Advanced Microeconomics

Final Winter 2018/2019

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	$\sum_{mid}$	$\sum$	Grade
PUNKTE:									

Problem 1 (8 points)

Consider the following two-person game.

		player 2	
		l	r
player 1	u	$(3,2)$	$(1,3)$
	d	$(4,4)$	$(3,2)$

Both players discount the future with discount rate $\delta \in (0,1)$.

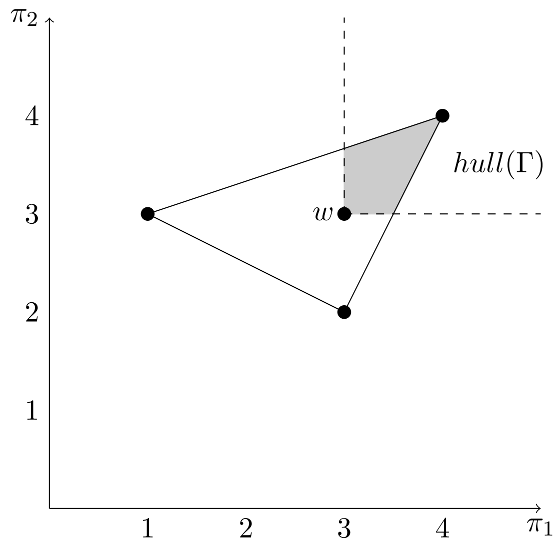
(a) Determine the worst punishment point.

(b) Consider the infinite repetition of the game where δ is assumed to be sufficiently high. Illustrate the set of possible payoffs in equilibrium in an appropriate figure. *Hint: Indicate the payoff of player 1 on the x-axis and the payoff of player 2 on the y-axis.*

Solution

(a) For player 1, the worst punishment is given by $w_1 = \min_{a_2} \max_{a_1} g_1(a_1, a_2)$, where a_i, $i = 1, 2$, denotes the strategy of player i and $g_1(a_1, a_2)$ denotes the payoff of player 1 if player 1 and 2 choose a_1, a_2, respectively. If player 2 plays l, player 1 will choose d $(3 < 4)$. If player 2 plays r, player 1 chooses d $(1 < 3)$. Therefore, player 2 chooses l for punishment of player 1 $(3 < 4)$ and thus $w_1 = 3$. Analogously, we receive $w_2 = 3$. The worst-punishment point is given by $w = (3, 3)$.

(b) An illustration of the set of equilibria in the infinitely repeated game is given below. Every payoff vector (π_1, π_2) that is in the convex hull of the game and to the north-east of the worst-punishment point w can be obtained in equilibrium for a sufficiently large δ.


π_2
4
3
2
1
w
$hull(\Gamma)$
1
2
3
4
π_1

Problem 2 (10 points)

Let (N, v) be a cooperative game with players $N = \{1, 2, 3\}$ and the coalition function

$$v(K) = \begin{cases} 0 & K \in \{\emptyset\} \\ 1, & K \in \{\{1\}, \{2\}, \{3\}, \{1, 2\}\} \\ 4, & K \in \{\{1, 3\}\} \\ 5, & K \in \{\{2, 3\}, \{1, 2, 3\}\} \end{cases}$$

(a) Determine the Shapley payoffs.

(b) Does the Shapley payoff vector lie in the core? *Hint: Consider the coalition* $K = \{2, 3\}$.

Solution

(a) There are six rank orders $(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)$ denoted by ρ_1 to ρ_6. The marginal contributions of player 1 are given by

$$\begin{aligned} MC_1^{\rho_1} = MC_1^{\rho_2} &= v(\{1\}) - v(\emptyset) = 1 \\ MC_1^{\rho_3} &= v(\{1, 2\}) - v(\{2\}) = 0 \\ MC_1^{\rho_4} = MC_1^{\rho_6} &= v(\{1, 2, 3\}) - v(\{2, 3\}) = 0 \\ MC_1^{\rho_5} &= v(\{1, 3\}) - v(\{3\}) = 3. \end{aligned}$$

The Shapley value of player 1 is thus given by

$$Sh_1(v) = \frac{1}{3!} \sum_{i=1}^{3!} MC_1^{\rho_i} = \frac{1}{6}(1 + 1 + 0 + 0 + 3 + 0) = \frac{5}{6}.$$

The marginal contributions of player 2 are given by

$$\begin{aligned} MC_2^{\rho_1} &= v(\{1, 2\}) - v(\{1\}) = 0 \\ MC_2^{\rho_2} = MC_2^{\rho_5} &= v(\{1, 2, 3\}) - v(\{1, 3\}) = 1 \\ MC_2^{\rho_3} = MC_2^{\rho_4} &= v(\{2\}) - v(\emptyset) = 1 \\ MC_2^{\rho_6} &= v(\{2, 3\}) - v(\{3\}) = 4. \end{aligned}$$

The Shapley value of player 2 is thus given by

$$Sh_2(v) = \frac{1}{3!} \sum_{i=1}^{3!} MC_2^{\rho_i} = \frac{1}{6}(0 + 1 + 1 + 1 + 1 + 4) = \frac{4}{3}.$$

The value of the grand coalition is given by $v(N) = 5$. By efficiency, $\sum_{i=1}^{3} Sh_i(v) = 5 = v(N)$. Thus, one obtains

$$Sh_3(v) = 5 - Sh_1(v) - Sh_2(v) = \frac{17}{6}.$$

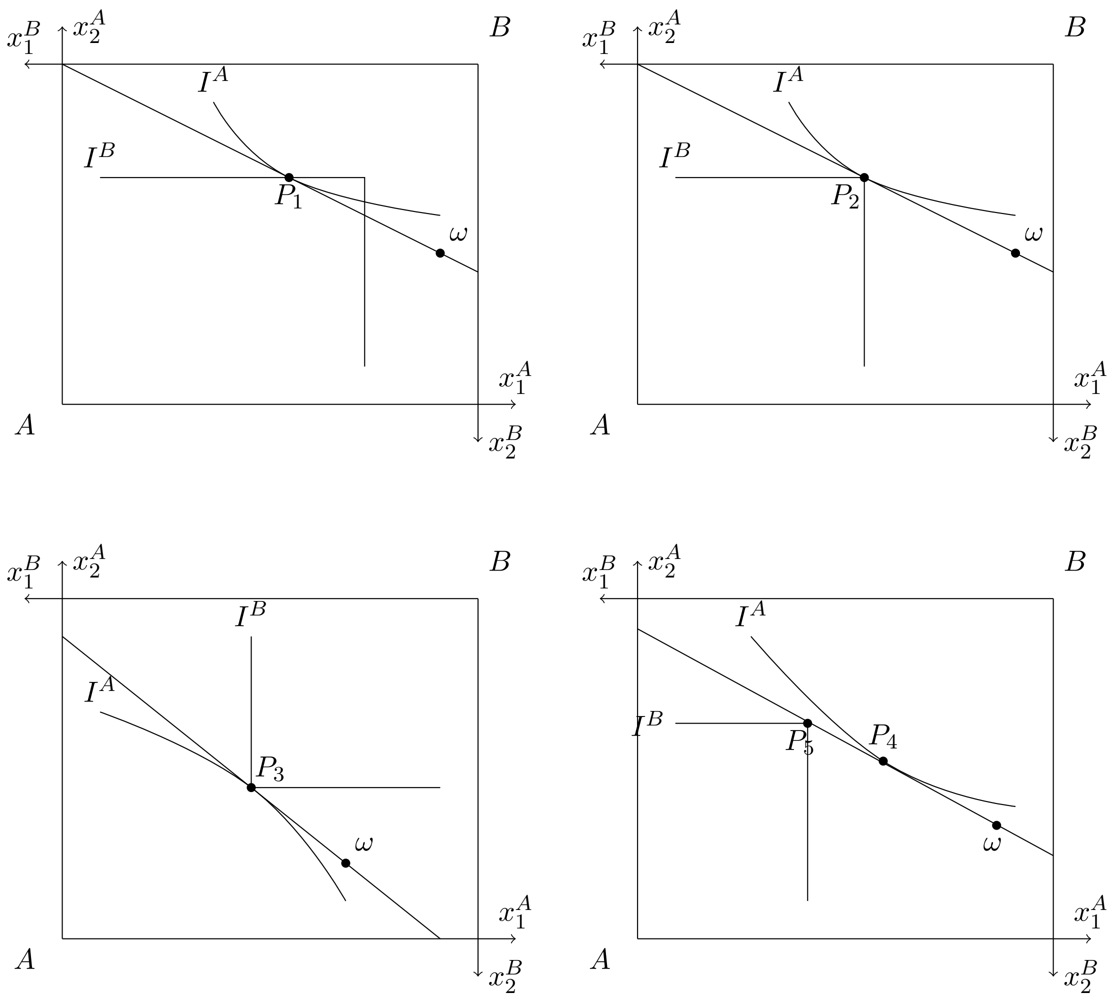
(b) The Shapley payoff vector is found to be $Sh(v) = \left(\frac{5}{6}, \frac{4}{3}, \frac{17}{6}\right)$. It is blockable by the coalition $\{2, 3\}$ due to

$$Sh_2(v) + Sh_3(v) = \frac{25}{6} < 5 = v(\{2, 3\}).$$

Hence, the Shapley payoff vector does not lie in the core.

Problem 3 (12 points)

Which P_i, $i = 1, 2, 3, 4, 5$ are a Walras allocation? The indifference curve of agent A is given by I^A, the indifference curve of agent B by I^B. Preferences are monotonic. The endowments of the agents are denoted by ω.



Solution:

P_1 is not a Walras allocation. Along the budget line, player B can be made better off if he consumes less of good 1 and more of good 2. Hence, P_1 is not a household optimum of agent B.

Since (i) the preferences of both agents are convex, (ii) $MRS_A = MOC$ holds in P_2, (iii) P_2 is the corner point of B's L-shaped indifference curve, and (iv) P_2 lies on the budget line of both agents, P_2 is a household optimum of agent A and a household optimum of agent B. Since both household optima lie in the same point, there is no excess demand. Therefore, P_2 is a Walras allocation.

P_3 is not a Walras allocation. Agent B prefers ω over P_3. Hence, P_3 is not a household optimum.

P_4 and P_5 are not a Walras allocation. The household optima do not lie in the same point ($P_4 \neq P_5$). There is excess demand for good 1.

Problem 4 (12 points)

Consider an exchange economy with two agents A and B. Consider $u_A(x_1^A, x_2^A) = \max(x_1^A, x_2^A)$ as utility function of agent A. The preferences of agent B are lexicographic with good 1 as the important good.

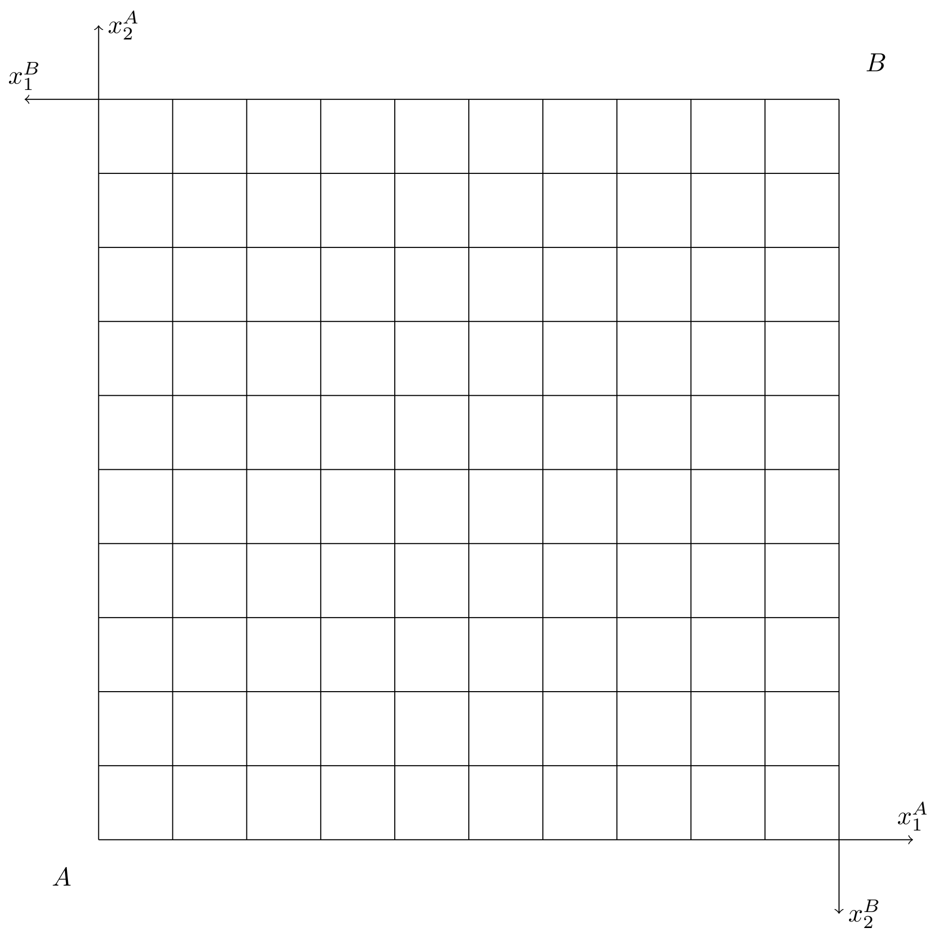
The initial endowment is given by

$$\omega^A = (6, 4) \text{ and } \omega^B = (4, 6).$$

(a) Use the graphic below to illustrate the initial endowments, the indifference curves of both agents that run through the endowment, the better sets and the exchange lens with respect to ω.

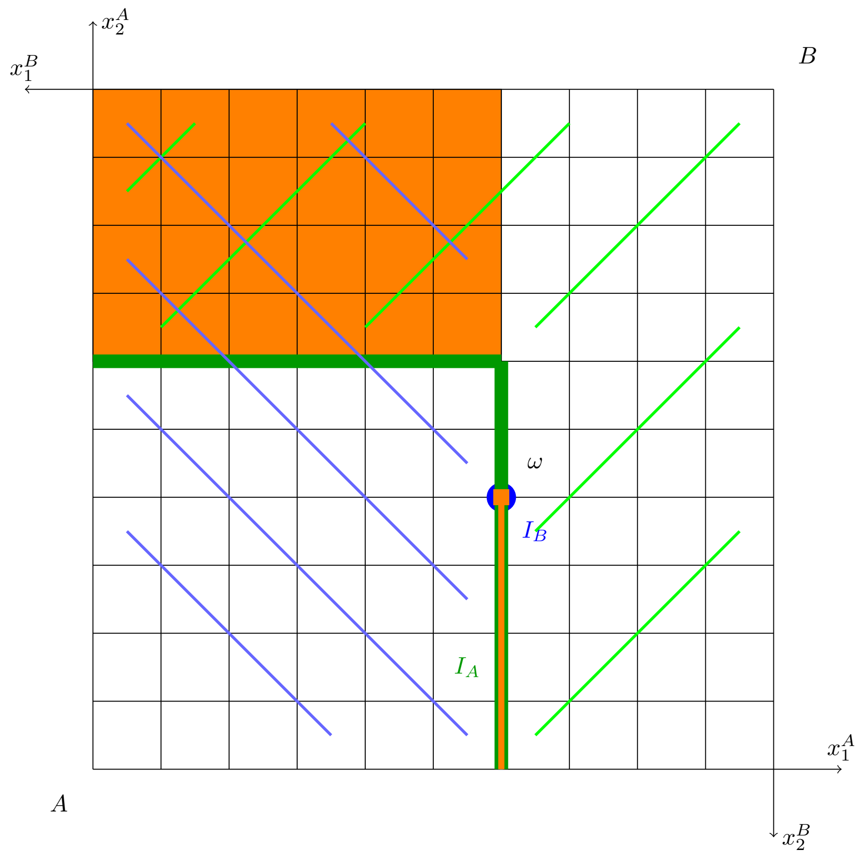
(b) Is point $C = ((6, 0), (4, 10))$ a Pareto improvement over the initial endowment? Is it Pareto efficient?

(c) Draw and explain the contract curve.



Solution

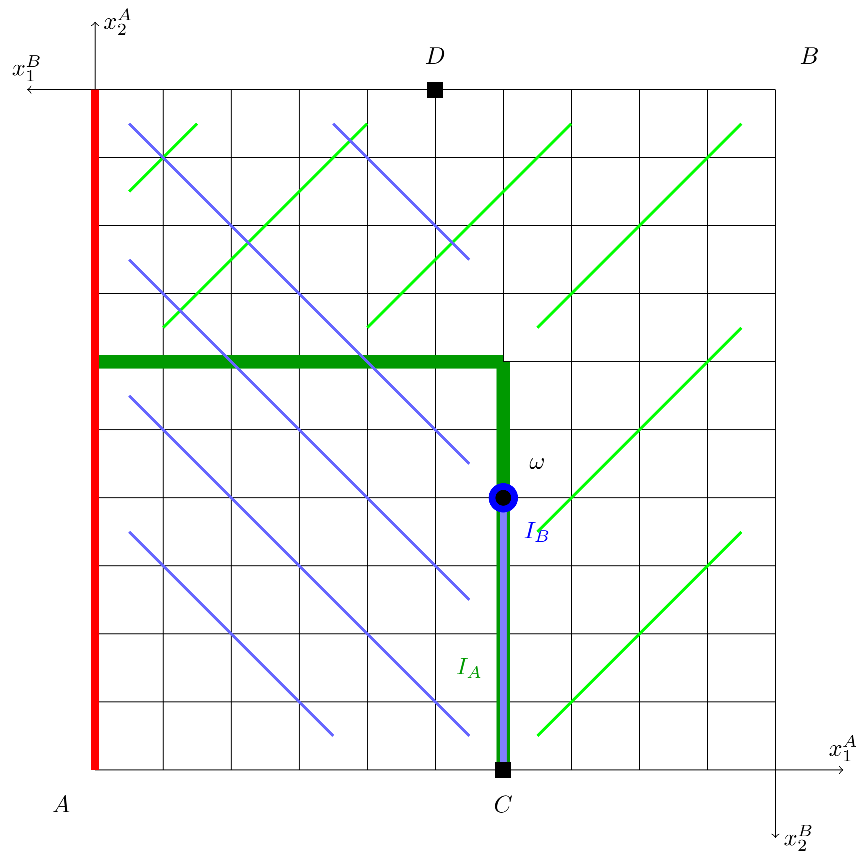
(a) The initial endowment is displayed at ω. The indifference curve of agent B is exactly the initial endowment (blue). The indifference curve of agent A is given as the green line. The better set of agent A is above and right of the indifference curve (light green), while the better set of agent B is left to the initial endowment (more of the important good 1) as well as the straight line below the initial endowment (same amount of good 1, but at least more of good 2). The better set of B is illustrated in light blue. The exchange lens with respect to ω is given as the orange area where at least one agent can improve without putting the other at a worse position (rectangle between (0,10), (6,10), (6,6), (0,6) excluding the line from $(6,6)$ to $(6,10)$; as well as the line from (6,4) to (6,0) including the initial endowment point.



(b) Point C is a Pareto improvement compared to the initial endowment because agent B is better off (x_1^B stays constant and $x_2^B > \omega_2^B$) while agent A stays on the same indifference curve. It is not Pareto efficient since both agents can improve by exchanging for instance one unit of good 1 (for agent B) for 10 units of good 2 (for agent 2).

The achieved point D ((5,10),(5,0)) is a Pareto improvement compared to point C because agent B has more of the important good (5>4) and agent A can achieve a utility of 10 compared to 6. Since a Pareto improvement is possible, point C cannot be Pareto efficient.

(c) The contract curve is the locus of all Pareto efficient allocations (red line). Only allocations for which agent B gets the maximum of the the import good are Pareto efficient; otherwise agent B is willing to give up all his units of good 2 in order to get more of good 1. If agent B is willing to give up all units of good 2, agent A can achieve the maximal utility of 10 (10 units of good 1) and thus giving good 1 to agent B can not make agent A worse off. If we assume that agent B has 10 units of the important good 1 then he will also start considering the amount of good 2 and is thus never willing to exchange anything if $x_1^B = 10$. The contract curve is thus given by the red line ((0,0),(10,10)) to ((0,10),(10,0)).



Problem 5 (6 points)

Consider five firms in a Cournot oligopoly. In equilibrium, price is given by $p^C = 4$, quantities by $x_1 = x_2 = 2$, $x_3 = x_4 = 1$ and $x_5 = 4$ and the price elasticity is $\epsilon_{X,p} = -4$. Determine the marginal costs of firm 2 in equilibrium.

Solution

In equilibrium marginal revenue equals marginal costs, that is $MC_2 = MR_2$. We can use the Amoroso-Robinson relation to find the marginal costs of firm 2.

$$R_2 = p(X) \cdot x_2$$

$$\begin{aligned}
MC_2 = MR_2 = \frac{\partial R_2}{\partial x_2} &= p(X) + \frac{dp(X)}{dX}\frac{dX}{dx_2}x_2 \\
&= p(X)\left[1 + \frac{dp(X)}{dX}\frac{x_2}{p(X)}\right] \\
&= p(X)\left[1 + \frac{dp(X)}{dX}\frac{X}{p(X)}\frac{x_2}{X}\right] \\
&= p(X)\left[1 + \frac{1}{\epsilon_{X,p}}s_2\right] \\
&= 4\left[1 + \frac{1}{-4}\cdot\frac{2}{10}\right] \\
&= \frac{19}{5} = 3.8
\end{aligned}$$

Problem 6 (12 points)

Two firms 1 and 2 compete in quantities. Firm 1 is the leader and firm 2 is the follower. $p(X) = 90 - X$ is the inverse demand function with $X = x_1 + x_2$. Firm 1 faces the cost function $C_1(x_1) = \frac{1}{2}x_1^2$, while firm 2's cost function is given by $C_2(x_2) = 30x_2$.

(a) Write down one example of a non-constant strategy of firm 2.

(b) Is the entry of firm 2 blockaded?

(c) Is it profitable for the leader to deter firm 2 from entry? *Hint: You need not calculate the Stackelberg equilibrium.*

Solution

(a) One strategy could be: $x_2(x_1) = x_1 \qquad \forall x_1$.

(b) The entry of firm 2 is blockaded if the monopoly price of firm 1 is equal or smaller than the average costs of firm 2: $AC_2 \geq p^M(C_1)$.

$$\Pi_1(x_1) = (90 - x_1)\,x_1 - \frac{1}{2}x_1^2$$

$$\begin{aligned}\frac{\partial \Pi_1}{\partial x_1} = 90 - 2x_1 - x_1 &\overset{!}{=} 0\\ 90 &= 3x_1\\ x_1^M &= 30\\ p^M &= 90 - 30 = 60\end{aligned}$$

Since $p^M(C_1) = 60 < 30 = AC_2$, the entry of firm 2 is not blockaded.

(c) We know that in the Stackelberg case, firm 1 will always make positive profits because of the quadratic cost function. Marginal costs of firm 1 are given by $MC_1 = \frac{\partial C_1(x_1)}{\partial x_1} = x_1$. So for $x_1 < 30$, marginal costs of firm 1 are smaller than marginal costs of firm 2 $(MC_2 = 30)$. The Stackelberg leader can thus always realize positive profits in the Stackelberg competition.

It is now sufficient to check the profits of firm 1 in case of deterrence.
Deterrence:

By backward induction, we first derive the reaction function of firm 2:

$$\Pi_2(x_1, x_2) = (90 - x_1 - x_2 - 30)\,x_2$$

$$\frac{\partial \Pi_2}{\partial x_2} = 60 - x_1 - 2x_2 \overset{!}{=} 0$$
$$x_2^R(x_1) = 30 - \frac{1}{2}x_1$$

Firm 2 will not enter the market if the best response is to play $x_2^R = 0$.

$$\begin{aligned} x_2^R(x_1) &\overset{!}{=} 0 \\ 30 - \frac{1}{2}x_1 &= 0 \\ x_1^L &= 60 \end{aligned}$$

Corresponding price and profit of firm 1 are given by

$$\begin{aligned} p^L &= 90 - 60 = 30 \\ \Pi_1^L &= 30 * 60 - \frac{1}{2}(60)^2 = 1800 - 1800 = 0 \end{aligned}$$

It is thus not profitable for firm 1 to deter firm 2 from entry $\left(\Pi_1^L = 0 < \Pi_1^S\right)$.

Alternative: We can also compare profits for the Stackelberg and the deterrence case.

Stackelberg:
We already derived the reaction function of firm 2: $x_2^R(x_1) = 30 - \frac{1}{2}x_1$.
The reduced profit function of firm 1 is given by

$$\begin{aligned} \Pi_1^r(x_1) = \Pi_1\left(x_1, x_2^R(x_1)\right) &= \left(90 - x_1 - x_2^R(x_1)\right)x_1 - \frac{1}{2}x_1^2 \\ &= \left(60 - \frac{1}{2}x_1\right)x_1 - \frac{1}{2}x_1^2, \end{aligned}$$

whose maximization leads to

$$\begin{aligned} \frac{\partial \Pi_1^r}{\partial x_1} = 60 - x_1 - x_1 &\overset{!}{=} 0 \\ 60 &= 2x_1 \\ x_1^S &= 30 \\ x_2^R(x_1^S) &= 30 - 15 = 15 \\ p^S &= 90 - (30 + 15) = 45 \\ \Pi_1^S &= 45 * 30 - 450 = 900 \end{aligned}$$

We can affirm that firm 1 realizes positive profits in the Stackelberg competition: $\Pi_1^S = 900 > 0 = \Pi_1^L$.