Advanced Microeconomics

Final Winter 2016/2017

24th February 2017

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	$\sum$	$\sum_{\text{mid}}$	Grade
PUNKTE:									

Problem 1 (12 points)

Consider the following two-person game. Find all equilibria in pure and properly mixed strategies and illustrate the reaction functions graphically!

		player 2	
		l	r
player 1	u	$(0,2)$	$(6,2)$
	d	$(3,2)$	$(0,0)$

Solution:

Let α denote the probability that player 1 plays u, and β the probability that player 2 plays l. We calculate the utilities of player 1 for playing a pure strategy if player 2 plays a mixed strategy and obtain

$$\begin{aligned} u_1(u,\beta) &= 6(1-\beta), \\ u_1(d,\beta) &= 3\beta. \end{aligned}$$

By comparing both utilities, we find

$$\begin{aligned} u_1(u,\beta) > u_1(d,\beta) &\iff \beta < \frac{2}{3}, \\ u_1(u,\beta) = u_1(d,\beta) &\iff \beta = \frac{2}{3}, \\ u_1(u,\beta) < u_1(d,\beta) &\iff \beta > \frac{2}{3}, \end{aligned}$$

which yields the reaction function of player 1

$$\alpha^R(\beta) = \begin{cases} 1, & \beta < \frac{2}{3} \\ [0,1], & \beta = \frac{2}{3} \\ 0, & \beta > \frac{2}{3}. \end{cases}$$

We calculate the utilities of player 2 for playing a pure strategy if player 1 plays a mixed strategy and obtain

$$\begin{aligned} u_2(\alpha,l) &= 2\alpha + 2(1-\alpha), \\ u_2(\alpha,r) &= 2\alpha. \end{aligned}$$

By comparing both utilities, we find

$$
\begin{aligned}
u_2(\alpha, l) < u_1(\alpha, r) \quad &\iff \quad \alpha < 1, \\
u_2(\alpha, l) = u_1(\alpha, r) \quad &\iff \quad \alpha = 1,
\end{aligned}
$$

which yields the reaction function of player 2

$$
\beta^R(\alpha) \quad = \quad \begin{cases} [0, 1], & \alpha = 1 \\ 1, & \alpha < 1. \end{cases}
$$

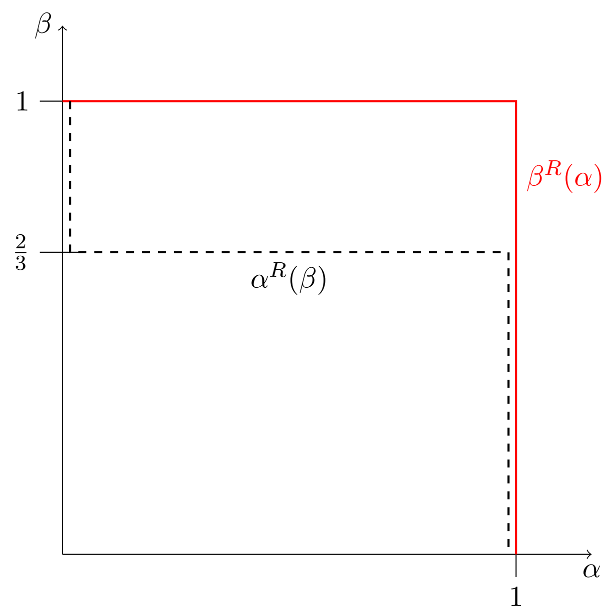
Determining the intersections yields two Nash equilibria in pure strategies

$$
(0, 1), (1, 0)
$$

and infinitly many Nash equilibria in properly mixed strategies given by

$$
\left\{ (1, \beta) : \beta \in \left(0, \frac{2}{3}\right] \right\}.
$$

A graphical illustration is given below.



Problem 2 (12 points)

Consider the following two-person game.

		player 2	
		l	r
player 1	u	$(2, 2)$	$(1, 0)$
	d	$(1, 3)$	$(4, 4)$

Both players discount the future with discount rate $\delta \in (0, 1)$.

(a) State all Nash-equilibria in pure strategies (no explanation required). (4)

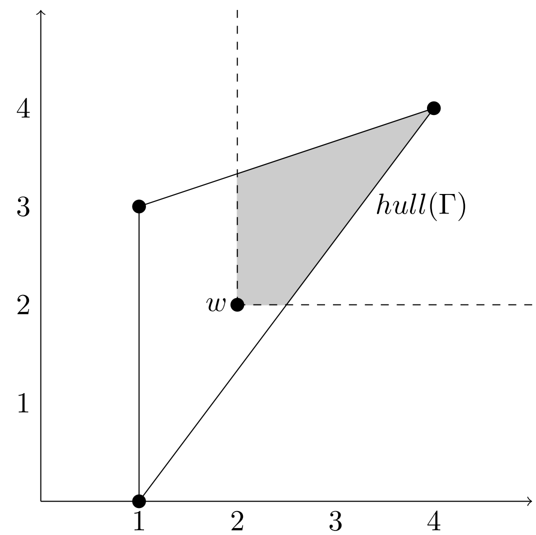
(b) Determine the worst punishment point (Explain!). Consider the appropriate infinite repetition of the game and illustrate the set of possible payoffs in equilibrium in an appropriate figure. Hint: Show the payoff of player 1 on the x-axis and the payoff of player 2 on the y-axis. (8)

Solution:

(a) The Nash equilibria are (u, l) and (d, r)

(b) For player 1 the worst punishment is given by $w_1 = \min_{a_2} \max_{a_1} g_1(a_1, a_2)$. If player 2 plays l, player 1 will choose u $(1 < 2)$. If player 2 plays r, player 1 chooses d $(1 < 4)$. Therefore, player 2 chooses l for punishment of player 1 $(2 < 4)$ and thus $w_1 = 2$. Analogously, we receive $w_2 = 2$. The worst-punishment point is given by $w = (2, 2)$.

An illustration of the set of equilibria in the infinitely repeated game is given below. Every payoff that is in the convex hull of the game and to the north-east of the worst-punishment point can be obtained in equilibrium for a sufficiently large δ.


4
3
2
1
1
2
3
4
w
hull(Γ)

Problem 3 (10 points)

Consider an exchange economy with two agents A and B. The preferences of the agents are represented by the following utility functions

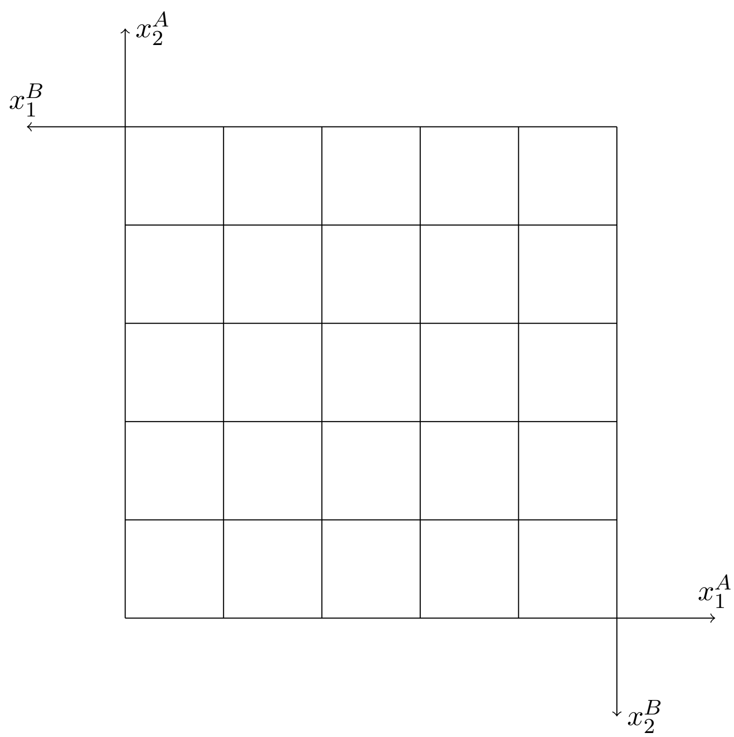
$$u^A\left(x_1^A, x_2^A\right) = \max\left\{x_1^A, x_2^A\right\},$$
$$u^B\left(x_1^B, x_2^B\right) = x_1^B + x_2^B.$$

The endowment is given by

$$\omega^A = (20, 30) \text{ and } \omega^B = (30, 20).$$

(a) Use the figure below to illustrate the endowments, the indifference curves of both agents that run through the endowment, the better set of agent A and the exchange lens with respect to ω.

(b) Is the allocation $P = ((10, 30), (40, 20))$ a Pareto improvement with respect to the endowment ω? Is P Pareto efficient?



Solution:

(b) Yes, P is a Pareto improvement with respect to ω. Agent A is indifferent and agent B is better off. However, P is not Pareto efficient. For example, moving further to the left to $Q = ((0, 30), (50, 20))$ leaves A indifferent and makes B better off. Hence, Q is a Pareto-improvement with respect to P and P is not Pareto-efficient. (The axes belonging to the coordinate system of A form the contract curve).

Problem 4 (10 points)

Consider the game (N, v), with $N = \{1, 2, 3\}$ and $v : 2^N \to \mathbb{R}$, defined by

$$v(K) = \begin{cases} \frac{|K|}{2} & |K| \text{ is even} \\ \frac{|K|-1}{2} & |K| \text{ is odd} \end{cases}$$

with $K \in 2^N$.

a) Calculate the Shapley payoffs!

b) Does the Shapley payoff vector lie in the core?

Solution:

a) There are six rank orders $(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)$ denoted by ρ_1 to ρ_6. The marginal contributions of player 1 are given by

$$\begin{aligned} MC_1^{\rho_1} = MC_1^{\rho_2} &= v(\{1\}) - v(\emptyset) = 0 \\ MC_1^{\rho_3} &= v(\{1, 2\}) - v(\{2\}) = 1 \\ MC_1^{\rho_4} = MC_1^{\rho_6} &= v(\{1, 2, 3\}) - v(\{2, 3\}) = 0 \\ MC_1^{\rho_5} &= v(\{1, 3\}) - v(\{3\}) = 1. \end{aligned}$$

The Shapley value of player 1 is thus given by

$$Sh_1(v) = \frac{1}{3!} \sum_{i=1}^{3!} MC_1^{\rho_i} = \frac{1}{6}(0 + 0 + 1 + 0 + 1 + 0) = \frac{1}{3}.$$

The marginal contributions of player 2 are given by

$$\begin{aligned} MC_2^{\rho_1} &= v(\{1, 2\}) - v(\{1\}) = 1 \\ MC_2^{\rho_2} = MC_2^{\rho_5} &= v(\{1, 2, 3\}) - v(\{1, 3\}) = 0 \\ MC_2^{\rho_3} = MC_2^{\rho_4} &= v(\{2\}) - v(\emptyset) = 0 \\ MC_2^{\rho_6} &= v(\{2, 3\}) - v(\{3\}) = 1. \end{aligned}$$

The Shapley value of player 2 is thus given by

$$Sh_2(v) = \frac{1}{3!} \sum_{i=1}^{3!} MC_2^{\rho_i} = \frac{1}{6}(1 + 0 + 1 + 0 + 0 + 0) = \frac{1}{3}.$$

The value of the grand coalition is given by $v(N) = 1$. By efficiency, $\sum_{i=1}^{3} Sh_i(v) = 1 = v(N)$. Thus, one obtains

$$Sh_3(v) = 1 - Sh_1(v) - Sh_2(v) = \frac{1}{3}.$$

Alternative solution: The above coalition function is symmetric due to $v(K) = f(|K|)$, f being a function $f : N \to \mathbb{R}$. Hence, the players are symmetric and $Sh_1(v) = Sh_2(v) = Sh_3(v)$ must hold. The value of the grand coalition is given by $v(N) = 1 = \sum_{i=1}^{3} Sh_i(v) = 3Sh_i(v)$ yielding $Sh_i(v) = \frac{1}{3}$ for all i.

b) The Shapley payoff vector is found to be $Sh(v) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. It is blockable by the coalition $\{1, 2\}$ due to

$$Sh_1(v) + Sh_2(v) = \frac{2}{3} < 1 = v(\{1, 2\}).$$

Hence, the Shaply payoff vector does not lie in the core.

Problem 5 (8 points)

Two firms 1 and 2 compete simultaneously in quantities. The inverse demand function is given by

$$p(X) = 40 - \frac{1}{2}X^2,$$

where $X = x_1 + x_2$. The cost function of firm 1 is given by

$$C_1(x_1) = \frac{1}{3}x_1^3,$$

while firm 2 has constant marginal and average costs of $c_2 > 0$.

a) Determine the monopoly price of firm 1 if firm 1 is the only supplier on the market.

b) Depending on c_2, determine whether the entry of firm 2 is blockaded!

Solution:

a) Firm 1's profit function is given by

$$\Pi_1(x_1) = \left(40 - \frac{1}{2}x_1^2\right) x_1 - \frac{1}{3}x_1^3$$

Profit maximization implies

$$\begin{aligned} \frac{d\Pi_1(x_1)}{dx_1} &= 40 - \frac{3}{2}x_1^2 - x_1^2 = 40 - \frac{5}{2}x_1^2 \stackrel{!}{=} 0 \\ x_1^2 &= 16 \\ \rightarrow x_1^M &= 4. \end{aligned}$$

The monopoly price of firm 1 is thus given by

$$p_1^M = p_1(x_1^M) = 40 - 8 = 32.$$

b) The entry of firm 2 is blockaded if $c_2 \geq p_1^M$. Thus, firm 2's entry is blockaded if $c_2 \geq 32$ and not blockaded if $c_2 < 32$.

Problem 6 (8 points)
Consider an exchange economy with two agents and at least two goods.

a) Assume that both agents have strictly monotonic preferences. Does $p \cdot z(p) = 0$ hold?

b) Assume $p \cdot z(p) = 0$ holds. Show that if all markets but one are cleared, the last one also clears or its price is zero (market clearance lemma).

Solution:

a) We show that

$$p \cdot z(p) = \sum_{i=1}^{2} p \cdot (x^i - w^i) = 0$$

holds, x^i being the household optimum and w^i being the initial endowment of agent i. *Proof:* x^i must be contained in the budget set. Hence, $p \cdot x^i > p \cdot w^i$ can be excluded. Assume $p \cdot x^i < p \cdot w^i$. The agent can afford bundles sufficiently close to x^i. Due to strict monotonicity, within the set of those affordable bundles, a bundle y^i exists that the household strictly prefers to x^i. This is a contradiction to x^i being a household optimum. Hence, $p \cdot x^i = p \cdot w^i$ which proves the claim.

b) Let n be the number of markets, p_j the price of good j, and $z_j(p)$ the excess demand on market j. Except k, $n-1$ markets are cleared. Thus $z_j(p)$=0, for all $j \neq k$. Hence,

$$0 = p \cdot z(p) = \sum_{j=1}^{n} p_j \cdot z_j(p) = \sum_{j \neq k} p_j \cdot z_j(p) + p_k \cdot z_k(p) = p_k \cdot z_k(p).$$

According to the above equation, either $p_k = 0$ or $z_k(p) = 0$ must hold.