Advanced Microeconomics

Final Winter 2017/2018

23th February 2018

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	$\sum$	$\sum_{\text{mid}}$	Grade
PUNKTE:									

Problem 1 (10 points)

Two firms simultaneously compete in quantities. The cost function of firm 1 is publicly observable and equal to $C_1(x_1) = x_1^2$. Firm 2 has constant marginal and average costs which are either $c_l = 4$ or $c_h = 8$. The cost of firm 2 are known by firm 2 but not observable for firm 1. However firm 1 knows that c_l occurs with probability $\frac{1}{4}$, while c_h occurs with probability $\frac{3}{4}$. The inverse demand function is given by

$$p(X) = 120 - X$$

It can be shown that firm 2's reaction function is equal to

$$x_2^R(x_1) = \begin{cases} 56 - \frac{1}{2}x_1, & \text{if } c = c_l \\ 52 - \frac{1}{2}x_1, & \text{if } c = c_h. \end{cases}$$

(a) Determine the reaction function of firm 1.

(b) Sketch the reaction functions and the Bayesian equilibrium.

Solution:

(a) Expected profit of firm 1 is given by

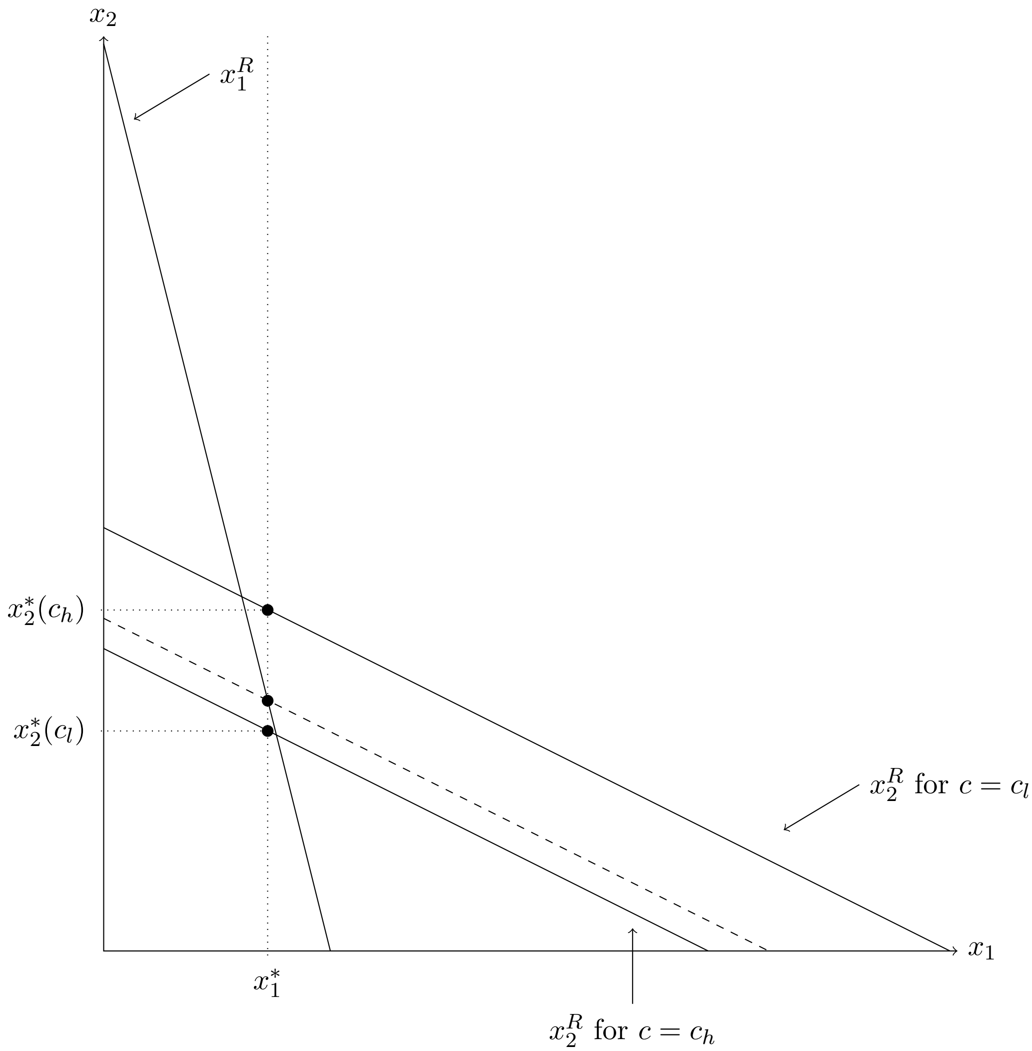
$$\begin{aligned} \Pi_1 =& \frac{1}{4}\left[\left(120 - x_1 - x_2^l\right)x_1 - x_1^2\right] \\ &+ \frac{3}{4}\left[\left(120 - x_1 - x_2^h\right)x_1 - x_1^2\right] \\ =& \left(120x_1 - 2x_1^2\right) - \left(\frac{1}{4}x_2^l + \frac{3}{4}x_2^h\right)x_1. \end{aligned}$$

We have

$$\frac{\partial \Pi_1}{\partial x_1} = 120 - 4x_1 - \left(\frac{1}{4}x_2^l + \frac{3}{4}x_2^h\right) \overset{!}{=} 0$$

and obtain the reaction function

$$x_1^R(x_2) = 30 - \frac{1}{4}\left(\frac{1}{4}x_2^l + \frac{3}{4}x_2^h\right).$$



(b)

Problem 2 (11 points)

Consider the following two-person game

		player 2	
		l	r
player 1	u	$(4,4)$	$(0,0)$
	d	$(0,0)$	$(3,4)$

and assume a discount factor of $\delta = 1$.

(a) State the Nash equilibria in pure strategies of this game. An explanation is not required.

(b) Give an example for a strategy of player 1 in the two-fold repetition. Explain the entries!

(c) Write down a subgame-perfect equilibrium in the two-fold repetition. Explain!

(d) Write down an equilibrium of the two-fold repetition which is not subgame perfect. An explanation is not required.

Solution:

(a) The Nash equilibria are given by (u,l) and (d,r).

(b) For example, a strategy of player 1 is given by

$$\left\lfloor \underbrace{u}_{\substack{\text{action at}\\ \text{the first stage}}}, \underbrace{u}_{\substack{\text{action at the}\\ \text{second stage}\\ \text{after } (u,l)}}, \underbrace{u}_{\substack{\text{action at the}\\ \text{second stage}\\ \text{after } (u,r)}}, \underbrace{u}_{\substack{\text{action at the}\\ \text{second stage}\\ \text{after } (d,l)}}, \underbrace{u}_{\substack{\text{action at the}\\ \text{second stage}\\ \text{after } (d,r)}} \right\rfloor$$

(c) An example is given by the strategy combination $(\lfloor u,u,u,u,u \rfloor, \lfloor l,l,l,l,l \rfloor)$. Both agents obtain their heighest payoff of 8. Hence, the given combination is an equilibrium. The equilibrium is subgame perfect because a stage equilibrium results in every subgame.

(d) The strategy combination $(\lfloor u, u, u, u, u \rfloor, \lfloor l, l, l, l, r \rfloor)$ is an equilibrium because agents realize their highest payoffs (8) and thus cannot deviate beneficially. This equilibrium is not subgame perfect because (u, r) is no equilibrium of the subgame which realizes after (d, r) is played at the first stage.

Problem 3 (12 points)

Consider an exchange economy with two agents A, B and only one good. The endowment is given by

$$\omega^A = 3, \omega^B = 7.$$

Sketch the exchange Edgeworth line including the endowment, the better set of agent B and the exchange lens with respect to ω for the following cases

(a) The preferences are given by $u^A\left(x^A\right) = 10 - x^A, u^B\left(x^B\right) = \frac{1}{1+x^B}$

(b) Agent A has strictly monotonic preferences, while for agent B the good is a neutral good.

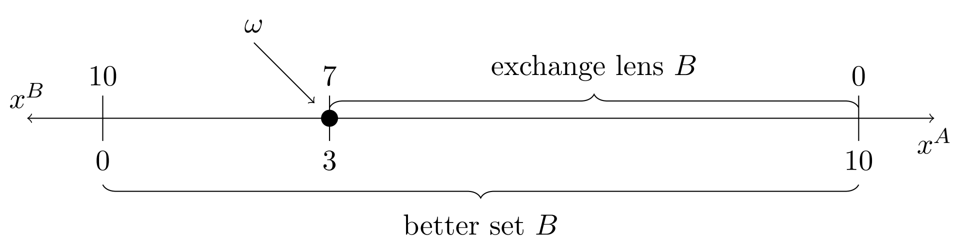
State the contract curve for each of the cases. Explain!

Solution:

(a)

Every allocation on the Edgeworth line is Pareto efficient because a shift to the right harms A and a shift to the left harms B. Thus, the contract curve is the whole Edgeworth line.

(b)



For agent B all allocations yield the same utility, while agent A's utility is increasing in the quantity of the good. Hence, $\left(x^A, x^B\right) = (10, 0)$ is the only Pareto-efficient point. Every other allocation can be improved in terms of Pareto-efficiency by a shift to the right. Thus, the contract curve is the allocation $(10, 0)$

Problem 4 (10 points)

Let (N, v) be a cooperative game with players $N = \{1, 2, 3\}$ and the coalition function

$$v(K) = \begin{cases} 0, & K \in \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}\} \\ 3, & K \in \{\{1,3\}\} \\ 4, & K \in \{\{2,3\}, \{1,2,3\}\} \end{cases}$$

Find the core.

Solution:

Feasibility and non-blockability by the grand coalition implies

$$x_1 + x_2 + x_3 = 4$$

Furthermore, non-blockability requires

$$x_1, x_2, x_3, x_1 + x_2 \geq 0$$

and

$$x_1 + x_3 \geq 3$$

as well as

$$x_2 + x_3 \geq 4.$$

We obtain

$$x_2 + x_3 = 4 - x_1 \overset{(x_1 \geq 0)}{\leq} 4 - 0 = 4$$

This inequality together with $x_2 + x_3 \geq 4$ yields

$$x_2 + x_3 = 4$$

and by using $x_1 + x_2 + x_3 = 4$ also

$$x_1 = 0.$$

According to $x_1 + x_3 \geq 3$, we then have

$$x_3 \geq 3.$$

The core is given by

$$C = \{(0, x_2, x_3) \in \mathbb{R}_+ \ : \ x_2 = 4 - x_3, x_3 \geq 3\}.$$

Problem 5 (8 points)

Two agents A, B trade two goods. Their endowments are given by

$$w^A = (20, 20) \text{ and } w^B = (10, 10).$$

The preferences of agent A satisfy local non-satiation. Agent B has strictly monotonic preferences. Assume that aggregate excess demand for good 1 is given by

$$z_1(p_1, p_2) = \frac{10p_1 - 60p_2}{p_1}$$

where $p_1, p_2 > 0$ denote the prices of good 1 and 2.

(a) Explain why $p \cdot z(p) = 0$ holds. *Hint: Show that you can apply Walras' law.*

(b) Determine the aggregate excess demand for good 2.

(c) Determine the price ratio $\frac{p_1}{p_2}$ such that the market for good 1 clears. Applying the market-clearance lemma, which prices clear market 2?

Solution:

(a) Since the preferences of agent B are strictly monotonic, local non-satiation holds for both agents. Then, by Walras' law, $p \cdot z(p) = 0$ holds.

(b) According to Walras' law, we have

$$p_1 \cdot \frac{10p_1 - 60p_2}{p_1} + p_2 \cdot z_2(p_1, p_2) = 0$$

and hence

$$z_2(p_1, p_2) = \frac{60p_2 - 10p_1}{p_2}.$$

(c) If the market for good 1 clears, we have

$$\frac{10p_1 - 60p_2}{p_1} = 0$$

and consequently

$$\frac{p_1}{p_2} = 6.$$

According to the market-clearance lemma (which can be applied because local non-satiation holds), if all but one market clear, the last market also clears. Consequently, the price ratio $\frac{p_1}{p_2} = 6$ also clears market 2.

Problem 6 (9 points)

Consider a polypsonistic labor market. An agent's productivity is denoted by t and uniformly distributed on the interval $[0,1]$. The reservation-wage function is given by

$$r(t) = \frac{1}{4}\left(t^2+1\right)$$

The principal cannot observe productivities and thus offers a constant wage w. An agent decides to work for the principal if and only if the offered wage is at least as large as her reservation wage.

(a) Which types of agents decide to work if the offered wage equals w?

(b) State the values of w for which an agent of type $t=0$ decides to work. Regarding efficiency, should an agent of type $t=0$ work?

(c) Show that the equilibrium wage equals $w^* = \frac{1}{2}$.

(d) Is the equilibrium efficient?

Hint: In equilibrium $w = E[t\ :\ t \in W]$ holds where W denotes the set of working agents.

Solution:

(a) An agent decides to work if

$$w \geq \frac{1}{4}\left(t^2+1\right)$$

holds. This condition is equivalent to

$$\sqrt{4w-1} \geq t.$$

Hence, agents with a productivity of $\sqrt{4w-1}$ and lower decide to work.

(b) An agent with productivity $t=0$ decides to work if

$$w \geq r(0) = \frac{1}{4}.$$

The agent should not work because his productivity is lower than his reservation wage:

$$t = 0 < \frac{1}{4} = r(0).$$

(c) The equilibrium wage can be determined as follows:

$$
\begin{aligned}
w &= E\left[t \ : \ t \in W\right] \\
&= E\left[t \ : \ t \leq \sqrt{4w-1}\right] \\
&= \frac{\int_0^{\sqrt{4w-1}} t\, dt}{\int_0^{\sqrt{4w-1}} dt} \\
&= \frac{\frac{1}{2}\sqrt{4w-1}^2}{\sqrt{4w-1}} \\
&= \frac{\sqrt{4w-1}}{2}. \\
\Rightarrow w^2 &= w - \frac{1}{4} \\
\Rightarrow w^2 - w + \frac{1}{4} &= 0 \\
\left(w - \frac{1}{2}\right)^2 &= 0
\end{aligned}
$$

We obtain $w^* = \frac{1}{2}$. **Alternatively:** Due to the uniform distribution on agents' productivities and the fact that a working agent's maximum productivity is $\sqrt{4w-1}$ for $w \in \left[\frac{1}{4}, \frac{1}{2}\right]$, the average productivity of the working agents is $\frac{\sqrt{4w-1}}{2}$, which must be equal to the wage w offered by the principal in equilibrium. Hence, we have

$$w = \frac{\sqrt{4w-1}}{2},$$

which is solved by $w^* = \frac{1}{2}$ (see equation above from line 5). **Remark:** The participation constraint is given by $t \leq \sqrt{4w^* - 1} = 1$. In equilibrium, therefore, all agents (but no more) work.

(d) The equilibrium where $w^* = \frac{1}{2}$ is not efficient because agents of type $t = 0$ work ($w^* = \frac{1}{2} > \frac{1}{4} = r(0)$), while they should not work ($t = 0 < \frac{1}{4} = r(0)$).