Advanced Microeconomics

Final Winter 2019/2020

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	7	$\sum_{mid}$	$\sum$	Grade
PUNKTE:										

Problem 1 (8 points)
Two firms 1 and 2 compete in quantities. Firm 1 is the leader and firm 2 is the follower. $p(X) = 130 - 2X$ is the inverse demand function with $X = x_1 + x_2$. Firm 1 faces the cost function $C_1(x_1) = 10x_1$, while firm 2's cost function is given by

$$C_2(x_2) = \begin{cases} 30x_2 + F, & x_2 > 0 \\ 0, & x_2 = 0 \end{cases}$$

Determine F such that entry of firm 2 is blockaded.

Solution
To check for blockade, we need to determine firm 1's monopolistic quantity. The profit function is given by $\Pi_1^M = (130 - 2x_1 - 10)\, x_1$. The first order condition $\frac{\partial \Pi_1}{\partial x_1} = 120 - 4x_1 \overset{!}{=} 0$ gives the monopolistic quantity of $x_1^M = 30$. Firm 2's profit is given by $\Pi_2(x_1^M, x_2) = (130 - 2x_1^M - 2x_2 - 30)x_2 - F$. Maximizing profits gives

$$\begin{aligned} \frac{\partial \Pi_2}{\partial x_2} = 100 - 60 - 4x_2 &\overset{!}{=} 0 \\ x_2^* &= 10. \end{aligned}$$

The entry of firm 2 is blockaded if

$$\begin{aligned} \Pi_2(x_1^M, x_2^*) = (130 - 2x_1^M - 2x_2^* - 30)x_2^* - F &\leq 0 \\ (100 - 60 - 20)10 - F &\leq 0 \\ F &\geq 200. \end{aligned}$$

So the entry is blockaded for $F \geq 200$.

Problem 2 (8 points)
Consider the following two-person game with mixed strategies. Determine both reaction functions and illustrate them graphically. Determine all equilibria.

		player 2	
		l	r
player 1	u	$(1,3)$	$(2,3)$
	b	$(5,2)$	$(1,2)$

Solution:
Let σ_1 denote the probability of player 1 to play u and let σ_2 denote the probability of player 2 to play l. Player 1 prefers u over b if

$$\begin{aligned} 1\sigma_2 + 2(1-\sigma_2) &\geq 5\sigma_2 + 1(1-\sigma_2) \\ 1-\sigma_2 &\geq 4\sigma_2 \\ 1 &\geq 5\sigma_2 \\ \sigma_2 &\leq \frac{1}{5} \end{aligned}$$

holds. We get

$$\sigma_1^R(\sigma_2) = \begin{cases} 1, & \sigma_2 < \frac{1}{5} \\ [0,1], & \sigma_2 = \frac{1}{5} \\ 0 & \sigma_2 > \frac{1}{5} \end{cases}.$$

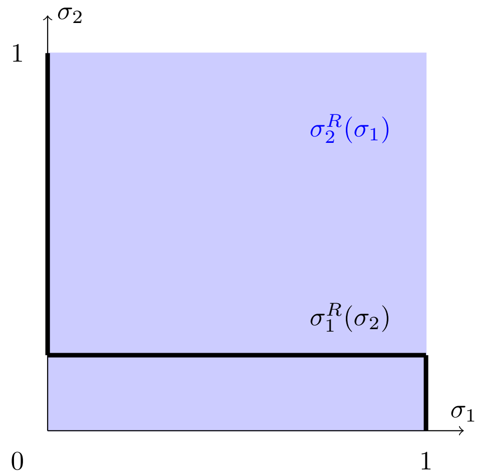
Player 2 is indifferent between playing l and r. Since

$$\begin{aligned} 3\sigma_1 + 2(1-\sigma_1) &\geq 3\sigma_1 + 2(1-\sigma_1) \\ 0 &\geq 0 \end{aligned}$$

holds. We get

$$\sigma_2^R(\sigma_1) = [0,1]$$

Hence, all equilibria in mixed strategies are given by $\left(\sigma_1^R(\sigma_2), \sigma_2\right), \sigma_2 \in [0,1]$. Especially, the two equilibria given by $(\sigma_1,\sigma_2) = (1,0)$ and $(\sigma_1,\sigma_2) = (0,1)$ are the only equilibria in pure strategies. A graphical illustration is given below.



σ_2
1
$\sigma_2^R(\sigma_1)$
$\sigma_1^R(\sigma_2)$
σ_1
0
1

Problem 3 (12 points)

A risk-neutral principal considers to hire an agent who undertakes a project for the principal. The hired agent can choose between two levels of effort $e \in \{0, 1\}$. If the agent chooses $e = 1$, the project yields $R_H = 15$ with probability 1. If the agent chooses $e = 0$, the project yields $R_H = 15$ with probability 1/3 and $R_L = 0$ with probability 2/3. The principal can observe the outcome of the project but cannot observe the agent's level of effort. Hence, the principal can condition wage payments $w_H \in \{2, 6\}$ or $w_L \in \{2, 6\}$ on the project's outcomes R_H and R_L, respectively. The vNM-utility of the hired agent is given by $u(w, e) = v(w) - c(e)$, where $v(2) = 2$, $v(6) = 5$ and $c(0) = 0$, $c(1) = 1$. The agent's reservation utility is given by $\bar{u} = 3.75$.

(a) State the agent's incentive constraint and his two participation constraints.

(b) Determine the optimal contract that is offered by the principal who incites the agent to choose $e = 1$.

Solution:

(a) Incentive constraint: The agent chooses a high level of effort ($e = 1$) if

$$\begin{aligned}
1 \cdot u(w_H, 1) &\geq \frac{1}{3} \cdot u(w_H, 0) + \frac{2}{3} \cdot u(w_L, 0) \\
v(w_H) - 1 &\geq \frac{1}{3} \cdot v(w_H) + \frac{2}{3} \cdot v(w_L) \\
\frac{2}{3} \cdot v(w_H) &\geq \frac{2}{3} \cdot v(w_L) + 1 \\
v(w_H) &\geq v(w_L) + \frac{3}{2},
\end{aligned}$$

where w_H is the wage payment if the project yields $R_H = 15$ and w_L is the wage payment if the project yields $R_L = 0$. The agent chooses a low level of effort ($e = 0$) if the inequality stated above holds with opposite sign. Participation constraints: The agent who chooses $e = 1$ participates if

$$\begin{aligned}
u(w_H, 1) &\geq \bar{u} \\
v(w_H) - 1 &\geq 3.75 \\
v(w_H) &\geq 4.75.
\end{aligned}$$

The agent who chooses $e = 0$ participates if

$$\begin{aligned}
\frac{1}{3} \cdot u(w_H, 0) + \frac{2}{3} \cdot u(w_L, 0) &\geq \bar{u} \\
\frac{1}{3} v(w_H) + \frac{2}{3} v(w_L) &\geq 3.75.
\end{aligned}$$

(b) The principal who incites the agent to choose $e = 1$ maximizes

$$\Pi(w_H, w_L, e) = 15 - w_H$$

subject to the incentive and participation constraints stated in **(a)**. The agent chooses $e = 1$ if $v(w_H) \geq v(w_L) + \frac{3}{2}$, which holds if $w_H = 6$ and $w_L = 2$. And, the agent participates if $v(w_h) \geq 4.75$, which holds if $w_H = 6$. Hence, the principal must offer $(w_H, w_L) = (6, 2)$.

Problem 4 (8 points)
Let (N, v) be a cooperative game with players $N = \{A, B, C\}$ and the coalition function

$$v(K) = \begin{cases} 0, & K = \emptyset \\ 5, & K \in \{\{A\}, \{B\}\} \\ 10, & K = \{C\} \\ 20, & K \in \{\{A, C\}, \{B, C\}\} \\ 25, & K = \{A, B\} \\ 30, & K = \{A, B, C\} \end{cases}$$

Determine the Shapley payoffs. *Hint: Do you find symmetric players?*

Solution
There are six rank orders $(A, B, C), (A, C, B), (B, A, C), (B, C, A), (C, A, B), (C, B, A)$ denoted by ρ_1 to ρ_6. The marginal contributions of player A are given by

$$\begin{aligned} MC_A^{\rho_1} = MC_A^{\rho_2} &= v(\{A\}) - v(\emptyset) = 5 \\ MC_A^{\rho_3} &= v(\{A, B\}) - v(\{B\}) = 20 \\ MC_A^{\rho_4} = MC_A^{\rho_6} &= v(\{A, B, C\}) - v(\{B, C\}) = 10 \\ MC_A^{\rho_5} &= v(\{A, C\}) - v(\{C\}) = 10 \end{aligned}$$

The Shapley value of player 1 is thus given by

$$Sh_A(v) = \frac{1}{3!} \sum_{i=1}^{3!} MC_A^{\rho_i} = \frac{1}{6}(5 + 5 + 20 + 10 + 10 + 10) = \frac{60}{6} = 10.$$

Player A and player B are symmetric because $v(\emptyset \cup \{A\}) = v(\emptyset \cup \{B\})$ and $v(\{C\} \cup \{A\}) = v(\{C\} \cup \{B\})$. Hence, one can apply the symmetry axiom, which yields

$$Sh_B(v) = Sh_A(\nu) = 10.$$

The value of the grand coalition is given by $v(N) = 30$. By efficiency, $\sum_{i \in N} Sh_i(v) = 30 = v(N)$. Thus, one obtains

$$Sh_C(v) = 30 - Sh_A(v) - Sh_B(v) = 10.$$

Problem 5 (11 points)
Consider an exchange economy with two agents A, B and two goods $1, 2$. The utility function of each agent $i \in \{A, B\}$ is given by

$$u^i(x_1^i, x_2^i) = x_1^i \cdot x_2^i.$$

The endowments are given given by

$$\omega^A = (12, 36) \quad \text{and} \quad \omega^B = (28, 24).$$

The price of good 2 is given by $p_2 = 2$.
Determine the Walras equilibrium.

Solution:
Preferences of agent $i \in \{A, B\}$ are monotonic because $\frac{\partial u^i}{\partial x_1^i}, \frac{\partial u^i}{\partial x_2^i} \geq 0$. The marginal rate of substitution is given by $MRS^i = \frac{MU_1^i}{MU_1^i} = \frac{x_2^i}{x_1^i}$. If x_1^i increases, x_2^i must decrease along the indifference curve due to monotonicity. Hence, the marginal rate of substitution is decreasing in x_1^i. Preferences are thus convex. The demand of agent i for good 1 is found by solving

$$MRS^i = \frac{x_2^i}{x_1^i} \stackrel{!}{=} \frac{p_1}{2} = \frac{p_1}{p_2} = MOC$$

for x_2^i, which yields $x_2^i = \frac{p_1 \cdot x_1^i}{2}$, and plugging that solution into the budget constraint

$$p_1\omega_1^i + 2\omega_2^i = p_1 x_1^i + 2x_2^i = 2p_1 x_1^i.$$

This yields $x_1^i(p_1, 2) = \frac{\omega_1^i}{2} + \frac{\omega_2^i}{p_1}$. We have $x_2^i(p_1, 2) = \frac{p_1 \cdot x_1^i(p_1, 2)}{2} = \frac{p_1 \cdot \omega_1^i}{4} + \frac{\omega_2^i}{2}$. Hence, excess demand for good 1 is given by

$$\begin{aligned} z_1(p_1, 2) &= \sum_{i \in \{A,B\}} \left(x_1^i(p_1, 2) - \omega_1^i\right) \\ &= \sum_{i \in \{A,B\}} \left(\frac{\omega_2^i}{p_1} - \frac{\omega_1^i}{2}\right) \\ &= \frac{\omega_2^A + \omega_2^B}{p_1} - \frac{\omega_1^A + \omega_1^B}{2} \\ &= \frac{60}{p_1} - 20. \end{aligned}$$

The market clearing price of that market solves $z_1(p_1^*, 2) = \frac{60}{p_1^*} - 20 \stackrel{!}{=} 0$ and is therefore given by $p_1^* = 3$. Since preferences $u^i(x_1^i + \epsilon, x_2^i + \epsilon) = (x_1^i + \epsilon)(x_2^i + \epsilon)$ are strictly increasing in $\epsilon \in R^+$ for every $x_1^i, x_2^i \in \mathbb{R}_0^+$, they satisfy local non-satiation. Hence, we can apply

Walras' law: $p_1^* z_1(p_1^*, 2) + 2 z_2(p_1^*, 2) \stackrel{!}{=} 0$. Since the market of good 1 is cleared at prices $p_1^* = 3$, $p_2 = 2$, the market of good 2 must be cleared, too. The Walras equilibrium is given by $\left((p_1, p_2), \left(\left(x_1^A, x_2^A\right), \left(x_1^B, x_2^B\right)\right)\right)$ where $p_1 = 3$, $p_2 = 2$ and

$$
\begin{aligned}
x_1^A &= \frac{12}{2} + \frac{36}{3} = 6 + 12 = 18, \\
x_2^A &= \frac{3 \cdot 12}{4} + \frac{36}{2} = 9 + 18 = 27, \\
x_1^B &= \frac{28}{2} + \frac{24}{3} = 14 + 8 = 22, \\
x_2^B &= \frac{3 \cdot 28}{4} + \frac{24}{2} = 21 + 12 = 33.
\end{aligned}
$$

Problem 6 (7 points)
Consider an exchange economy with two agents A and B. The preferences of both agents are lexicographic with good 2 as the important good.
The initial endowment is given by

$$\omega = \left((\omega_1^A, \omega_2^A), (\omega_1^B, \omega_2^B)\right)$$

with $\omega_i^A + \omega_i^B > 0$, $i \in \{1, 2\}$.

(a) Determine the contract curve (i.e., all Pareto-efficient allocations).

(b) Determine ω's exchange lens.

(c) Determine the core.

Solution:
(a) In an exchange economy where both agents have lexicographic preferences and care for the same good, each feasible allocation (no excess demand) is Pareto-efficient, because no agent can improve without making the other agent worse off. The contract curve is the locus of all Pareto-efficient allocations (the whole Edgeworth-box). Hence, the contract curve is given by $\left((x_1^A, x_2^A), (\omega_1^A + \omega_1^B - x_1^A, \omega_2^A + \omega_2^B - x_2^A)\right)$, $x_1^A \in [0, \omega_1^A + \omega_1^B]$, $x_2^A \in [0, \omega_2^A + \omega_2^B]$. **(b)** The exchange lens of ω consists of the initial endowment only. **(c)** The core (intersection of contract curve and exchange lens) is thus given by the initial endowment ω.

Problem 7 (6 points)
Consider the utility function

$$U(x_1, x_2) = x_1^2 + x_2^3.$$

Assume $p_1, p_2 > 0$. Derive Hicksian demand!

Solution:

Marginal utility $\frac{\partial U}{\partial x_i} \geq 0$, $i = 1, 2$, is non-negative. Preferences are thus monotonic. The marginal rate of substitution is given by

$$MRS = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{2x_1}{3x_2^2}.$$

If x_1 increases, x_2 decreases along the indifference curve due to monotonicity. The MRS thus increases if x_1 increases (and if x_2 decreases). Preferences are thus concave. Therefore, the solution to the household's optimization problem is a corner solution. We compare expenditure $e(\bar{U}, p)$ for the two corner solutions $\left(\sqrt{\bar{U}}, 0\right)$ and $\left(0, \sqrt[3]{\bar{U}}\right)$. The household strictly prefers $\left(\sqrt{\bar{U}}, 0\right)$ over $\left(0, \sqrt[3]{\bar{U}}\right)$ if

$$\begin{aligned} p_1 \sqrt{\bar{U}} &< p_2 \sqrt[3]{\bar{U}}) \\ p_1^6 \bar{U} &< p_2^6 \\ \bar{U} &< \left(\frac{p_2}{p_1}\right)^6. \end{aligned}$$

Hicksian demand is thus given by

$$\chi\left(\bar{U}, p_1, p_2\right) = \begin{cases} \left(\sqrt{\bar{U}}, 0\right), & \bar{U} < \left(\frac{p_2}{p_1}\right)^6 \\ \left\{\left(\sqrt{\bar{U}}, 0\right), \left(0, \sqrt[3]{\bar{U}}\right)\right\}, & \bar{U} = \left(\frac{p_2}{p_1}\right)^6 \\ \left(0, \sqrt[3]{\bar{U}}\right), & \bar{U} > \left(\frac{p_2}{p_1}\right)^6 \end{cases}.$$