Advanced Microeconomics

Final Exam Winter 2011/2012

You have to accomplish this test within **60 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	7	8	$\sum$

Problem 1 (5 points)

Onno owns a ship worth 500 million Euro. With a probability of $p = 0.1$, the ship sinks and its worth becomes 0. Onno is risk **loving**. (No more specific information on his risk attitudes is available.) Would he sell the ship for 475 million Euro?

Solution

We have $E([500, 0; 0.9.0.1]) = 0.9 \cdot 500 + 0.1 \cdot 0 = 450$ and $450 < 475$. Hence a risk-neutral Onno would prefer the degenerated lottery $[475; 1]$ against $[500, 0; 0.9.0.1]$. However, since Onno is risk-loving, i.e.

$$[500, 0; 0.9.0.1] \succsim [450; 1]$$

he is willing to pay for risk, i.e. there is some risk premium RP such that

$$[500, 0; 0.9.0.1] \sim [450 - RP; 1]$$

where $RP < 0$ (risk loving!).

Thus, if $450 - RP < 475$, i.e. $RP > -25$, then Onno would sell. If however $RP < -25$, Onno would not sell. Hence, the information given does not suffice to answer the question.

Problem 2 (10 points)

Consider the following strategic form game, where the payoffs/utilities of player A are the left numbers in the matrix entries.

Player A \ Player B	$b1$	$b2$	$b3$
$a1$	5,4	3,3	3,3
$a2$	2,2	4,5	2,1
$a3$	1,3	2,4	1,6

a) Successively delete strictly dominated strategies as long as this is possible (i.e., apply iterative strict dominance)! Provide **all necessary inequalities**! *Hint: You may use the matrices below to indicate the deleted strategies (one strategy per step only).*

Step 1

Player A \ Player B	$b1$	$b2$	$b3$
$a1$	5,4	3,3	3,3
$a2$	2,2	4,5	2,1
$a3$	1,3	2,4	1,6

Inequalities :

Step 2

Player A \ Player B	$b1$	$b2$	$b3$
$a1$	5,4	3,3	3,3
$a2$	2,2	4,5	2,1
$a3$	1,3	2,4	1,6

Inequalities :

Step 3

Player A \ Player B	$b1$	$b2$	$b3$
$a1$	5,4	3,3	3,3
$a2$	2,2	4,5	2,1
$a3$	1,3	2,4	1,6

Inequalities :

Step 4

Player A \ Player B	$b1$	$b2$	$b3$
$a1$	5,4	3,3	3,3
$a2$	2,2	4,5	2,1
$a3$	1,3	2,4	1,6

Inequalities :

b) Determine the Nash equilibria in **pure** strategies (if any) of the **original** game!

Solution

(a) We know that the order of eliminating strictly dominated strategies does not affect the outcome.

Strategy $a3$ is strictly dominated by $a1$, because

$$5 > 1, 3 > 2, 3 > 1.$$

After deletion of $a3$ we have

Player B

Player A		$b1$	$b2$	$b3$
	$a1$	5,4	3,3	3,3
	$a2$	2,2	4,5	2,1

Here $b3$ is strictly dominated by $b1$, because

$$4 > 3, 2 > 1.$$

After deletion of $b3$ we have

Player B

Player A		$b1$	$b2$
	$a1$	5,4	3,3
	$a2$	2,2	4,5

In this game, none of A's strategies is dominated because

$$5 > 2, \text{ but } 3 < 4,$$

and similarly for B,

$$5 > 2, \text{ but } 3 < 4.$$

(b) The game

Player B

Player A		$b1$	$b2$
	$a1$	5,4	3,3
	$a2$	2,2	4,5

has two pure-strategy equilibria (a1,b1), because

$$5 > 2, 4 > 3$$

and (a2,b2), because

$$4 > 3, 5 > 2$$

This already shows that there are no other pure-strategy equilibria.

Problem 3 (12 points)

A firm produces one good with a technology given by the production function $y = f(x) = x^{\frac{1}{3}}$. The factor price w and the price p for the good are fixed.

a) Explore whether the production function exhibits increasing returns to scale.

b) Determine the cost function!

Problem 3 (continuation)

c) Determine the demand function for the input factor!

d) How much will the firm produce?

Solution

(a) We consider a scale variation with a factor $t > 1$ and find

$$\begin{aligned} f(t \cdot x) &= \sqrt[3]{t \cdot x} \\ &= \sqrt[3]{t}\sqrt[3]{x} \\ &= t^{\frac{1}{3}} \cdot f(x), \end{aligned}$$

i.e., we have decreasing returns to scale. A counterexample may also confirm that the production function does not exhibit increasing returns to scale.

(b) Input x yields output $x^* = y^3$.

In order to obtain the cost function, we evaluate the minimum cost combination by the factor price,

$$\begin{aligned} C(y) &= w \cdot x^* \\ &= w \cdot y^3. \end{aligned}$$

(c) We employ the conditions for the optimal factor use

$$p \cdot \frac{1}{3} \cdot x^{-\frac{2}{3}} = p\frac{df}{dx} = w,$$

and solve this for x giving

$$\begin{aligned} x^{-\frac{2}{3}} &= \frac{3w}{p} \\ x &= \left(\frac{3w}{p}\right)^{-\frac{3}{2}} = \left(\frac{p}{3w}\right)^{\frac{3}{2}}. \end{aligned}$$

(d) We employ the condition for profit maximization and the solution of b)

$$p \overset{!}{=} MC = \frac{dC}{dy} = 3y^2 \cdot w$$

and solve for y,

$$y = \sqrt{\frac{p}{3w}}.$$

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Problem 4 (7 points)
Consider the game (N, v) given by $N = \{1, 2, 3\}$ and $v : 2^N \to \mathbb{R}$, defined by

$$v(K) = \begin{cases} 0, & K = \emptyset \\ 1, & K \in \{\{1\}, \{3\}\} \\ 2, & K \in \{\{2\}, \{1,3\}, \{1,2\}, \{2,3\}\} \\ 3, & K \in \{1,2,3\} \end{cases}.$$

Calculate the Shapley payoffs for all players!
Hint: Players 1 *and* 3 *are symmetric.*

Solution
We have to calculate the marginal contributions for all orders of N :

rank order	MC_1	MC_2	MC_3
123	1	1	1
132	1	1	1
213	0	2	1
231	1	2	0
312	1	1	1
321	1	1	1

The Shapley value ist the average of all these marginal contributions. Hence the solution is:

$$\begin{aligned} Sh_1(N,v) &= \frac{5}{6} = Sh_3(N,v) \\ Sh_2(N,v) &= \frac{4}{3}. \end{aligned}$$

Problem 5 (10 points)
Consider the following two person game! Calculate all equilibria in pure and properly mixed strategies! Illustrate both reaction functions graphically!

player 2

		A	B
player 1	C	$3, 2$	$3, 0$
	D	$0, 4$	$4, 5$

Solution
If α denotes the probability of chosing strategy C and β of chosing the strategy A :

$$\begin{aligned}\Pi_1(\alpha,\beta) &= 3\alpha\beta+3\alpha(1-\beta)+4(1-\alpha)(1-\beta)\\ \Pi_2(\alpha,\beta) &= 2\alpha\beta+4(1-\alpha)\beta+5(1-\alpha)(1-\beta)\end{aligned}$$

By the first order conditions we obtain

$$\begin{aligned}\frac{\partial\Pi_1}{\partial\alpha}(\alpha,\beta) &= 3\beta+3(1-\beta)-4(1-\beta)\\ &= 4\beta-1.\\ \frac{\partial\Pi_2}{\partial\beta}(\alpha,\beta) &= 2\alpha+4(1-\alpha)-5(1-\alpha)\\ &= 3\alpha-1\end{aligned}$$

$$\alpha^R(\beta)=\left\{\begin{array}{cc} 1 & \beta>\frac{1}{4}\\ [0,1] & \beta=\frac{1}{4}\\ 0 & \beta<\frac{1}{4}\end{array}\right\}$$

$$\beta^R(\alpha)=\left\{\begin{array}{cc} 1 & \alpha>\frac{1}{3}\\ [0,1] & \alpha=\frac{1}{3}\\ 0 & \alpha<\frac{1}{3}\end{array}\right\}$$

Nash equilibria in pure strategies are (C,A) and (D,B) . There is one equilibrium in properly mixed strategies, namely $\left(\frac{1}{3},\frac{1}{4}\right)$.

Problem 6 (7 points)
Show the market clearence theorem: If $p >> 0$ and all markets but one are cleared, all markets are cleared!

Assume: $p \cdot x^i = p \cdot \omega^i$ for every consumer $i \in N$ *(Walras' law)!*

Solution

If there are l markets and $l-1$ are cleared, the excess demand on these markets is 0.

Without loss of generality, markets $g = 1, ..., l-1$ are cleared. Applying Walras's law we get

$$p \sum_{i \in N} \left(x^i - \omega^i \right) = 0$$

This is equivalent to

$$\sum_{j=1}^{l} p_j \underbrace{\left(\sum_{i \in N} x^i_j - \sum_{i \in N} \omega^i_j \right)}_{= z_j(p)} = 0$$

Using that the markets $1, ..., l-1$ are cleared:

$$0 = pz(p) = p_l z_l(p).$$

$p >> 0$, hence $p_l > 0$ and $z_l(p) = 0$. Therefore the excess demand of the market l is zero and hence by definition the market is cleared.

Problem 7 (3 points)
Calculate the Herfindahl index for four firms supplying the quantities $q_1 = 6$, $q_2 = 3$, $q_3 = 1$, and $q_4 = 0$!

Solution

The quantities sum up to 10. This yields

$$H = \left(\frac{6}{10}\right)^2 + \left(\frac{3}{10}\right)^2 + \left(\frac{1}{10}\right)^2 + \left(\frac{0}{10}\right)^2 = \frac{23}{50}$$

Problem 8 (6 points)
Consider the production of two goods A and B with input factors capital (C) and labour (L) and with marginal rates of technical substitution $\left|\frac{dC}{dL}\right| = MRTS$ obeying

$$MRTS_A = 7 > 3 = MRTS_B.$$

Show that this situation is not efficient by pointing to a Pareto- improving factor reallocation!

Solution
If it holds that

$$MRST_A = 7 > 3 = MRST_B$$

and we increase the use of labour for the production of good A by a small unit, we have to decrease the use of labour for the production of good B by a small unit. Otherwise for producing the same amounts of goods A and B we have to reallocate the second input factor. For the production of A, according to the MRST, we have decrease the capital use for good A and still producing the same amount in comparison to the starting point. Otherwise only 3 additional units of capital are required to produce the same amount of good B.This means we save 4 units of capitel and can use them to increase the production of both goods or only one of them. This means the starting point was pareto- inefficient.