Advanced Microeconomics

Adverse selection

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Part G. Contracts and principal-agent theories

1. Adverse selection
2. Hidden action

Introduction

- 2001 Nobel prize: George A. Akerlof, A. Michael Spence, and Joseph E. Stiglitz
- Informational asymmetries that are already present before the players decide whether or not to accept the contract or which contract to accept
- For example,
  - the ability of a worker is known to the worker (agent) but not to the firm (principal) who considers to hire the worker,
  - the car driver (agent) is better informed than the insurance company (principal) about the driver's accident-proneness,
  - the owner of a used car for sale (agent) may have a very good idea about the quality of that car while the potential buyer (principal) does not.
- The problem is: for a given wage, a given insurance premium, or a given price for a used car, the badly qualified workers, the high-risk insurees and the owners of bad cars are more eager to enter into a contract than the opposite types of agents.

Introduction

- At first sight, the informational asymmetry is a problem for the principal. **But:** It is the agent who needs to convince the principal that he is of a "good type"
- Two different classes of models

Screening

Principal chooses menu of contracts.	Nature chooses the agent's type.	Agent chooses signal.	Agent decides which contract (if any) to accept.

Signaling

Nature chooses the agent's type.	Agent chooses signal.	Principal observes the signal, but not the type and chooses contract.	Agent decides whether to accept or not.

A polypsonistic labor market

The market model

- The model is originally due to *Akerlof (1970)* who considers the market for used cars
- Many identical firms with a binary (0 or 1 worker) unit demand for labor
- The workers differ in their productivities:
  - The marginal productivity t of any worker is constant. We address the worker with productivity t as worker t.
  - There is a continuum of workers on the type interval $T := [\underline{t}, \overline{t}]$, $0 \leq \underline{t} < \overline{t} < \infty$.
  - Worker t's marginal productivity is the same in every firm
  - $r(t)$ is the opportunity cost of employment for worker t

Definition

A polypsonistic labor market is a tuple $\Gamma = \left(P, T, \ \tau, r, A, (u_t)_{t\in T}, u_P\right)$

- P is the principal,
- $T = [\underline{t}, \overline{t}]$ is the set of agents respectively types,
- W the set of working agents,
- τ is a probability (or density) distribution on T,
- $r : T \to \mathbb{R}$ is the reservation-wage function
- $A = \{\mathbf{y}, \mathbf{n}\}$ is the action set for each agent,
  where $\mathbf{y}$ means ("yes" or "accept") and $\mathbf{n}$ ("no" or "decline")
- $u_t : \mathbb{R} \times A \to \mathbb{R}$ is agent t's payoff function defined by

$$u_t(w, a) = \begin{cases} w, & a = \mathbf{y} \\ r(t), & a = \mathbf{n} \end{cases}$$

Definition

$u_P : \mathbb{R} \times 2^T \to \mathbb{R}$ is the principal's payoff function defined by

$$u_P(w, W) = \underbrace{E[t : t \in W]}_{\substack{\text{average productivity} \\ \text{of working agents}}} - w = \int_{t \in W} t \frac{\tau(t)}{\int_{\bar{t} \in W} \tau(\bar{t})\, d\bar{t}} dt - w.$$

Definition

Let Γ be a polypsonistic labor market. An agent strategy is a function $s : T \to A$. The function $s^w : T \to A$, given by

$$s^w(t) = \begin{cases} \mathbf{y}, & w \geq r(t) \\ \mathbf{n}, & w < r(t) \end{cases}$$

is called the optimal agent strategy at wage rate w.

A polypsonistic labor market

Observable productivity

- The profit function is

$$u_P(w, W) = \int_{t \in W} (t - w(t)) \frac{\tau(t)}{\int_{\overline{t} \in W} \tau(\overline{t})\, d\overline{t}} dt$$

- In equilibrium, every worker is paid his marginal product:

$$w^*(t) = t, t \in T$$

- Therefore,

$$s^{w^*(t)=t}(t) = \begin{cases} \mathbf{y}, & t \geq r(t) \\ \mathbf{n}, & t < r(t) \end{cases}$$

  so that those workers get employed who produce less on their own than in the firm.
- Pareto efficient outcome
- Firms do not make any profit

A polypsonistic labor market

Unobservable productivity

All the workers obtain the same wage

Definition

Let Γ be a polypsonistic labor market and s^w the optimal agent strategy. Then,

$$T(w) := \{t \in T : s^w(t) = \mathbf{y}\} = \{t \in T : w \geq r(t)\}$$

is called the labor supply.

Definition

Let Γ be a polypsonistic labor market under nonobservability. A wage $\widehat{w}$ and the worker set $\hat{W}$ form an equilibrium if

- $t \in \hat{W} \Leftrightarrow s^{\widehat{w}}(t) = \mathbf{y}$ and
- $\widehat{w} = E\left[t : t \in \hat{W}\right]$.

- Thus, our definition of equilibrium has two requirements
  - At wage rate $\widehat{w}$, the agents who want to be employed are employed and the other agents are not, $\hat{W} = T(\widehat{w})$
  - The polypsonist's expected payoff is zero
- Consider

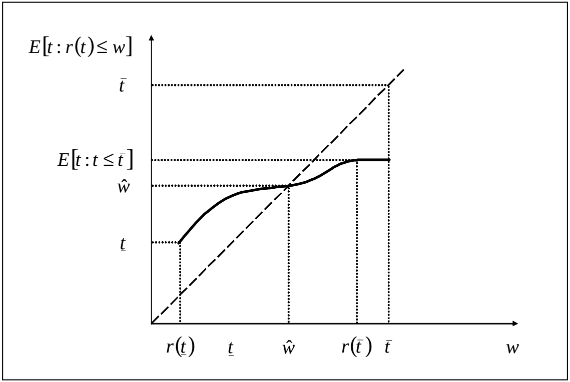
$$
\begin{aligned}
\widehat{w} &= E\left[t : t \in \hat{W}\right] \\
&= E\left[t : t \in T(\widehat{w})\right] \\
&= E\left[t : \widehat{w} \geq r(t)\right]
\end{aligned}
$$

- Fixed-point equation:
  - wage rate $\widehat{w}$
  - workers $T(\widehat{w}) = \hat{W}$ want to be employed
  - average productivity of $\widehat{w}$

Example:

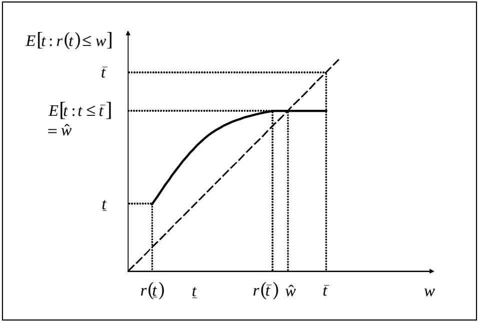
- $r(t) \leq t$ for all $t \in T$; all agents should be employed
- r is a monotonicly increasing function; productive workers have better outside options

  - Figure: $r(\overline{t}) > E[t : t \leq \overline{t}]$
  - $w \uparrow$, types with higher reservation wage and higher productivity work

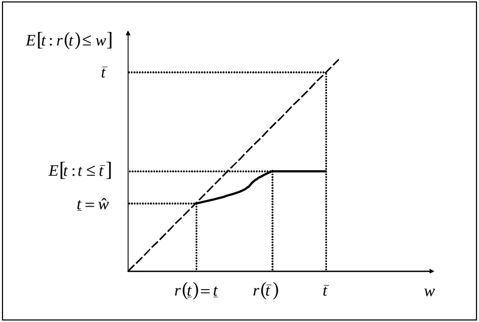


A polypsonistic labor market

Inefficient equilibrium?



- Figure: $r(\bar{t}) < E[t : t \leq \bar{t}]$
- There are not many badly qualified workers so that the curve of average productivity is rather steep
- Despite asymmetric information, we might obtain an efficient outcome
- Although the wage $r(\bar{t})$ is sufficient to employ all the workers, the average productivity is above this wage rate



The market collapses wholesale; a numerical example:

- $r(t) = \alpha t$ with $\frac{1}{2} < \alpha < 1$
- t distributed equally on $[0, 2]$

A polypsonistic labor market

Inefficient equilibrium?

In this case:

- $r\left(\underline{t}\right) = r\left(0\right) = 0 = \underline{t}$
- $r\left(t\right) = \alpha t < t$ for all $t > 0$ (i.e., all the agents should go to work)
- $E\left[t : r\left(t\right) \leq w\right] = E\left[t : \alpha t \leq w\right] = E\left[t : t \leq \frac{w}{\alpha}\right] = \frac{1}{2}\frac{w}{\alpha} < w$
  (the most productive worker has reservation price $w = r\left(t\right) = \alpha t$ and thus the productivity $\frac{w}{\alpha}$)

- Signaling is an action undertaken by the informed party
- Just saying "I am a good type" does not help
- Useful signals need to sort good and bad types
- We use a simple two-type model to shed light on screening and signaling

- Labor is the only input and the many identical firms have a binary unit demand for labor
- Assumption: workers need to work, but are free to choose the best wage offered to them
- Two types of workers:
  - high productivity $t_h > 0$
  - low productivity $t_l > 0$
- Disutility from schooling $c(t)\,a$ (a = workers' time in schools and universities) differs:
  - high productivity workers have $c_h := c(t_h)$
  - low-productivity workers have $c_l := c(t_l)$

  Assumption: $c_l > c_h > 0$

Definition

A polypsonistic labor market with education is a tuple $\Gamma = \left(P, T, \tau, A, (u_t)_{t \in T}, u_P\right)$ where

- P is the principal,
- $T = \{t_h, t_l\}$ is the set of types (agents),
- τ is a probability distribution on T with $\tau_h := \tau(t_h)$ and $\tau_l := \tau(t_l)$ denoting the respective portions of workers,
- $A = \mathbb{R}_+$ is the action set for each agent with actions $a \in A$ denoting the number of school years,
- $u_t : \mathbb{R} \times A \to \mathbb{R}$, $t \in \{t_h, t_l\}$, defined by $u_t(w, a) = w - c(t)\, a$
- $u_P : \{t_h, t_l\} \times \mathbb{R} \to \mathbb{R}$ defined by $u_P(t, w) = t - w$

- The equilibirum wages are

$$\begin{aligned} w^*(t_h) &= t_h > \tau_h t_h + \tau_l t_l \text{ and} \\ w^*(t_l) &= t_l < \tau_h t_h + \tau_l t_l \end{aligned}$$

  Every worker gets his marginal productivity
- There is no need to suffer the burden of education and we have a Pareto-efficient outcome

A polypsonistic labor market with education

Unobservable productivity

- Two types of equilibria:
  - separating – different types are treated differently
  - pooling – all the types are treated the same
- Pooling equilibrium = equal pay:
  - Zero-profit condition implies

$$w^* = \tau_h t_h + \tau_l t_l$$

  - all workers choose $a^* = 0$
  - Good workers might have an incentive to be screened or to signal their good quality (e.g., by applying to universities with tough micro courses)

1. The principals choose wage contracts, i.e. functions $w : A \to \mathbb{R}$
   We restrict attention to binary wage contracts (strategies) $s_P = (a^+, w_h, w_l)$ defined by

$$w(a) = \begin{cases} w_h, & a \geq a^+ \\ w_l, & a < a^+ \end{cases}$$

2. Nature chooses each worker's type, t_h or t_l
3. The workers decide about their education efforts $a \in A$ as a function of

$$\left(\underbrace{t}_{\text{type}}, \underbrace{a^+}_{\substack{\text{minimal} \\ \text{schooling}}}, \underbrace{w_h}_{\text{high wage}}, \underbrace{w_l}_{\text{low wage}} \right)$$

4. The workers decide which firm to choose

Definition (Screening equilibria)

Let Γ be a polypsonistic labor market with education and screening. The strategy combination $(\hat{s}_P, \hat{s}) = ((\hat{a}^+, \hat{w}_h, \hat{w}_l), \hat{s})$ (with the principal strategy $\hat{s}_P$ and the agent strategy $\hat{s}$) is a separating equilibrium

- if the principal differentiates wages and maximizes his profits which are zero, i.e., if $\hat{s}_P = (\hat{a}^+, \hat{w}_h, \hat{w}_l)$ obeys $\hat{w}_h = t_h$ and $\hat{w}_l = t_l$,
- if the different types act differently: $\hat{s}(t, \hat{a}^+, \hat{w}_h, \hat{w}_l) \begin{cases} \geq \hat{a}^+, & t = t_h \\ < \hat{a}^+ & t = t_l \end{cases}$, and
- if the agents maximize their payoff, i.e., if $\hat{s}(t, \hat{a}^+, w_h, w_l) = \arg\max_{a \in A} \begin{cases} w_h - c(t)\, a, & a \geq a^+ \\ w_l - c(t)\, a, & a < a^+ \end{cases}$ holds for all $(t, a^+, w_h, w_l) \in T \times A \times \mathbb{R} \times \mathbb{R}$.

$(\hat{s}_P, \hat{s})$ is a separating equilibrium —>

$$\hat{s}\left(t, \hat{a}^{+}, \hat{w}_h, \hat{w}_l\right) = \begin{cases} \hat{a}^{+}, & t = t_h \\ 0, & t = t_l \end{cases}$$

- Do the agents indeed seperate themselves?
- Agent t_h prefers a^+ years of schooling over 0 years if

$$\underbrace{w_h - c_h a^+}_{\substack{\text{The productive} \\ \text{agent's payoff} \\ \text{in case of} \\ \text{extensive schooling}}} \geq \underbrace{w_l - c_h \cdot 0}_{\substack{\text{The productive} \\ \text{agent's payoff} \\ \text{in case of} \\ \text{no schooling}}}$$

  or

$$a^+ \leq \frac{w_h - w_l}{c_h}$$

  holds
- The salary difference $w_h - w_l$ outweights the productive type's cost of education $c_h a^+$.

The unproductive agent t_l is happy not to choose any education in case of

$$\underbrace{w_h - c_l a^+}_{\substack{\text{The unproductive}\\ \text{agent's payoff}\\ \text{in case of}\\ \text{extensive schooling}}} \leq \underbrace{w_l - c_l \cdot 0}_{\substack{\text{The unproductive}\\ \text{agent's payoff}\\ \text{in case of}\\ \text{no schooling}}}$$

or, solving again for a^+,

$$a^+ \geq \frac{w_h - w_l}{c_l}.$$

The interval

$$\frac{w_h - w_l}{c_l} \leq a^+ \leq \frac{w_h - w_l}{c_h}$$

is non-empty.

Lemma (Screening equilibria)

Let Γ be a polypsonistic labor market with education and screening. The separating equilibria $(\hat{s}_P, \hat{s}) = ((\hat{a}^+, \hat{w}_h, \hat{w}_l), \hat{s})$ are given by

- $\hat{s}_P = (\hat{a}^+, t_h, t_l)$ *where* $\hat{a}^+$ *fulfills* $\frac{t_h - t_l}{c_l} = \hat{a}^+$ *and*
- $\hat{s}(t, \hat{a}^+, \hat{w}_h, \hat{w}_l) = \begin{cases} \hat{a}^+, & t = t_h \\ 0, & t = t_l \end{cases}$

$\hat{a}^+ > \frac{w_h - w_l}{c_l}$ cannot hold in equilibrium because then an $\varepsilon > 0$ exists such that $\hat{a}^+ - \varepsilon > \frac{w_h - w_l}{c_l}$. An ε-firm can offer the threshold $\hat{a}^+ - \varepsilon$ and the wage $t_h - \frac{1}{2}c_h\varepsilon$. Productive workers seek employment with this ε-firm by

$$\left(w_h - \frac{1}{2}c_h\varepsilon\right) - c_h\left(\hat{a}^+ - \varepsilon\right) = w_h - c_h\hat{a}^+ + \frac{1}{2}c_h\varepsilon > w_h - c_h\hat{a}^+,$$

and the ε-firm makes a positive profit of $\frac{1}{2}c_h\varepsilon$.

- Given the principals' education threashold $\hat{a}^+$, the agents cannot do any better than following their strategy $\hat{s}$. The productive workers choose $a = a^+$ and the unproductive ones choose $a = 0$
- The equilibria specified in the lemma are subgame perfect. The agents choices are optimal given any principal strategy, not just an equilibrium principal strategy

A polypsonistic labor market with education and signalling

- In a somewhat similar fashion, a polypsonistic labor market with education and **signaling** can also be constructed. In a suchlike model, the workers, not the principals, are the first movers
- The productive workers can choose the minimum level of education $\hat{a}^+$ necessary to have unproductive agents shy away from education
- Screening and signalling may be beneficial for productive workers, but need not. (You can find out that productive workers benefit from education and the wage w_h if $\tau_l > c_h / c_l$.)