Advanced Microeconomics

Winter 2010/2011

28th February 2011

You have to accomplish this test within **120 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

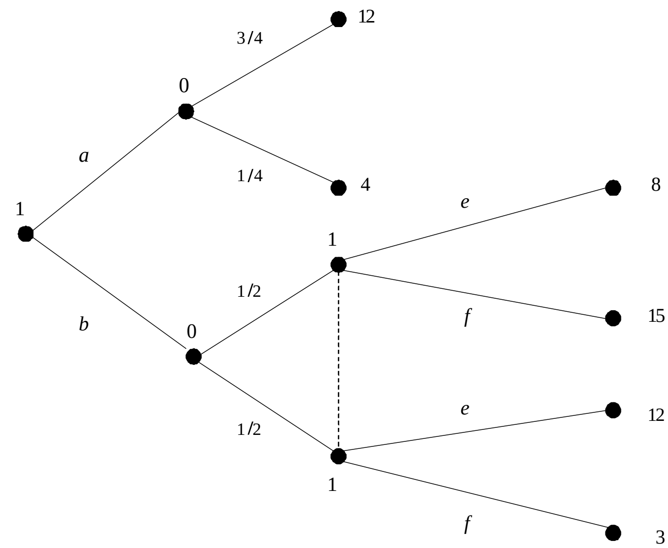
UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	7	8	9	10	11	12	$\sum$

Problem 1 (15 points)
Consider the following decision problem:



Nodes, indicated by '0' refer to nature, those indicated by '1' to the decider.

(a) How many subtrees does this decision tree have?

(b) Provide a list of all pure strategies of player 1!

(c) Is this a situation with perfect recall? Justify!

(d) Determine all optimal strategies, pure and properly mixed ones!

Solution

(a) The whole tree and the two trees starting when nature moves are subtrees.

(b) Pure Strategies are $[a, e]$, $[a, f]$, $[b, e]$ and $[b, f]$, where the first entry indicates the action, which was chosen in the first node and the second indicates the action at the uncertainty node.

(c) This is a situation with perfect recall. The only non trivial information set is the set after action b by player 1 and the move of the nature. The experience of player 1 for both nodes is the first node and his/her action b.. This means perfect recall.

(d) In the upper subtree player 1 gets an expected payoff of $\frac{3}{4} \cdot 12 + \frac{1}{4} \cdot 4 = 10$, when he choses action a. If he choses action b, he gets an expected payoff for action e of $\frac{1}{2} \cdot 8 + \frac{1}{2} \cdot 12 = 10$.If he choses b and f he gets an expected payoff of $\frac{1}{2} \cdot 15 + \frac{1}{2} \cdot 3 = 9$. So the optimal strategies are $[a, e]$, $[a, f]$ and $[b, e]$.

Problem 2 (15 points)
Consider the utility function $u(x_1, x_2) = \ln x_1 + 4x_2$, prices $p_1 = 1$ and $p_2 = 4$ and income 21. The price for good 1 changes to $p_1 = 2$. Calculate the compensating variation and the equivalent variation.

Solution

The household optimum before the price increase is calculated via:

$$MRS = \frac{\frac{1}{x_1}}{4} = \frac{1}{4x_1} \stackrel{!}{=} \frac{1}{4} = \frac{p_1}{p_2}.$$

So $x_1 = 1$ and $x_2 = \frac{21-1}{4} = 5$.
After the price increase we calculate

$$MRS = \frac{\frac{1}{x_1}}{4} = \frac{1}{4x_1} \stackrel{!}{=} \frac{2}{4} = \frac{p_1}{p_2}$$

and this yields to $x_1 = 1/2, x_2 = \frac{m-1}{4} = 5$.
For the compensating variation, we have to calculate

$$U^{old} = \ln 1 + 4 \cdot 5 \stackrel{!}{=} \ln(1/2) + m + cv - 1$$

and therefore $|\ln(1/2)| = CV$.
For the equivalent variation, we have to calculate

$$U^{new} = \ln 1/2 + 4 \cdot 5 \stackrel{!}{=} \ln(1) + m + ev - 1$$

and therefore $|\ln(1/2)| = EV$.

Problem 3 (10 points)
Consider the following two person game! Calculate all equilibria in pure and properly mixed strategies! Illustrate both reaction functions graphically!

		player 2	
		C	D
player 1	C	$1, 1$	$3, 2$
	D	$2, 4$	$0, 2$

Solution

Let p be the probability of player 1, that he plays action C and q the probability of player 2, that he plays action C.

The expected payoffs are:

$$\begin{aligned} u_1(p,q) &= pq + 3p(1-q) + 2(1-p)q \\ u_2(p,q) &= pq + 2p(1-q) + 4(1-p)q + 2(1-q)(1-p) \end{aligned}$$

Differentiate w.r.t. p respectively q :

$$\begin{aligned} \frac{\partial u_1}{\partial p}(p,q) &= q + 3(1-q) - 2q = 3 - 4q \\ \frac{\partial u_2}{\partial q}(p,q) &= p - 2p + 4(1-p) - 2(1-p) = 2 - 3p \end{aligned}$$

Therefore

$$p = \begin{cases} 1 & \text{if} \quad q < \frac{3}{4} \\ 0 & \text{if} \quad q > \frac{3}{4} \\ \in [0,1] & \text{if} \quad q = \frac{3}{4} \end{cases}$$

$$q = \begin{cases} 1 & \text{if} \quad p < \frac{2}{3} \\ 0 & \text{if} \quad p > \frac{2}{3} \\ \in [0,1] & \text{if} \quad p = \frac{2}{3} \end{cases}$$

Therefore there are 3 mixed equilibria: $p_1 = 1, q_1 = 0$, $p_2 = 0, q_2 = 1$ and $p_3 = \frac{3}{4}$, $q_3 = \frac{2}{3}$.

Problem 4 (10 points)

Andy and Bruno both like ice cream (good 1) and chocolate (good 2). Their initial endowments are $(\omega_1^A, \omega_2^A) = (8, 45)$ and $(\omega_1^B, \omega_2^B) = (32, 45)$. Andy's preferences are represented by the utility function $u_A(x_1, x_2) = x_1 x_2$. Bruno's preferences are described by $u_B(x_1, x_2) = x_1^2 \sqrt{x_2}$.

(a) Are the endowments Pareto-efficient?

(b) Determine the exchange lense for the given endowments!

(c) Calculate all Pareto- efficient allocations!

Solution

(a)

$$\begin{aligned} MRS_A &= \frac{x_2^A}{x_1^A} = \frac{45}{8} \\ MRS_B &= 4\frac{x_2^B}{x_1^B} = 4 \cdot \frac{45}{32} \end{aligned}$$

Therefore the endowments are Pareto-efficient.

(b) The exchange lense consists of the endowment, cause it is pareto- efficient.

(c)

$$MRS_A = \frac{x_2^A}{x_1^A} \overset{!}{=} 4\frac{x_2^B}{x_1^B} = 4\frac{90 - x_2^A}{40 - x_1^A}$$

Optimality requires

$$40x_2^A \overset{!}{=} 360x_1^A - 3x_1^A x_2^A$$

and therefore

$$x_1^A = \frac{40x_2^A}{360 - 3x_2^A},\ 0 \le x_2 \le 90$$

or

$$x_2^A = \frac{360x_1^A}{40 + 3x_1^A},\ 0 \le x_1 \le 40.$$

Problem 5 (10 points)
Form the derivative of the duality equation

$$\chi_g(p,\bar{U}) = x_g(p, e(p,\bar{U}))$$

with respect to p_g and derive the Slutsky equation for money income with the help of Shephard's lemma! Under which conditions is x_g a Giffen good?

Solution

Differentiated w.r.t p_g gives

$$\frac{\partial \chi_g}{\partial p_g} = \frac{\partial x_g}{\partial p_g} + \frac{\partial x_g}{\partial m}\frac{\partial e}{\partial p_g} \ \textbf{2Punkte}$$

and applying Shepard's lemma

$$\frac{\partial e\left(p,\bar{U}\right)}{\partial p_g} = \chi_g \ \textbf{2Punkte}$$

yields

$$\begin{aligned} \frac{\partial \chi_g}{\partial p_g} &= \frac{\partial x_g}{\partial p_g} + \frac{\partial x_g}{\partial m}\chi_g \ \textbf{1Punkt} \\ &\Longrightarrow \frac{\partial x_g}{\partial p_g} = \frac{\partial \chi_g}{\partial p_g} - \frac{\partial x_g}{\partial m}\chi_g \ \textbf{1Punkt}. \end{aligned}$$

Giffen goods satisfy $\partial x_g/\partial p_g > 0$ **1Punkt**, what requires $\frac{\partial \chi_g}{\partial p_g} > \frac{\partial x_g}{\partial m}\chi_g$ **1Punkt**. In particular, g is inferior/not normal ($\frac{\partial x_g}{\partial m} < 0$ **1Punkt**) since $\frac{\partial \chi_g}{\partial p_g} \leq 0$ **1Punkt**.

Problem 6 (10 points)
State the Independence Axiom from vNM-Theory on preferences over lotteries. Now consider the following lotteries:

$$\begin{aligned} L_1 &= [3000, 0; 1, 0] \text{ and } L_2 = \left[4000, 0; \frac{4}{5}, \frac{1}{5}\right] \\ L_3 &= \left[3000, 0; \frac{1}{4}, \frac{3}{4}\right] \text{ and } L_4 = \left[4000, 0; \frac{1}{5}, \frac{4}{5}\right] \end{aligned}$$

In experiments, a majority of people express $L_1 \prec L_2$ and $L_3 \succ L_4$. Show that these choices contradict the Independence Axiom! *Hint: Use the lottery that gives payoff* 0 *with probability* 1.

Solution
Independence axiom **I**: For any L_i, L_j, L_k and any p

$$L_i \succsim L_j \iff [L_i, L_k; p, 1-p] \succsim [L_j, L_k; p, 1-p]$$

3Punkte
Denote $L_0 = [0; 1]$. Appearently, we have $L_3 = \left[3000, 0; \frac{1}{4}, \frac{3}{4}\right] = \left[L_1, L_0; \frac{1}{4}, \frac{3}{4}\right]$. **3Punkte**

Further, $L_4 = \left[4000, 0; \frac{1}{5}, \frac{4}{5}\right] = \left[L_2, L_0; \frac{1}{4}, \frac{3}{4}\right]$ **1Punkt** what can be seen by

$$\begin{aligned} \left[L_2, L_0; \frac{1}{4}, \frac{3}{4}\right] &= \left[4000, 0, 0; \frac{1}{4} \cdot \frac{4}{5}, \frac{1}{4} \cdot \frac{1}{5}, \frac{3}{4} \cdot 1\right] \\ &= \left[4000, 0, 0; \frac{1}{5}, \frac{1}{20}, \frac{3}{4}\right] \\ &= \left[4000, 0, 0; \frac{1}{5}, \frac{4}{5}\right]. \end{aligned}$$ **2Punkte**

Thus, by **I**, $L_1 \prec L_2 \iff L_3 \prec L_4$. **1Punkt** Therefore, many people do not obey **I**.

Problem 7 (5 points)
Wrong answers are punished with negative points. You do *not* need to reason your answers in this problem!

		true	false
i)	The revelation principle implies: to tell the truth is a dominant strategy in every message game of a direct mechanism.		
ii)	If no direct mechanism that induces truthtelling can implement a given social choice rule, no direct mechanism can.		
iii)	The Clarke-Groves-mechanism is not direct.		

Solution

		true	false
i)	The revelation principle implies: to tell the truth is a dominant strategy in every message game of a direct mechanism.		x
ii)	If no direct mechanism that induces truthtelling can implement a given social choice rule, no direct mechanism can.	x	
iii)	The Clarke-Groves-mechanism is not direct.		x

Problem 8 (15 points)
Let there be an amount ω of a good ($\ell = 1$) and $n \geq 3$ agents. Each agent i has transitive preferences $\succsim_i$ on the set of feasible allocations

$$\left\{ \mathbf{x} \in \mathbb{R}_+^n \mid \sum_{i=1}^{n} x_i \leq \omega \right\}.$$

Note that an agent's preference relation orders the allocations, not x_i.

(a) Assume that each agent i has a preference relation that is represented by

$$u_i(\mathbf{x}) = x_i + \frac{1}{2} \sum_{j \neq i} x_j$$

Determine all Pareto- optimal allocations.

(b) We say an allocation $\mathbf{x}$ beats $\mathbf{y}$, denoted by $\mathbf{x}\mathcal{B}\mathbf{y}$, if $\mathbf{x} \succsim_i \mathbf{y}$ holds for all n agents. Is the relation $\mathcal{B}$ transitive, given any individual transitive preference relation on the set of feasible allocations? Is $\mathcal{B}$ complete?

Solution

(a) Every allocation satisfying $\sum_{i=1}^{n} x_i = \omega$ is Pareto- efficient **2Punkt**.[1] Reducing an x_i by some amount Δ can compensated only by inclining $\sum_{j \neq i} x_j$ by at least 2Δ what is not feasible **1Punkt** + **1Punkt**. Further, if $\sum_{i=1}^{n} x_i < \omega$, $\mathbf{x} + \frac{1}{n}(\boldsymbol{\omega} - \mathbf{x})$ is an Pareto improvement **1Punkt**. +Verständnis von Pareto-Konzepten **2Punkte**

(b) Yes**1Punkt**. Let $\mathbf{x}\mathcal{B}\mathbf{y}$ and $\mathbf{y}\mathcal{B}\mathbf{z}$. **1Punkt**That means $\mathbf{x} \succsim_i \mathbf{y}$ and $\mathbf{y} \succsim_i \mathbf{z}$ for all i.**1Punkt** Hence, by transitivity of $\succsim_i$, $\mathbf{x} \succsim_i \mathbf{z}$ for all i, and therefore $\mathbf{x}\mathcal{B}\mathbf{z}$.**1Punkt** The relation is not comlete**1Punkt** as e.g. only half of the agents might rank $\mathbf{x} \succsim_i \mathbf{y}$ while only a half things $\mathbf{x} \succsim_i \mathbf{y}$. **1Punkt** + Def. Transitivität **1Punkte** + Def. Vollständigkeit **1Punkte**

[1] $x_i = \frac{1}{n}$ damit P-o, gibt einen punkt
$x_i = x_j$ reicht nicht

Problem 9 (10 points)
Determine the Nash equilibria in pure strategies of the following strategic game (Tamino) involving two players $i = 1, 2$ with strategy sets $S_i = [0, \infty)$ and payoff functions

$$u_i(s_i, s_j) = \begin{cases} -s_i, & s_i < s_j \\ \frac{w_i}{2} - s_i, & s_i = s_j \\ w_i - s_j, & s_i > s_j. \end{cases}$$

where $w_1 \geq w_2 > 0$.

Solution:

If $s_j < w_i$, player i maximizes his payoff by waiting longer than player j. In this case he obtains utility $w_i - s_j > 0$. If $s_j \geq w_i$, player i cannot obtain a positive payoff. Thus, he maximizes his utility by choosing $s_i = 0$. Thus, $(w_2 + x, 0)$ with $x \geq 0$ are the only equilibria for $w_1 > w_2$. For player 1 a change in his strategy c.p. changes his payoff only if $s_1 = 0$. In this case his utility decreases ($w_1 > \frac{w_1}{2}$) – there is no incentive to deviate for player 1. If player 2 changes his strategy, his utility gets negative – there is also no incentive to deviate for player 2. If $w_1 = w_2$, additionally the strategy combinations $(0, w_1 + x)$ with $x \geq 0$ constitute equilibria. The argumentation proceeds analogously.

Problem 10 (5 points)
Use the Herfindahl index to show that the concentration on a market increases whenever two firms merge.

Problem 11 (10 points)
Sketch the use of Brouwer's fixed-point theorem for proving the existence of a Walras equilibrium.

Problem 12 (5 points)
What is a correlated equilibrium?