Advanced Microeconomics

Final Winter 2015/2016

26th February 2016

You have to accomplish this test within **60 minutes.**

MATRIKEL-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

Anforderungen/Requirements:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!
Unterstreichen Sie Ihre Lösungen!/Underline your solutions!

	1	2	3	4	5	6	$\sum$	$\sum_{\text{mid}}$	Grade
PUNKTE:									

Problem 1 (12 points)

Consider the following two person game. Find all equilibria in pure and properly mixed strategies and illustrate the reaction function graphically!

player 2

		l	r
player 1	u	$(2,1)$	$(1,0)$
	d	$(0,0)$	$(1,2)$

Solution:

Let α denote the probability that player 1 plays u, and β the probability that player 2 plays l. We calculate the utilities of player 1 for playing a pure strategy if player 2 plays a mixed strategy and obtain

$$\begin{aligned} u_1(u,\beta) &= 2\beta + 1(1-\beta), \\ u_1(d,\beta) &= 0 + 1(1-\beta). \end{aligned}$$

By comparing both utilities, we find

$$\begin{aligned} u_1(u,\beta) > u_1(d,\beta) &\iff \beta > 0, \\ u_1(u,\beta) = u_1(d,\beta) &\iff \beta = 0, \end{aligned}$$

which yields the reaction function of player 1

$$\alpha^R(\beta) = \begin{cases} 1, & \beta > 0 \\ [0,1], & \beta = 0. \end{cases}$$

We calculate the utilities of player 2 for playing a pure strategy if player 1 plays a mixed strategy and obtain

$$\begin{aligned} u_2(\alpha,l) &= \alpha + 0, \\ u_2(\alpha,r) &= 0 + 2(1-\alpha). \end{aligned}$$

By comparing both utilities, we find

$$\begin{aligned} u_2(\alpha,l) < u_1(\alpha,r) &\iff \alpha < \frac{2}{3}, \\ u_2(\alpha,l) = u_1(\alpha,r) &\iff \alpha = \frac{2}{3}, \\ u_2(\alpha,l) > u_1(\alpha,r) &\iff \alpha > \frac{2}{3}, \end{aligned}$$

which yields the reaction function of player 2

$$\beta^R(\alpha) \quad = \quad \begin{cases} 0, & \alpha < \frac{2}{3} \\ [0,1], & \alpha = \frac{2}{3} \\ 1, & \alpha > \frac{2}{3}. \end{cases}$$

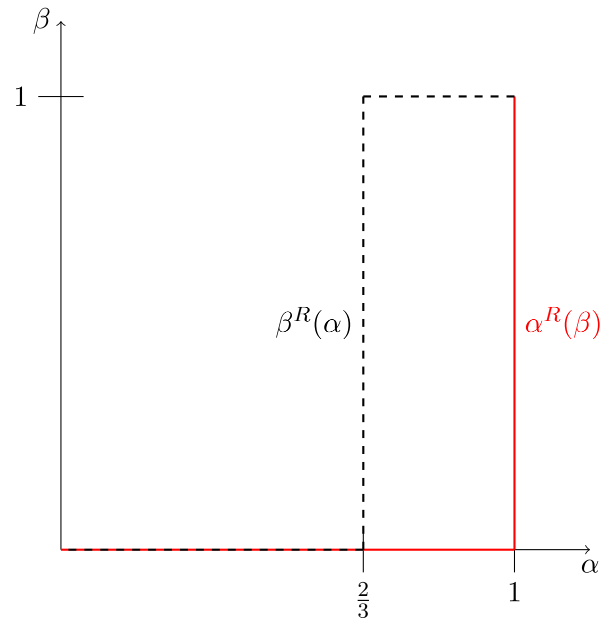
Determining the intersections yields two Nash equilibria in pure strategies

$$(0,0), (1,1)$$

and infinitly many Nash equilibria in mixed strategies given by

$$\left\{(\alpha, 0) : \alpha \leq \frac{2}{3}\right\}.$$

A graphical illustration may look like



Alternativ Solution:

Let α denote the probability that player 1 plays u, and β the probability that player 2 plays l. The players' utilities are given by

$$\begin{aligned} u_1(\alpha, \beta) &= 2\alpha\beta + \alpha(1-\beta) + (1-\alpha)(1-\beta), \\ u_2(\alpha, \beta) &= \alpha\beta + 2(1-\alpha)(1-\beta). \end{aligned}$$

We form the partial derivatives

$$\begin{aligned}
\frac{\partial u_1}{\partial \alpha} &= 2\beta + 1 - \beta - 1 + \beta = 2\beta, \\
\frac{\partial u_2}{\partial \beta} &= \alpha - 2 + 2\alpha = 3\alpha - 2
\end{aligned}$$

and find

$$\begin{aligned}
\frac{\partial u_1}{\partial \alpha} > 0 &\iff \beta > 0, \\
\frac{\partial u_1}{\partial \alpha} = 0 &\iff \beta = 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial u_2}{\partial \beta} < 0 &\iff \alpha < \frac{2}{3}, \\
\frac{\partial u_2}{\partial \beta} = 0 &\iff \alpha = \frac{2}{3}, \\
\frac{\partial u_2}{\partial \beta} > 0 &\iff \alpha > \frac{2}{3}.
\end{aligned}$$

Finnally, we obtain the reaction functions

$$\begin{aligned}
\alpha^R(\beta) &= \begin{cases} 1, & \beta > 0 \\ [0,1], & \beta = 0, \end{cases} \\
\beta^R(\alpha) &= \begin{cases} 0, & \alpha < \frac{2}{3} \\ [0,1], & \alpha = \frac{2}{3} \\ 1, & \alpha > \frac{2}{3}. \end{cases}
\end{aligned}$$

Determining the intersections yields two Nash equilibria in pure strategies

$$(0,0), (1,1)$$

and infinitly many Nash equilibria in mixed strategies given by

$$\left\{ (\alpha, 0) : \alpha \leq \frac{2}{3} \right\}.$$

Problem 2 (10 points)

A principal employs an agent, who is responsible for an investment project. The project can result in success or failure. The probability for a success is given by the agent's non-observable effort $e \in [0,1]$. If the project is successful, the principal receives a benefit R and nothing in case of failure. The agent receives a wage w if the project is successful and otherwise nothing. He faces effort costs $c(e) = \frac{1}{2}e^2$; his reservation payoff is given by $\overline{u} = 0$. The principal maximizes the (expected) difference of benefit and wage, while the agent maximizes the (expected) difference of wage and effort costs.

Derive the participation constraint and the incentive constraint! What is the optimal contract from the principal's point of view? What are the principal's and the agent's (expected) payoffs resulting from this contract?

Solution:

Let e^* denote the effort level that the principal wants to induce. The participation constraint is given by

$$e^* \cdot w - \frac{1}{2}(e^*)^2 \geq 0.$$

The incentive constraint is given by

$$e^* = \arg\max_e e \cdot w - \frac{1}{2}e^2$$

or equivalently

$$e^* w - \frac{1}{2}(e^*)^2 \geq ew - \frac{1}{2}e^2 \text{ for all } e \in [0,1].$$

The incentive constraint is satisfied if $e^* = w$. Then, the participation constraint is also fulfilled. The principal chooses the wage w such that he maximizes his expected payoff

$$e^* \cdot R - e^* \cdot w = wR - w^2$$

and thus the optimal contract is given by

$$w^* = \frac{R}{2}.$$

The principal induces the effort level $e^* = \frac{R}{2}$ and obtains the expected payoff $\frac{R^2}{4}$. The agent obtains the expected payoff $\frac{R^2}{8}$.

Problem 3 (12 points)

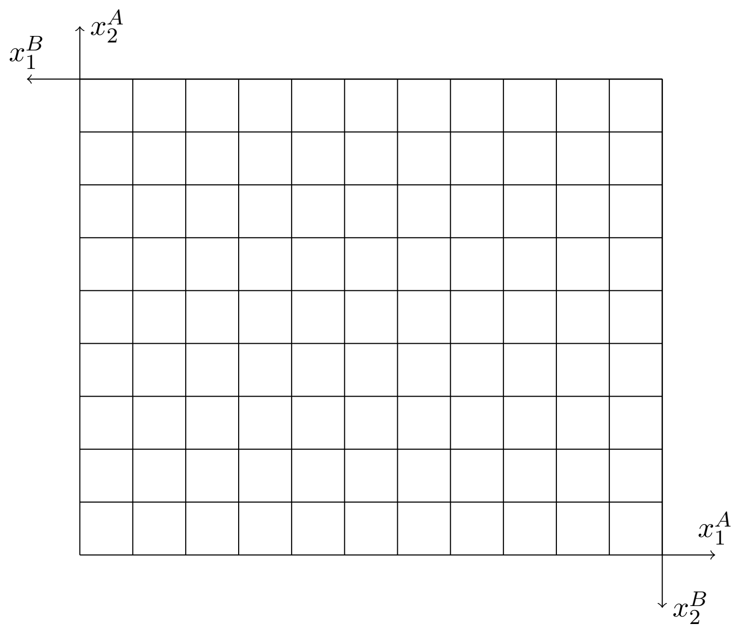
Consider an exchange economy with two agents A and B. The preferences of the agents are represented by the following utility functions

$$u^A\left(x_1^A, x_2^A\right) = x_1^A - x_2^A,$$
$$u^B\left(x_1^B, x_2^B\right) = x_1^B + x_2^B.$$

The endowment is given by

$$\omega^A = (7, 3) \text{ and } \omega^B = (4, 6).$$

Use the figure below to illustrate the endowments, the indifference curves of both agents that run through the endowment, the better set of agent A and the exchange lens with respect to ω. Determine extensively and illustrate all Pareto-efficient allocations.



Solution:

Any allocation with $x_2^A > 0$ cannot be Pareto efficient. In this case, if agent A gives some units of good 2 to agent B, this is a Pareto-improvement: Agent A's utility increases, because good 2 is a bad, and agent B's utility increases, because his preferences are monotonic.

All allocations satisfying $x_2^A = 0$ are Pareto efficient. The utility of both agents is increasing in the amount of good one. Hence, exchanging good 1 makes one agent worse off. Giving good 2 from agent B to agent A makes both agents worse off.

Problem 4 (8 points)

Consider the game (N, v) given by $N = \{1, 2, 3\}$ and $v : 2^N \to \mathbb{R}$, defined by

$$v(K) = \begin{cases} 1, & \{1,2\} \subseteq K \\ \frac{1}{2}, & \{1\} \subseteq K \\ 0, & \text{otherwise} \end{cases}.$$

Calculate the Shapley payoffs for all players!

Solution:

There are six rank orders: $(1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)$. The marginal contribution of player 1 is given by

$$\begin{aligned} MC_1^1 = MC_1^2 &= v(\{1\}) - v(\{\emptyset\}) = \frac{1}{2} - 0, \\ MC_1^3 &= v(\{1,2\}) - v(\{2\}) = 1 - 0, \\ MC_1^4 = MC_1^6 &= v(\{1,2,3\}) - v(\{2,3\}) = 1 - 0, \\ MC_1^5 &= v(\{1,3\}) - v(\{3\}) = \frac{1}{2} - 0. \end{aligned}$$

Player 1's payoff is thus given by

$$Sh_1(\nu) = \frac{1}{3!} \sum_{i=1}^{3!} MC_1^i = \frac{1}{6} \left(\frac{1}{2} + \frac{1}{2} + 1 + 1 + 1 + \frac{1}{2} \right) = \frac{1}{6} \frac{9}{2} = \frac{3}{4}.$$

Player 3 is null player. His marginal contribution is given by

$$MC_3^i = 0\,, \forall i.$$

Player 3's payoff is thus given by

$$Sh_3(\nu) = \frac{1}{3!} \sum_{i=1}^{3!} MC_1^i = \frac{1}{6} (0 + 0 + 0 + 0 + 0 + 0) = 0.$$

By efficiency, $\sum_{i=1}^{3} Sh_i(\nu) = v(N) = 1$. Therefore, we have

$$\begin{aligned} Sh_1(\nu) + Sh_2(\nu) + Sh_3(\nu) = 1. \\ \iff Sh_2(\nu) = 1 - Sh_1(\nu) = \frac{1}{4}. \end{aligned}$$

Alternative Solution:

Player 3 is null player. By the null player axiom,

$$Sh_3(\nu) = 0.$$

For players 1 and 2, two rank orders are left $(1, 2)$ and $(2, 1)$. Hence, player 1 obtains

$$Sh_1(\nu) = \frac{1}{2}\left(\frac{1}{2} - 0\right) + \frac{1}{2}(1 - 0) = \frac{3}{4}.$$

By efficiency $\sum_{i=1}^{3} Sh_i(\nu) = v(N) = 1$, player 2 obtains

$$Sh_2(\nu) = 1 - Sh_1(\nu) = \frac{1}{4}.$$

Problem 5 (6 points)

Consider a Cournot-competition market with n firms. We have price-elasticity $\epsilon_{Y,p} = -2$, price $p^C = 4$, and constant marginal costs $MC = 3$. Firm i's output is given by y_i. Overall output is given by $Y = \sum_{i=1}^{n} y_i$. Determine the equilibrium market share of firm 1.

Hint: Calculate marginal revenues from $R(y_i) = p(Y)y_i$.

Solution:

Cournot competition leads to Amoroso-Robinson relation

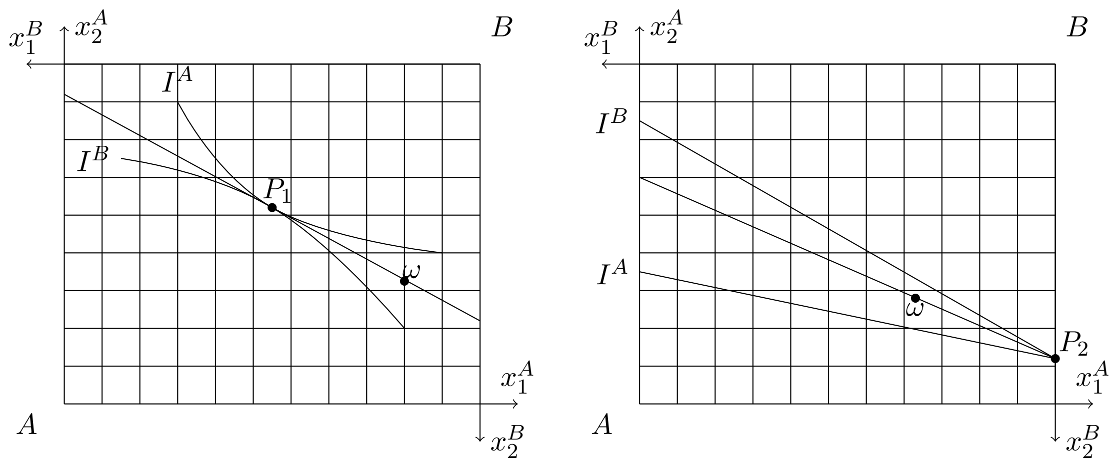
$$\begin{aligned} MR_1 &= \frac{\partial p(Y)}{\partial Y}\frac{\partial Y}{\partial y_1}y_1 + p(Y) = MC \\ &= p(Y)\left[\frac{\partial p(Y)}{\partial Y}\frac{Y}{p(Y)}\frac{y_1}{Y} + 1\right] \\ &= p(Y)\left[\frac{1}{\epsilon_{Y,p}}s_1 + 1\right]. \end{aligned}$$

Inserting leads to

$$\begin{aligned} 3 &= 4\left[s_1\frac{1}{-2} + 1\right] \iff \\ 3 - 4 &= \frac{-4}{2}s_1 \iff \\ s_1 &= \frac{1}{2}. \end{aligned}$$

Problem 6 (12 points)

Which P_i, $i = 1, 2, 3, 4, 5$ is a Walras allocation? Explain! The indifference curve of agent A is given by I^A, the indifference curve of agent B by I^B. Preferences are strictly monotonic. The endowment of the agents is denoted by ω.



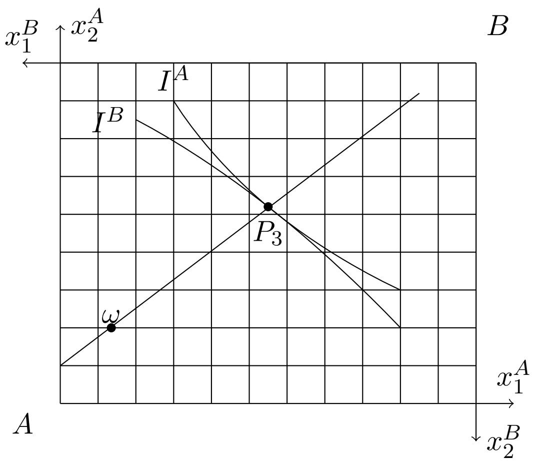
Solution:

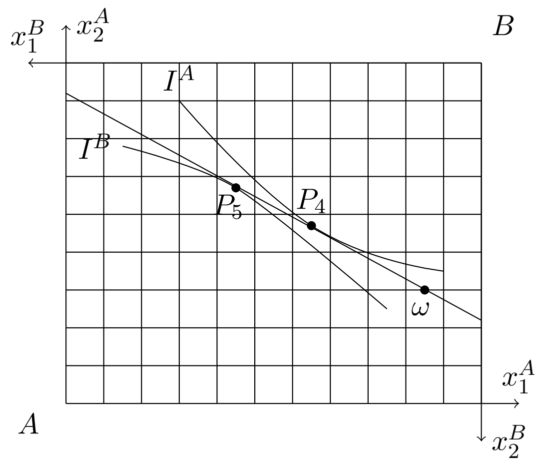
P_1 is a Walras allocation because indifference curves are convex,

$$MRS^A = \frac{p_1}{p_2} = MRS^B$$

holds, and both household optima lie in the same point given by P_1 (no excess demand). P_2 is not a Walras allocation. Player A prefers ω over P_2. Hence, P_2 is not a household optimum.

Problem 6 (continuation)





Solution:

P_3 is not a Walras allocation. Player B prefers ω over P_3. Hence, P_3 is not a household optimum. P_4 and P_5 are not a Walras allocation. The household optima do not lie in the same point ($P_4 \neq P_5$). There is excess demand for good 1.