Solution for Problem 1
The players 3 and 5 are null players (**1 PKT**). By the null player axiom,

$$\mathrm{Sh}_3(v) = \mathrm{Sh}_5(v) = 0. \qquad (\mathbf{1\ PKT})$$

The players $1, 2$, and 3 are symmetric (**1 PKT**). By the symmetry axiom,

$$\mathrm{Sh}_1(v) = \mathrm{Sh}_2(v) = \mathrm{Sh}_4(v) \qquad (\mathbf{1\ PKT})$$

Moreover, by efficiency (**1 PKT**), $\sum_{i\in N} \mathrm{Sh}_i(v) = v(N) = 1$ (**1 PKT**). Therefore, we have

$$\begin{aligned} 1 &= \mathrm{Sh}_1(v) + \mathrm{Sh}_2(v) + \mathrm{Sh}_4(v) + \mathrm{Sh}_3(v) + \mathrm{Sh}_5(v) \\ &= \mathrm{Sh}_1(v) + \mathrm{Sh}_2(v) + \mathrm{Sh}_4(v) \\ &= 3 \cdot \mathrm{Sh}_1(v). \qquad (\mathbf{1\ PKT}) \end{aligned}$$

Thus, the payoffs are $\mathrm{Sh}_1(v) = \mathrm{Sh}_2(v) = \mathrm{Sh}_4(v) = 1/3$ and $\mathrm{Sh}_3(v) = \mathrm{Sh}_5(v) = 0.$ (**1 PKT**)

Solution for Problem 2

(a) The utility functions of A and B are strictly increasing in at least one good. Consider a bundle (ω_1, ω_2). By choosing a small amount more of the good a for A and b for B we find a better bundle for the agent which lies in an $\varepsilon-$ ball around (ω_1, ω_2), such that local non-satiation holds. We now can imply Walras' Law and $p \cdot z(p) = 0$ holds.

(b) $p \cdot z(p) = 0$ implies

$$p_b z_b = -p_a z_a$$

and therefore

$$z_b = -\frac{p_a}{p_b} \frac{p_a^2 - 4p_a p_b}{p_b^2} = \frac{4p_a^2 p_b - p_a^3}{p_b^3}$$

(c) The apple market is cleared if the excess demand is zero.

$$\frac{p_a^2 - 4p_a p_b}{p_b^2} = 0 \iff p_a = 4p_b \iff \frac{p_a}{p_b} = 4.$$

For the price ratio $\frac{p_a}{p_b} = 4$ the apple market is cleared.

The market clearance lemma now says (remember local non-satiation), if the apple market is cleared, the banana market is cleared, too. Therefore the same price ratio clears the banana market.

Another way to obtain the solution is to calculate the prize ratios, such that $z_b = 0$:

$$\begin{aligned} \frac{4p_a^2 p_b - p_a^3}{p_b^3} &= 0 \iff \\ 4p_a^2 p_b - p_a^3 &= 0 \iff \\ 4p_b &= p_a. \end{aligned}$$

Solution for Problem 3

(a)

	contract 1	contract 2	reject both contracts
u_{low}	6	5	0
u_{high}	-3	1	0

The high-skilled agent chooses contract 2, the low-skilled agent chooses contract 1.Therefore the types are revealed and both are hired.

(b)

	contract 1	contract 2	reject both contracts
u_{low}	4	5	0
u_{high}	-1	-2	0

The high-skilled agent chooses none of the contracts, the low-skilled agent chooses contract 2. Again, the types are revealed.

(c)

	contract 1	contract 2	reject both contracts
u_{low}	11	6	0
u_{high}	4	0	0

The high-skilled agent and the low-skilled agent choose the first contract. The types are not revealed.

Solution for Problem 4

a) We have 4 subgames. The first subgame is the whole game. The other subgames have their initial nodes at the firm 2's decision nodes. One strategy is for instance:

$$x_2 = \begin{cases} 0; & x_1 = 0 \\ 4; & x_1 = 4 \\ 6; & x_1 = 6 \end{cases}.$$

b) The profit function of firm 2 is given by:

$$\pi_2(x_1, x_2) = (12 - 2X)\, x_2 - \frac{1}{2} x_2^2.$$

First-order condition:

$$\frac{\partial \pi_2}{\partial x_2} = 12 - 2X - 2x_2 - 2x_2 \overset{!}{=} 0.$$

Reaction function of firm 2:

$$x_2^R(x_1) = \begin{cases} 2 - \frac{1}{3}x_1; & x_1 \le 6 \\ 0 & x_1 > 6 \end{cases}.$$

Firm 1's profit function is given by:

$$\pi_1(x_1, x_2) = (12 - 2X)\, x_1 - \frac{1}{4} x_1^2.$$

We have to compare:

$$\begin{aligned} \pi_1\left(0, x_2^R(0)\right) &= 0 \\ \pi_1\left(4, x_2^R(4)\right) &= \left(12 - 2\left(4 + \frac{2}{3}\right)\right) 4 - 4 = \frac{20}{3} \\ \pi_1\left(6, x_2^R(6)\right) &= (12 - 12) \cdot 6 - \frac{36}{4} < 0 \end{aligned}$$

As a result, the subgame perfect equilibrium is given by:

$$\left(x_1, x_2^R(0), x_2^R(4), x_2^R(6)\right) = \left(4, 2, \frac{2}{3}, 0\right)$$

c) By b) we obtain, that if firm 1 chooses $x_1 = 0$ or $x_1 = 4$, firm 2 chooses a positive output and there is no deterrence. If firm 1 chooses $x_1 = 6$, firm 2 does not produce. Hence $x_1^L = 6$. But $\pi_1\left(6, x_2^R(6)\right) = (12 - 12) \cdot 6 - \frac{36}{4} < 0$, therefore deterrence is not beneficial.

Solution for Problem 5

(a) We have

$$\tau\left(c_B^L \mid c_A^H\right) = \frac{\Pr\left(c_B^L, c_A^H\right)}{\Pr\left(c_A^H\right)} = \frac{\Pr\left(c_B^L, c_A^H\right)}{\Pr\left(c_B^L, c_A^H\right) + \Pr\left(c_B^H, c_A^H\right)}, \qquad (\textbf{2 PKT})$$

giving

$$\tau\left(c_B^L \mid c_A^H\right) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{8}} = \frac{4}{5}. \qquad (\textbf{1 PKT})$$

(b) A player's strategy describes what he does when he knows his type, i.e., firm A's strategy assigns an action to every type $t_A \in \left\{c_A^L, c_A^H\right\}$. The firm is a price setter. Thus, it choose a price $p_A \in [0, +\infty)$. A strategy of firm A is a function (**1 PKT**)

$$s_A : T_A \longrightarrow A_A$$

where $T_A = \left\{c_A^L, c_A^H\right\}$ (1 **PKT**) and $A_A = [0, +\infty)$ (1 **PKT**)

A Bayesian equilibrium is a strategy combination $s^* = (s_A^*, s_B^*)$ where both strategies are best answers to the other strategy (1 **PKT**), i.e.,

$$s_A^*(t_A) \in \arg\max_{a_A \in A_A} \left[\tau\left(c_B^L \mid t_A\right) \cdot u_A\left(a_A, s_B^*\left(c_B^L\right), t_A\right) + \tau\left(c_B^H \mid t_A\right) \cdot u_A\left(a_A, s_B^*\left(c_B^H\right), t_A\right)\right]$$

for both $t_A \in \left\{c_A^L, c_A^H\right\}$ and

$$s_B^*(t_B) \in \arg\max_{a_B \in A_B} \left[\tau\left(c_A^L \mid t_B\right) \cdot u_B\left(a_B, s_A^*\left(c_A^L\right), t_B\right) + \tau\left(c_A^H \mid t_B\right) \cdot u_B\left(a_B, s_A^*\left(c_A^H\right), t_B\right)\right]$$

for both $t_B \in \left\{c_B^L, c_B^H\right\}$. (1 **PKT**)

Solution for Problem 6

(a) Example: $\lfloor A, A, A, A, A \rfloor = \lfloor$action stage1, action in stage 2 if stage 1 was AC, .. was AD, was BC, was $BD\rfloor$ (**1 PKT**)

Thus, player 1 has $2^5 = 32$ strategies. (**1 PKT**)

(b) In the stage game, $\lfloor B, D \rfloor$ is the only Nash equilibrium. (**2 PKT**)

We use backwards induction (**2 PKT**):

At stage 2, the only Nash equilibrium is obtained if player 1 chooses B and player 2 chooses D (independent of the outcome in the first stage). (**1 PKT**)

In the first stage, they may anticipate this, i.e., they know that the other player will play his stage game equilibrium strategy in the second stage, (irrespective of the first stage outcome). Therefore, both players also play their stage game equilibrium strategy on the first stage. (**1 PKT**)

Thus, the subgame perfect equilibrium is $(\lfloor B, B, B, B, B \rfloor, \lfloor D, D, D, D, D \rfloor)$. (**1 PKT**)

Solution for Problem 7

$$\begin{aligned} Q &= q_1 + q_2 + q_3 = 600 \\ H &= \left(\frac{q_1}{Q}\right)^2 + \left(\frac{q_2}{Q}\right)^2 + \left(\frac{q_3}{Q}\right)^2 \\ &= \frac{1}{36} + \frac{1}{9} + \frac{1}{4} = \frac{14}{36} = \frac{7}{18} \end{aligned}$$