good 2 to B, B can give nothing in return and thus A is also worse of. There is no Pareto-improvement and thus these points are Pareto-optimal.

Problem 3 (12 points)
Consider the following two person game with mixed strategies. Calculate both reaction functions and illustrate them graphically. Determine all equilibria in pure and properly mixed strategies.

		player 2	
		o	u
player 1	l	$(2,2)$	$(0,1)$
	r	$(2,1)$	$(0,2)$

Solution

First of all we determine the payoff functions:

$$\begin{aligned} u_1(\sigma_1,\sigma_2) &= 2\sigma_1\sigma_2 + 2(1-\sigma_1)\sigma_2, \\ u_2(\sigma_1,\sigma_2) &= 2\sigma_1\sigma_2 + \sigma_1(1-\sigma_2) + (1-\sigma_1)\sigma_2 + 2(1-\sigma_1)(1-\sigma_2). \end{aligned}$$

We compute the reaction function of agent 1 by examining the derivative of the payoff function:

$$\frac{\partial u_1}{\partial \sigma_1} = 2\sigma_2 - 2\sigma_2 = 0$$

and get $\sigma_1^R(\sigma_2) = [0,1]$. Now, consider the payoff function of agent 2. We have

$$\frac{\partial u_2}{\partial \sigma_2} = 2\sigma_1 - \sigma_1 + (1-\sigma_1) - 2(1-\sigma_1) = 2\sigma_1 - 1$$

which yields

$$\sigma_2^R(\sigma_1) = \begin{cases} 0, & \sigma_2 < \frac{1}{2} \\ [0,1], & \sigma_2 = \frac{1}{2} \\ 1, & \sigma_2 > \frac{1}{2}. \end{cases}$$

Since it does not matter for player 1 which strategy he chooses, all strategy combinations lying on the reaction function of agent 2 are Nash-equilibria. In particular $(0,0)$ and $(1,1)$ are the pure strategy equilibria.

Problem 4 (10 points)
Show the free-goods lemma: "Assume local non-satiation and weak monotonicity for all households. If $\left[\widehat{p}, \left(\widehat{x}^i\right)_{i=1,\ldots,n}\right]$ is a Walras equilibrium and the excess demand for a good j is negative, this good must be free ($p_j = 0$)."

Hint:

i) *Can the price be negative?*

ii) *Can the price be positive? You may use Walras' Law: "Given local non-satiation, the value of the excess demand is zero".*

Solution:

Assume the price is not negative, thus we have either $p_j < 0$ or $p_j > 0$.

i) If $p_j < 0$, a household can "buy" additional units of j without being worse of (weak monotonicity). Household has additional funding for preferred bundles (non-satiation). Contradiction to Walras equilibrium!

ii) If $p_j > 0$. Walras' Law implies

$$\begin{aligned} 0 &= p \cdot z(p) \\ &= p_j \cdot z_j(p) + \sum_{i \neq j} p_i \cdot z_i(p) \end{aligned}$$

As $z_j(p) < 0$ and $p_j > 0$ we obtain

$$0 < \sum_{i \neq j} p_i \cdot z_i(p), \text{ but}$$

$p_i \geq 0$ (because of i) and $z_i(p) \leq 0$ (Walras equilibrium) imply

$$\sum_{i \neq j} p_i \cdot z_i(p) \leq 0, \text{ Contradiction!!}$$

Hence $p_j = 0$.

Problem 5 (6 points)
Consider a Cournot-competition market with four firms, whose equilibrium outputs are $y_1 = 2, y_2 = 3$, $y_3 = 4$ and $y_4 = 1$. The price on the market is $p = 5$ and the marginal revenue of firm 1, given the four outputs, is $MR_1 = 4$. Determine the price-elasticity of demand on this market!

Solution:
Cournot competition leads to

$$\begin{aligned} MR_i &= \frac{\partial p(Y)}{\partial y_i} y_i + p(Y) \\ &= p(Y)\left[\frac{\partial p(Y)}{\partial y_i}\frac{y_i}{Y}\frac{Y}{p(Y)} + 1\right] \\ &= p(Y)\left[s_i \cdot \frac{1}{\varepsilon_{Y,p}} + 1\right]. \end{aligned}$$

Inserting leads to

$$\begin{aligned} 4 &= 5\left(\frac{2}{10} \cdot \frac{1}{\varepsilon_{Y,p}} + 1\right) \iff \\ \varepsilon_{Y,p} &= -1. \end{aligned}$$

Problem 6 (12 points)
Consider the following hidden action model with the set of actions $E = \{e_1, e_2\}$ and two outcomes $x_H = 10$ and $x_L = 0$. The conditional probabilities of the outcomes, due to the effort, are given by the table

	x_H	x_L
e_1	$\frac{2}{3}$	$\frac{1}{3}$
e_2	$\frac{1}{2}$	$\frac{1}{2}$

.

The principal maximizes the (expected) difference of output and wage, the agent maximizes the (expected) difference of the wage and the effort costs. The effort costs are given by $c(e_1) = 3$ and $c(e_2) = 2$. The agent's reservation payoff is $\overline{u} = 0$.

a) What is the optimal contract $(w(e_1), w(e_2))$, when effort is observable?

b) Assume effort is not observable. Derive the participation constraint and the incentive constraint for the agent, given action e_1 is preferred by the principal!

Solution:

a) If effort is observable, the expected profit of principal is given by

$$\pi_P = \begin{cases} \frac{2}{3} \cdot 10 - w(e_1) & , e_1 \text{ is preferred} \\ \frac{1}{2} \cdot 10 - w(e_2) & , e_2 \text{ is preferred} \end{cases} . \tag{1}$$

Thus e_1 is preferred if

$$w(e_1) - w(e_2) \leq \frac{5}{3}$$

is possible.

The incentive and participation constraints are:

$$\begin{array}{cl} \begin{array}{c} w(e_1) - 3 \geq w(e_2) - 2, \\ w(e_1) - 3 \geq 0. \end{array} & , e_1 \text{ is preferred} \\ \begin{array}{c} w(e_1) - 3 \leq w(e_2) - 2, \\ w(e_2) - 2 \geq 0. \end{array} & , e_2 \text{ is preferred} \end{array} .$$

The latter one and the lower part of (1) are not compatible. Therefore the principal prefers e_1 and an optimal contract is given by $w(e_1) = 3$ and $\frac{4}{3} \leq w(e_2) < 2$.

b) If effort is not observable and e_1 is preferred by the principal, we obtain the participation constraint by

$$\frac{2}{3} \cdot w_H + \frac{1}{3} \cdot w_L - 3 \geq 0.$$

The incentive constraint is given by

$$\frac{2}{3} \cdot w_H + \frac{1}{3} \cdot w_L - 3 \geq \frac{1}{2} \cdot w_H + \frac{1}{2} \cdot w_L - 2.$$