Advanced Microeconomics

Adverse selection

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Part G. Contracts and principal-agent theories

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2. Hidden action

Adverse selection

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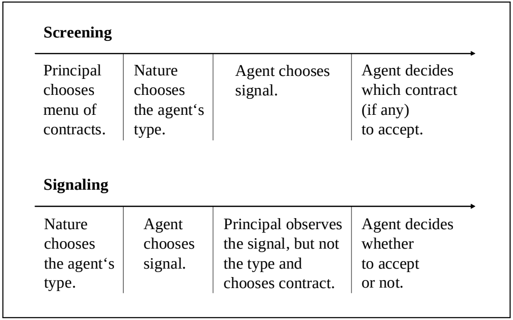
Introduction

- 2001 Nobel prize: George A. Akerlof, A. Michael Spence, and Joseph E. Stiglitz
- Informational asymmetries that are already present before the players decide whether or not to accept the contract or which contract to accept
- For example,
  - the ability of a worker is known to the worker (agent) but not to the firm (principal) who considers to hire the worker,
  - the car driver (agent) is better informed than the insurance company (principal) about the driver's accident-proneness,
  - the owner of a used car for sale (agent) may have a very good idea about the quality of that car while the potential buyer (principal) does not.
- The problem is: for a given wage, a given insurance premium, or a given price for a used car, the badly qualified workers, the high-risk insurees and the owners of bad cars are more eager to enter into a contract than the opposite types of agents.

Introduction

- At first sight, the informational asymmetry is a problem for the principal. **But:** It is the agent who needs to convince the principal that he is of a "good type"
- Two different classes of models



A polypsonistic labor market

The market model

- The model is originally due to *Akerlof (1970)* who considers the market for used cars
- Many identical firms with a binary (0 or 1 worker) unit demand for labor
- The workers differ in their productivities:
  - The marginal productivity t of any worker is constant. We address the worker with productivity t as worker t.
  - There is a continuum of workers on the type interval $T := [\underline{t}, \overline{t}]$, $0 \leq \underline{t} < \overline{t} < \infty$.
  - Worker t's marginal productivity is the same in every firm
  - $r(t)$ is the opportunity cost of employment for worker t

A polypsonistic labor market

The market model

Definition

A polypsonistic labor market is a tuple $\Gamma = (P, T, \tau, r, A, (u_t)_{t \in T}, u_P)$

- P is the principal,
- $T = [\underline{t}, \overline{t}]$ is the set of agents respectively types,
- W the set of working agents,
- τ is a probability (or density) distribution on T,
- $r : T \to \mathbb{R}$ is the reservation-wage function
- $A = \{\mathbf{y}, \mathbf{n}\}$ is the action set for each agent,
  where **y** means ("yes" or "accept") and **n** ("no" or "decline")
- $u_t : \mathbb{R} \times A \to \mathbb{R}$ is agent t's payoff function defined by

$$u_t(w, a) = \begin{cases} w, & a = \mathbf{y} \\ r(t), & a = \mathbf{n} \end{cases}$$

A polypsonistic labor market

The market model

Definition

$u_P : \mathbb{R} \times 2^T \rightarrow \mathbb{R}$ is the principal's payoff function defined by

$$u_P(w, W) = \underbrace{E[t : t \in W]}_{\substack{\text{average productivity} \\ \text{of working agents}}} - w = \int_{t \in W} t \frac{\tau(t)}{\int_{\bar{t} \in W} \tau(\bar{t})\, d\bar{t}} dt - w.$$

Definition

Let Γ be a polypsonistic labor market. An agent strategy is a function $s : T \rightarrow A$. The function $s^w : T \rightarrow A$, given by

$$s^w(t) = \begin{cases} \mathbf{y}, & w \geq r(t) \\ \mathbf{n}, & w < r(t) \end{cases}$$

is called the optimal agent strategy at wage rate w.

A polypsonistic labor market

Observable productivity

- The profit function is

$$u_P(w, W) = \int_{t \in W} (t - w(t)) \frac{\tau(t)}{\int_{\bar{t} \in W} \tau(\bar{t})\, d\bar{t}} dt$$

- In equilibrium, every worker is paid his marginal product:

$$w^*(t) = t, t \in T$$

- Therefore,

$$s^{w^*(t)=t}(t) = \begin{cases} \mathbf{y}, & t \geq r(t) \\ \mathbf{n}, & t < r(t) \end{cases}$$

  so that those workers get employed who produce less on their own than in the firm.

- Pareto efficient outcome
- Firms do not make any profit

A polypsonistic labor market

Unobservable productivity

All the workers obtain the same wage

Definition

Let Γ be a polypsonistic labor market and s^w the optimal agent strategy. Then,

$$T(w) := \{t \in T : s^w(t) = \mathbf{y}\} = \{t \in T : w \geq r(t)\}$$

is called the labor supply.

Definition

Let Γ be a polypsonistic labor market under nonobservability. A wage $\widehat{w}$ and the worker set $\hat{W}$ form an equilibrium if

- $t \in \hat{W} \Leftrightarrow s^{\widehat{w}}(t) = \mathbf{y}$ and
- $\widehat{w} = E\left[t : t \in \hat{W}\right]$.

A polypsonistic labor market

Unobservable productivity

- Thus, our definition of equilibrium has two requirements
  - At wage rate $\widehat{w}$, the agents who want to be employed are employed and the other agents are not, $\hat{W} = T\left(\widehat{w}\right)$
  - The polypsonist's expected payoff is zero
- Consider

$$\begin{aligned} \widehat{w} &= E\left[t : t \in \hat{W}\right] \\ &= E\left[t : t \in T\left(\widehat{w}\right)\right] \\ &= E\left[t : \widehat{w} \geq r\left(t\right)\right] \end{aligned}$$

- Fixed-point equation:
  - wage rate $\widehat{w}$
  - workers $T\left(\widehat{w}\right) = \hat{W}$ want to be employed
  - average productivity of $\widehat{w}$

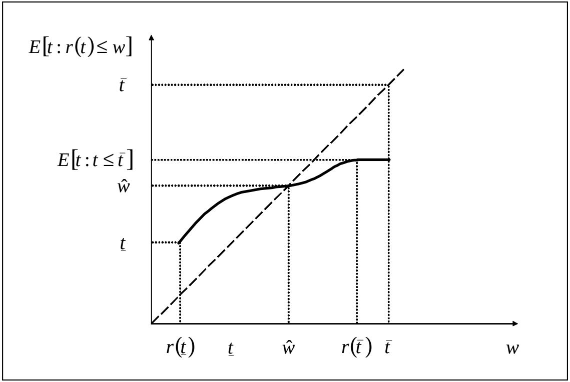
A polypsonistic labor market

Inefficient equilibrium?

Example:

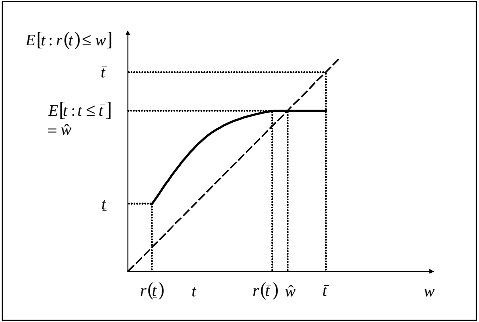
- $r(t) \leq t$ for all $t \in T$; all agents should be employed
- r is a monotonicly increasing function; productive workers have better outside options

- Figure: $r(\bar{t}) > E[t : t \leq \bar{t}]$
- $w \uparrow$, types with higher reservation wage and higher productivity work
- employing $\bar{t}$?



A polypsonistic labor market

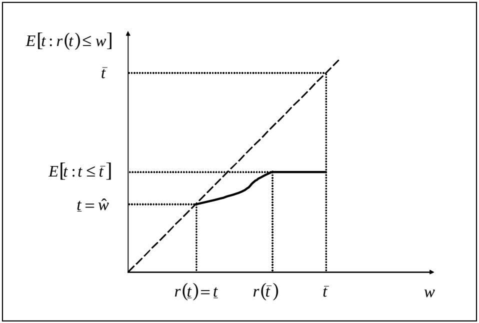
Inefficient equilibrium?



- Figure: $r(\bar{t}) < E[t : t \leq \bar{t}]$
- There are not many badly qualified workers so that the curve of average productivity is rather steep
- Despite asymmetric information, we might obtain an efficient outcome
- Although the wage $r(\bar{t})$ is sufficient to employ all the workers, the average productivity is above this wage rate

A polypsonistic labor market

Inefficient equilibrium?



The market collapses wholesale; a numerical example:

- $r(t) = \alpha t$ with $\frac{1}{2} < \alpha < 1$
- t distributed equally on $[0, 2]$

A polypsonistic labor market

Inefficient equilibrium?

In this case:

- $r\left(\underline{t}\right) = r\left(0\right) = 0 = \underline{t}$
- $r\left(t\right) = \alpha t < t$ for all $t > 0$ (i.e., all the agents should go to work)
- $E\left[t : r\left(t\right) \leq w\right] = E\left[t : \alpha t \leq w\right] = E\left[t : t \leq \frac{w}{\alpha}\right] = \frac{1}{2}\frac{w}{\alpha} < w$
  (the most productive worker has reservation price $w = r\left(t\right) = \alpha t$ and thus the productivity $\frac{w}{\alpha}$)

A polypsonistic labor market with education

Introduction

- Signaling is an action undertaken by the informed party
- Just saying "I am a good type" does not help
- Useful signals need to sort good and bad types
- We use a simple two-type model to shed light on screening and signaling

A polypsonistic labor market with education

The market model

- Labor is the only input and the many identical firms have a binary unit demand for labor
- Assumption: workers need to work, but are free to choose the best wage offered to them
- Two types of workers:
  - high productivity $t_h > 0$
  - low productivity $t_l > 0$
- Disutility from schooling $c(t)\, a$ ($a =$ workers' time in schools and universities) differs:
  - high productivity workers have $c_h := c(t_h)$
  - low-productivity workers have $c_l := c(t_l)$

  Assumption: $c_l > c_h > 0$

A polypsonistic labor market with education

The market model

Definition

A polypsonistic labor market with education is a tuple
$\Gamma = \left(P, T, \tau, A, (u_t)_{t \in T}, u_P\right)$ where

- P is the principal,
- $T = \{t_h, t_l\}$ is the set of types (agents),
- τ is a probability distribution on T with $\tau_h := \tau(t_h)$ and $\tau_l := \tau(t_l)$ denoting the respective portions of workers,
- $A = \mathbb{R}_+$ is the action set for each agent with actions $a \in A$ denoting the number of school years,
- $u_t : \mathbb{R} \times A \to \mathbb{R}$, $t \in \{t_h, t_l\}$, defined by $u_t(w, a) = w - c(t)\, a$
- $u_P : \{t_h, t_l\} \times \mathbb{R} \to \mathbb{R}$ defined by $u_P(t, w) = t - w$

A polypsonistic labor market with education

Observable productivity

- The equilibirum wages are

$$\begin{aligned} w^*(t_h) &= t_h > \tau_h t_h + \tau_l t_l \text{ and} \\ w^*(t_l) &= t_l < \tau_h t_h + \tau_l t_l \end{aligned}$$

  Every worker gets his marginal productivity
- There is no need to suffer the burden of education and we have a Pareto-efficient outcome

A polypsonistic labor market with education

Unobservable productivity

- Two types of equilibria:
  - separating – different types are treated differently
  - pooling – all the types are treated the same
- Pooling equilibrium = equal pay:
  - Zero-profit condition implies

$$w^* = \tau_h t_h + \tau_l t_l$$

  - all workers choose $a^* = 0$
  - Good workers might have an incentive to be screened or to signal their good quality (e.g., by applying to universities with tough micro courses)

A polypsonistic labor market with education and screening

Sequence and strategies

1. The principals choose wage contracts, i.e. functions
   $w : A \to \mathbb{R}$
   We restrict attention to binary wage contracts (strategies)
   $s_P = (a^+, w_h, w_l)$ defined by

$$w(a) = \begin{cases} w_h, & a \geq a^+ \\ w_l, & a < a^+ \end{cases}$$

2. Nature chooses each worker's type, t_h or t_l
3. The workers decide about their education efforts $a \in A$ as a function of

$$\left(\underbrace{t}_{\text{type}}, \underbrace{a^+}_{\substack{\text{minimal} \\ \text{schooling}}}, \underbrace{w_h}_{\text{high wage}}, \underbrace{w_l}_{\text{low wage}} \right)$$

4. The workers decide which firm to choose

Definition (Screening equilibria)

Let Γ be a polypsonistic labor market with education and screening. The strategy combination $(\hat{s}_P, \hat{s}) = ((\hat{a}^+, \hat{w}_h, \hat{w}_l), \hat{s})$ (with the principal strategy $\hat{s}_P$ and the agent strategy $\hat{s}$) is a separating equilibrium

- if the principal differentiates wages and maximizes his profits which are zero, i.e., if $\hat{s}_P = (\hat{a}^+, \hat{w}_h, \hat{w}_l)$ obeys $\hat{w}_h = t_h$ and $\hat{w}_l = t_l$,
- if the different types act differently: $\hat{s}(t, \hat{a}^+, \hat{w}_h, \hat{w}_l) \begin{cases} \geq \hat{a}^+, & t = t_h \\ < \hat{a}^+ & t = t_l \end{cases}$, and
- if the agents maximize their payoff, i.e., if $\hat{s}(t, \hat{a}^+, w_h, w_l) = \arg\max_{a \in A} \begin{cases} w_h - c(t)\, a, & a \geq a^+ \\ w_l - c(t)\, a, & a < a^+ \end{cases}$ holds for all $(t, a^+, w_h, w_l) \in T \times A \times \mathbb{R} \times \mathbb{R}$.

A polypsonistic labor market with education and screening

Separating equibria

$(\hat{s}_P, \hat{s})$ is a separating equilibrium —>

$$\hat{s}\left(t, \hat{a}^{+}, \hat{w}_h, \hat{w}_l\right) = \begin{cases} \hat{a}^{+}, & t = t_h \\ 0, & t = t_l \end{cases}$$

A polypsonistic labor market with education and screening

Separating equibria

- Do the agents indeed seperate themselves?
- Agent t_h prefers a^+ years of schooling over 0 years if

$$\underbrace{w_h - c_h a^+}_{\substack{\text{The productive}\\ \text{agent's payoff}\\ \text{in case of}\\ \text{extensive schooling}}} \geq \underbrace{w_l - c_h \cdot 0}_{\substack{\text{The productive}\\ \text{agent's payoff}\\ \text{in case of}\\ \text{no schooling}}}$$

or

$$a^+ \leq \frac{w_h - w_l}{c_h}$$

holds

- The salary difference $w_h - w_l$ outweights the productive type's cost of education $c_h a^+$.

A polypsonistic labor market with education and screening

Separating equibria

The unproductive agent t_l is happy not to choose any education in case of

$$\underbrace{w_h - c_l a^+}_{\substack{\text{The unproductive}\\ \text{agent's payoff}\\ \text{in case of}\\ \text{extensive schooling}}} \quad \leq \quad \underbrace{w_l - c_l \cdot 0}_{\substack{\text{The unproductive}\\ \text{agent's payoff}\\ \text{in case of}\\ \text{no schooling}}}$$

or, solving again for a^+,

$$a^+ \geq \frac{w_h - w_l}{c_l}.$$

The interval

$$\frac{w_h - w_l}{c_l} \leq a^+ \leq \frac{w_h - w_l}{c_h}$$

is non-empty.

Lemma (Screening equilibria)

Let Γ be a polypsonistic labor market with education and screening. The separating equilibria $(\hat{s}_P, \hat{s}) = ((\hat{a}^+, \hat{w}_h, \hat{w}_l), \hat{s})$ are given by

- $\hat{s}_P = (\hat{a}^+, t_h, t_l)$ *where* $\hat{a}^+$ *fulfills* $\frac{t_h - t_l}{c_l} = \hat{a}^+$ *and*
- $\hat{s}(t, \hat{a}^+, \hat{w}_h, \hat{w}_l) = \begin{cases} \hat{a}^+, & t = t_h \\ 0, & t = t_l \end{cases}$

$\hat{a}^+ > \frac{w_h - w_l}{c_l}$ cannot hold in equilibrium because then an $\varepsilon > 0$ exists such that $\hat{a}^+ - \varepsilon > \frac{w_h - w_l}{c_l}$. An ε-firm can offer the threshold $\hat{a}^+ - \varepsilon$ and the wage $t_h - \frac{1}{2}c_h\varepsilon$. Productive workers seek employment with this ε-firm by

$$\left(w_h - \frac{1}{2}c_h\varepsilon\right) - c_h\left(\hat{a}^+ - \varepsilon\right) = w_h - c_h\hat{a}^+ + \frac{1}{2}c_h\varepsilon > w_h - c_h\hat{a}^+,$$

and the ε-firm makes a positive profit of $\frac{1}{2}c_h\varepsilon$.

A polypsonistic labor market with education and screening

Separating equibria: proof

- Given the principals' education threashold $\hat{a}^+$, the agents cannot do any better than following their strategy $\hat{s}$. The productive workers choose $a = a^+$ and the unproductive ones choose $a = 0$
- The equilibria specified in the lemma are subgame perfect. The agents choices are optimal given any principal strategy, not just an equilibrium principal strategy

A polypsonistic labor market with education and signalling

- In a somewhat similar fashion, a polypsonistic labor market with education and **signaling** can also be constructed. In a suchlike model, the workers, not the principals, are the first movers
- The productive workers can choose the minimum level of education $\hat{a}^+$ necessary to have unproductive agents shy away from education
- Screening and signalling may be beneficial for productive workers, but need not. (You can find out that productive workers benefit from education and the wage w_h if $\tau_l > c_h / c_l$.)