Advanced Microeconomics

General equilibrium theory I: the main results

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Part F. Perfect competition and competition policy

1. General equilibrium theory I: the main results
2. General equilibrium theory II: criticism and applications
3. Introduction to competition policy and regulation

General equilibrium theory I: the main results

1. Introduction
2. Exchange theory: positive theory
3. Exchange and production economy: positive theory
4. Normative theory

General equilibrium theory

Introduction

- Allocation of goods takes place in two different modes:
  - the first of which being person-to-person;
  - the second mode is impersonal trading, expounded by General Equilibrium Theory (GET).
- GET envisions a market system with perfect competition.
- All agents (households and firms) are price takers.
- Under which conditions are there prices such that
  - all actors behave in a utility, or profit, maximizing way and
  - the demand and supply schedules can be fulfilled simultaneously?

—> Walras equilibrium

General equilibrium theory

Assumptions

- The goods are private and there are no external effects.
- The individuals interact via market transactions only.
- The individuals take prices as given.
- There are no transaction costs.
- The goods are homogeneous but there can be many goods.
- The preferences are monotonic and convex (and, of course, transitive, reflexive, and symmetric).

Nobel prices in GET

In 1972

'for their pioneering contributions to general economic equilibrium theory and and welfare theory'

1/2 John R Hicks (Oxford University), and

1/2 Kenneth Arrow (Harvard University).

In 1982

'for having incorporated new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium'

- Gerard Debreu (University of California, Berkeley).

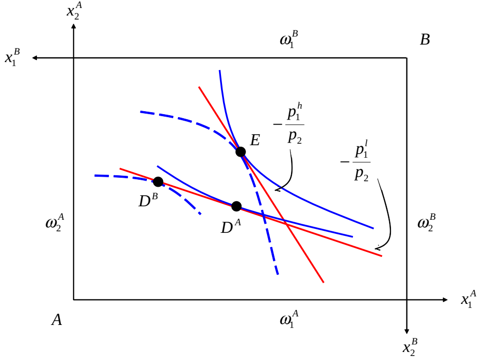
In 1988

'for his pioneering contributions to the theory of markets and efficient utilization of resources'

- Maurice Allais (Ecole Nationale Supérieure des Mines de Paris).

Exchange theory: positive theory

Exchange Edgeworth box: prices and equilibria



The low price p_1^l is not possible in a Walras equilibrium, because there is excess demand for good 1 at this price:

$$x_1^A + x_1^B > \omega_1^A + \omega_1^B .$$

Definition (exchange economy)

An exchange economy is a tuple

$$\mathcal{E} = \left(N, G, \left(\omega^i\right)_{i \in N}, \left(\precsim^i\right)_{i \in N}\right)$$

consisting of

- the set of agents $N = \{1, 2, ..., n\}$,
- the finite set of goods $G = \{1, ..., \ell\}$,

and for every agent $i \in N$

- an endowment $\omega^i = \left(\omega^i_1, ..., \omega^i_\ell\right) \in \mathbb{R}^\ell_+$, and
- a preference relation $\precsim^i$.

Exchange theory: positive theory

Definition of an exchange economy

The total endowment of an exchange economy is given by $\omega = \sum_{i \in N} \omega^i$.

Definition

Consider an exchange economy $\mathcal{E}$.

- A bundle $(y^i)_{i \in N} \in \mathbb{R}_+^{\ell \cdot n}$ is an allocation.
- An allocation $(y^i)_{i \in N}$ is called feasible if $\sum_{i \in N} y^i \leq \sum_{i \in N} \omega^i$ holds.

Exchange theory: positive theory

Definition

Assume an exchange economy $\mathcal{E}$, a good $g \in G$ and a price vector $p \in \mathbb{R}^\ell$. If every household $i \in N$ has a unique household optimum $x^i\left(p, \omega^i\right)$, good g's excess demand is denoted by $z_g\left(p\right)$ and defined by

$$z_g\left(p\right) := \sum_{i=1}^{n} x_g^i\left(p, \omega^i\right) - \sum_{i=1}^{n} \omega_g^i.$$

The corresponding excess demand for all goods $g = 1, ..., \ell$ is the vector

$$z\left(p\right) := \left(z_g\left(p\right)\right)_{g=1,\ldots,\ell}.$$

The value of the excess demand is given by

$$p \cdot z\left(p\right).$$

Exchange theory: positive theory

Excess Demand and Market Clearance

Lemma (Walras' law)

Every consumer demands a bundle of goods obeying $p \cdot x^i \leq p \cdot \omega^i$ where local nonsatiation implies equality. For all consumers together, we have

$$p \cdot z(p) = \sum_{i=1}^{n} p \cdot \left(x^i - \omega^i\right) \leq 0$$

and, assuming local-nonsatiation, $p \cdot z(p) = 0$.

Definition

A market g is called cleared if excess demand $z_g(p)$ on that market is equal to zero.

Abba (A) and Bertha (B) consider buying two goods 1 and 2, and face the price p for good 1 in terms of good 2. Think of good 2 as the numéraire good with price 1. Abba's and Bertha's utility functions, u_A and u_B, respectively, are given by $u_A\left(x_1^A, x_2^A\right) = \sqrt{x_1^A} + x_2^A$ and $u_B\left(x_1^B, x_2^B\right) = \sqrt{x_1^B} + x_2^B$. Endowments are $\omega^A = (18, 0)$ and $\omega^B = (0, 10)$. Find the bundles demanded by these two agents. Then find the price p that fulfills $\omega_1^A + \omega_1^B = x_1^A + x_1^B$ and $\omega_2^A + \omega_2^B = x_2^A + x_2^B$.

Lemma (Market clearance)

In case of local nonsatiation,

1. *if all markets but one are cleared, the last one also clears or its price is zero,*
2. *if at prices $p \gg 0$ all markets but one are cleared, all markets clear.*

Proof.

If $\ell - 1$ markets are cleared, the excess demand on these markets is 0. Without loss of generality, markets $g = 1, ..., \ell - 1$ are cleared. Applying Walras's law we get

$$0 = p \cdot z(p) = p_\ell z_\ell(p).$$

□

Definition

A price vector $\widehat{p}$ and the corresponding demand system $\left(\widehat{x}^i\right)_{i=1,\dots,n} = \left(x^i\left(\widehat{p},\omega^i\right)\right)_{i=1,\dots,n}$ is called a Walras equilibrium if

$$\sum_{i=1}^n \widehat{x}^i \leq \sum_{i=1}^n \omega^i$$

or

$$z\left(\widehat{p}\right) \leq 0$$

holds.

Definition

A good is called free if its price is equal to zero.

Lemma (free goods)

Assume local nonsatiation and weak monotonicity for all households. If $\left[\widehat{p}, \left(\widehat{x}^i\right)_{i=1,...,n}\right]$ *is a Walras equilibrium and the excess demand for a good is negative, this good must be free.*

Proof.

Assume, to the contrary, that $p_g > 0$ holds. We obtain a contradiction to Walras' law for local nonsatiation:

$$\begin{aligned}
0 \underbrace{=}_{\text{Walras' law}} p \cdot z(p) &= \underbrace{p_g z_g(p)}_{<0} + \sum_{\substack{g'=1,\\ g'\neq g}}^{\ell} p_{g'} z_{g'}(p) \quad (z_g(p) < 0) \\
&< \sum_{\substack{g'=1,\\ g'\neq g}}^{\ell} \underbrace{p_{g'}}_{\substack{\geq 0 \\ \text{(local nonsatiation and} \\ \text{weak monotonicity)}}} \underbrace{z_{g'}(p)}_{\substack{\leq 0 \\ \text{(definition} \\ \text{Walras equilibrium)}}} \\
&\leq 0.
\end{aligned}$$

$\square$

Definition

A good is desired if the excess demand at price zero is positive.

Lemma (desiredness)

We obtain $z(\widehat{p}) = 0$ if

- *all goods are desired*
- *local nonsatiation and weak monotonicity hold and*
- *$\widehat{p}$ is a Walras equilibrium.*

Proof.

Suppose that there is a good g with $z_g(\widehat{p}) < 0$. Then g must be a free good according to the lemma on free goods and have a positive excess demand by the definition of desiredness, $z_g(\widehat{p}) > 0$. $\square$

Exchange theory: positive theory

Example: The Cobb-Douglas Exchange Economy with Two Agents

Parameters a_1 and a_2 and endowments $\omega^1 = (1, 0)$ and $\omega^2 = (0, 1)$,
Agent 1: $U_1(x_1, x_2) = x_1^{a_1} x_2^{1-a_1}$, $0 \leq a_1 \leq 1$ (a_2 is agent 2's parameter)

- Agent 1's demand for good 1:

$$x_1^1(p_1, p_2, \omega^1 \cdot p) = a_1 \frac{m}{p_1} = a_1 \frac{\omega^1 \cdot p}{p_1} = a_1.$$

- Agent 2's demand for good 1:

$$x_1^2(p_1, p_2, \omega^2 \cdot p) = a_2 \frac{\omega^2 \cdot p}{p_1} = a_2 \frac{p_2}{p_1}.$$

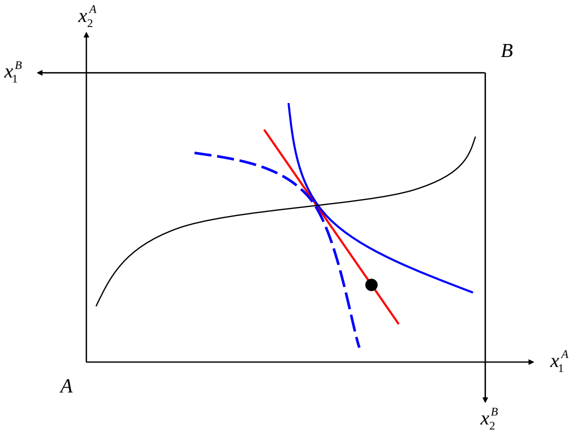
- Market 1 is cleared if

$$a_1 + a_2 \frac{p_2}{p_1} = 1 \text{ or } \frac{p_1}{p_2} = \frac{a_2}{1 - a_1}$$

- How about the market for good 2?

Exchange theory: positive theory

Example: The Cobb-Douglas Exchange Economy with Two Agents



Theorem (Existence of the Walras Equilibrium)

If the following conditions hold:

- *the preferences are strictly monotonic,*
  - *so that household optima exist (!) for strictly positive prices and*
  - *so that the value of the excess demand is zero (!),*

  and
- *aggregate excess demand is a continuous function (in prices),*

a Walras equilibrium exists.

Theorem (Brouwer's fixed-point theorem)

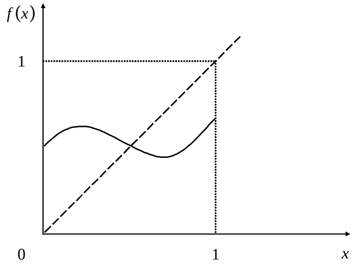
Suppose $f : M \to M$ is a function on the nonempty, compact (closed and bounded as a subset of $\mathbb{R}^\ell$) and convex set $M \subseteq \mathbb{R}^\ell$. If f is continuous, there exists $x \in M$ such that $f(x) = x$. x is called a fixed point.

Exchange theory: positive theory

Existence of the Walras equilibrium

Continuous function on the unit interval.

- $f(0) = 0$ or $f(1) = 1$
  —> fixed point is found
- $f(0) > 0$ and $f(1) < 1$
  —> the graph cuts the 45°-line
  —> fixed point is found



Real-life examples:

- rumpling a handkerchief
- stirring cake dough

Exchange theory: positive theory

Existence of the Walras equilibrium

Assume, one of the requirements for the fixed-point theorem does not hold. Show, by a counter example, that there is a function such that there is no fixed point. Specifically, assume that

a) M is not compact

b) M is not convex

c) f is not continuous.

Exchange theory: positive theory

Existence of the Walras equilibrium

Hans-Jürgen Podszuweit (found in Homo Oeconomicus, XIV (1997), p. 537):

Das Nilpferd hört perplex:
Sein Bauch, der sei konvex.
Und steht es vor uns nackt,
sieht man: Er ist kompakt.
Nimmt man 'ne stetige Funktion
von Bauch
in Bauch
– Sie ahnen schon –,
dann nämlich folgt aus dem
Brouwer'schen Theorem:
Ein Fixpunkt muß da sein.
Dasselbe gilt beim Schwein
q.e.d.

Exchange theory: positive theory

Existence of the Walras equilibrium

- Constructing a convex and compact set:
- Norm prices of the ℓ goods such that the sum of the nonnegative (!, we have strict monotonicity) prices equals 1. We can restrict our search for equilibrium prices to the $\ell - 1$-dimensional unit simplex:

$$S^{\ell-1} = \left\{ p \in \mathbb{R}_+^{\ell} : \sum_{g=1}^{\ell} p_g = 1 \right\}.$$

- $S^{\ell-1}$is nonempty, compact (closed and bounded as a subset of $\mathbb{R}^{\ell}$) and convex.
- Exercise: Draw $S^1 = S^{2-1}$.

The idea of the proof: First, we define a continuous function f on this (nonempty, compact and convex) set. Brouwer's theorem says that there is at least one fixed point of this function. Second, we show that such a fixed point fulfills the condition of the Walras equilibrium.

The abovementioned continuous function

$$f = \begin{pmatrix} f_1 \\ f_2 \\ . \\ . \\ . \\ f_\ell \end{pmatrix} : S^{\ell-1} \to S^{\ell-1}$$

is defined by

$$f_g(p) = \frac{p_g + \max(0, z_g(p))}{1 + \sum_{g'=1}^{\ell} \max(0, z_{g'}(p))}, g = 1, ..., \ell$$

Exchange theory: positive theory

Existence of the Walras equilibrium

f is continuous because every f_g, $g = 1, ..., \ell$, is continuous. The latter is continuous because z (according to our assumption) and max are continuous functions. Finally, we can confirm that f is well defined, i.e., that $f(p)$ lies in $S^{\ell-1}$ for all p from $S^{\ell-1}$:

$$\begin{aligned}
\sum_{g=1}^{\ell} f_g(p) &= \sum_{g=1}^{\ell} \frac{p_g + \max(0, z_g(p))}{1 + \sum_{g'=1}^{\ell} \max(0, z_{g'}(p))} \\
&= \frac{1}{1 + \sum_{g'=1}^{\ell} \max(0, z_{g'}(p))} \sum_{g=1}^{\ell} (p_g + \max(0, z_g(p))) \\
&= \frac{1}{1 + \sum_{g'=1}^{\ell} \max(0, z_{g'}(p))} \left(1 + \sum_{g=1}^{\ell} \max(0, z_g(p))\right) \\
&= 1.
\end{aligned}$$

Exchange theory: positive theory

Existence of the Walras equilibrium

The function f increases the price of a good g in case of $f_g(p) > p_g$, only, i.e. if

$$\frac{p_g + \max(0, z_g(p))}{1 + \sum_{g'=1}^{\ell} \max(0, z_{g'}(p))} > p_g$$

or

$$\frac{\max(0, z_g(p))}{\sum_{g'=1}^{\ell} \max(0, z_{g'}(p))} > \frac{p_g}{\sum_{g'=1}^{\ell} p_{g'}}$$

holds.

Interpretation: Increase price if its relative excess demand is greater than its relative price.

—> f = Walras auctioneer

—> tâtonnement

Exchange theory: positive theory

Existence of the Walras equilibrium

We now complete the proof: according to Brouwer's fixed-point theorem there is one $\widehat{p}$ such that

$$\widehat{p} = f\left(\widehat{p}\right),$$

from which we have

$$\widehat{p}_g = \frac{\widehat{p}_g + \max\left(0, z_g\left(\widehat{p}\right)\right)}{1 + \sum_{g'=1}^{\ell} \max\left(0, z_{g'}\left(\widehat{p}\right)\right)}$$

and finally

$$\widehat{p}_g \sum_{g'=1}^{\ell} \max\left(0, z_{g'}\left(\widehat{p}\right)\right) = \max\left(0, z_g\left(\widehat{p}\right)\right)$$

for all $g = 1, ..., \ell$.

Exchange theory: positive theory

Existence of the Walras equilibrium

Next we multiply both sides for all goods $g = 1, ..., \ell$ by $z_g(\widehat{p})$:

$$z_g(\widehat{p})\widehat{p}_g \sum_{g'=1}^{\ell} \max\left(0, z_{g'}(\widehat{p})\right) = z_g(\widehat{p}) \max\left(0, z_g(\widehat{p})\right)$$

and summing up over all g yields

$$\sum_{g=1}^{\ell} z_g(\widehat{p})\widehat{p}_g \sum_{g'=1}^{\ell} \max\left(0, z_{g'}(\widehat{p})\right) = \sum_{g=1}^{\ell} z_g(\widehat{p}) \max\left(0, z_g(\widehat{p})\right).$$

By Walras' law, the left-hand expression is equal to zero. The right-hand one consists of a sum of expressions, which are equal either to zero or to $\left(z_g(\widehat{p})\right)^2$. Therefore, $z_g(\widehat{p}) \leq 0$ for all $g = 1, ..., \ell$. This is what we wanted to show.

Exchange theory: positive theory

Existence of the Nash equilibrium

Theorem (Existence of Nash equilibria)

Any finite strategic game $\Gamma = (N, S, u)$ *(i.e.,* $|N| < \infty$ *and* $|S| < \infty$*) has a Nash equilibrium.*

- The proof follows Nash's (1951) second proof which rests upon Brouwer's fixed-point theorem and is somewhat similar to the proof of the Walras equilibrium. That is the reason why we present it now.
- See manuscript ...

Exchange and production economy: positive theory

Definition

A production and exchange economy is a tuple

$$\mathcal{E} = \left(N, M, G, \left(\omega^i\right)_{i \in N}, \left(\precsim^i\right)_{i \in N}, \left(Z^j\right)_{j \in M}, \left(\theta^i_j\right)_{\substack{i \in N,\\ j \in M}} \right)$$

consisting of

- the set of households $N = \{1, 2, ..., n\}$,
- the set of firms $M = \{1, 2, ..., m\}$,
- the set of goods $G = \{1, ..., \ell\}$,
- for every household $i \in N$
  - an endowment $\omega^i \in \mathbb{R}^\ell_+$ and a preference relation $\precsim^i$,
- for every firm $j \in M$ a production set $Z^j \subseteq \mathbb{R}^\ell$ and
- the economy's ownership structure $\left(\theta^i_j\right)_{\substack{i \in N,\\ j \in M}}$ where $\theta^i_j \geq 0$ for all $i \in N$, $j \in M$ and $\sum_{i=1}^n \theta^i_j = 1$ for all $j \in M$ hold.

Exchange and production economy: positive theory

Definition

Let $\mathcal{E}$ be a production and exchange economy. The production plans z^j, $j \in M$, and the consumption plans x^i, $i \in N$, are called feasible if they fulfill

- $z^j \in Z^j$ for all $j \in M$ and
- $\sum_{j \in M} z_g^j \geq \sum_{i \in N} \left(x_g^i - \omega_g^i \right)$ for all $g \in G$.

Exchange and production economy: positive theory

Definition

A price vector $\widehat{p} \in \mathbb{R}^{\ell}$, together with the corresponding production plans $\left(\hat{y}^j\right)_{j \in M}$ and consumption plans $\left(\widehat{x}^i\right)_{i \in N}$, is called a Walras equilibrium of a production and exchange economy $\mathcal{E}$ if

- the production and consumption plans are feasible,
- for every household $i \in N$, $\widehat{x}^i$ is a best bundle for consumer i from his budget set

$$B^i\left(\widehat{p}, \omega^i, \left(\theta^i_j\right)_{j \in M}\right) := \left\{x^i \in \mathbb{R}^{\ell}_+ : \widehat{p} \cdot x^i \leq \widehat{p} \cdot \omega^i + \sum_{j \in M} \theta^i_j \widehat{p} \cdot \hat{z}^j\right\}$$

  and
- for every firm $j \in M$, $\hat{z}^j$ is from $\arg\max_{z^j \in Z^j} \widehat{p} \cdot z^j$.

Normative theory

The first welfare theorem from the point of view of partial analysis

slope of	holding constant	algebraic expression
indifference curve	utility $U(x_1, x_2)$	$MRS = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}}$
isoquant	output $f(x_1, x_2)$	$MRTS = \frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}}$
transformation curve	cost $C(x_1, x_2)$	$MRT = \frac{\frac{\partial C}{\partial x_1}}{\frac{\partial C}{\partial x_2}}$

Normative theory

The first welfare theorem from the point of view of partial analysis

- A theoretical reason for the confidence of many economists in the efficiency of the market mechanism lies in the first theorem of welfare economics which states that a system of perfectly competitive markets is Pareto efficient.
- Partial analysis (we concentrate on one or two markets leaving the repercussions on and from other markets aside) concerns
  - exchange optimality (is it possible to make a consumer better off without making another one worse off?),
  - production optimality (is it possible to produce more of one good without producing less of any other good?), and
  - the optimal product mix (is it better to produce more of one good and less of another one?).

Normative theory

Exchange optimality

Assume two households A and B and two goods 1 and 2.

- First step:
  Along the contract curve or exchange curve,

$$\left|\frac{dx_2^A}{dx_1^A}\right| = MRS^A \stackrel{!}{=} MRS^B = \left|\frac{dx_2^B}{dx_1^B}\right|$$

- Second step:
  Household optimality means

$$MRS^A \stackrel{!}{=} \frac{p_1}{p_2} \stackrel{!}{=} MRS^B.$$

Thus, the Walras equilibrium implies exchange optimality.

Normative theory

Production optimality

Assume two goods 1 and 2 produced by factors of production C (capital) and L (labor).

- First step:
  Pareto efficiency implies

$$\left|\frac{dC_1}{dL_1}\right| = MRTS_1 \overset{!}{=} MRTS_2 = \left|\frac{dC_2}{dL_2}\right|.$$

- Second step:
  Cost minimization means

$$MRTS_1 \overset{!}{=} \frac{w}{r} \overset{!}{=} MRTS_2.$$

Thus, the Walras equilibrium implies production optimality.

Normative theory

Optimal product mix

- First step:
  Pareto optimality implies $MRS = MRT$.
- Second step:
  Individual utility and profit maximization implies

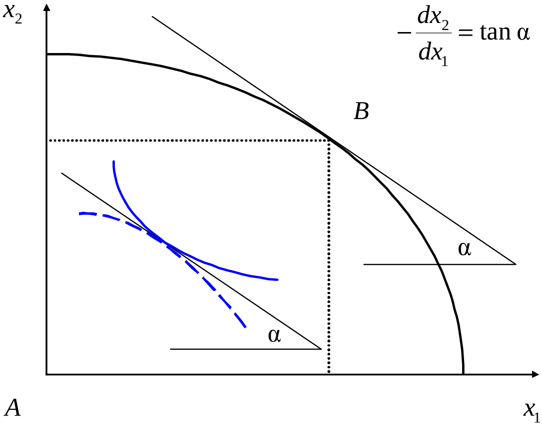
$$MRS = \left|\frac{dx_2}{dx_1}\right|^{\text{indifference curve}} \underbrace{\overset{!}{=}}_{\substack{\text{household}\\\text{optimum}}} \frac{p_1}{p_2} \underbrace{\overset{!}{=}}_{\substack{\text{profit}\\\text{maximization}}} \frac{MC_1}{MC_2}$$

$$= MRT = \left|\frac{dx_2}{dx_1}\right|^{\text{transformation curve}}$$

Thus, the Walras equilibrium implies an optimal product mix.

Normative theory

Summary



Here: $\omega_1^A = \omega_2^A = 0$

How about $\omega_1^A > 0, \omega_2^A > 0$ (two possibilities)

Normative theory

Summary

Pareto optimality requires	in case of perfect competition
$MRS^A \stackrel{!}{=} MRS^B$	$MRS^A \stackrel{!}{=} \frac{p_1}{p_2} \stackrel{!}{=} MRS^B$
$MRTS_1 \stackrel{!}{=} MRTS_2$	$MRTS_1 \stackrel{!}{=} \frac{w}{r} \stackrel{!}{=} MRTS_2$
$MRS \stackrel{!}{=} MRT$	$MRS \stackrel{!}{=} \frac{p_1}{p_2} \stackrel{!}{=} \frac{MC_1}{MC_2} = MRT$

Definition (blockable allocation, core)

Let $\mathcal{E} = \left(N, G, \left(\omega^i\right)_{i \in N}, \left(\precsim^i\right)_{i \in N}\right)$ be an exchange economy. A coalition $S \subseteq N$ is said to block an allocation $\left(y^i\right)_{i \in N}$, if an allocation $\left(z^i\right)_{i \in N}$ exists such that

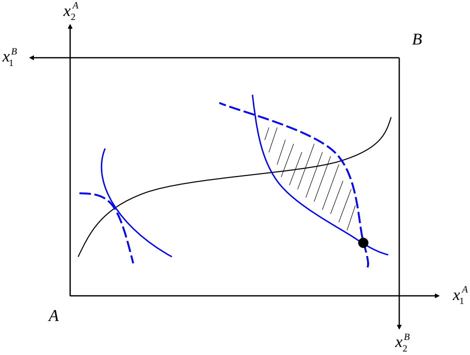
- $z^i \succsim^i y^i$ for all $i \in S$, $z^i \succ^i y^i$ for some $i \in S$ and
- $\sum_{i \in S} z^i \leq \sum_{i \in S} \omega^i$

hold.
An allocation is not blockable if there is no coalition that can block it. The set of all feasible and non-blockable allocations is called the core of an exchange economy.

Normative theory

General equilibrium analysis

- Core in the Edgeworth box: Every household (considered a one-man coalition) blocks any allocation that lies below the indifference curve cutting his endowment point.
- Therefore, the core is contained inside the exchange lense.
- Both households together block any allocation that is not Pareto efficient.
- Thus, the core is the intersection of the exchange lense and the contract curve.



Theorem

Assume an exchange economy $\mathcal{E}$ with local non-satiation and weak monotonicity. Every Walras allocation lies in the core.

Remember from household theory:

Lemma

Let $x^*(p, m)$ be a household optimum. Then local nonsatiation and weak monotonicity imply $p \geq 0$.

Normative theory

General equilibrium analysis - proof

- Consider a Walras allocation $\left(\widehat{x}^i\right)_{i\in N}$. The lemma above implies

$$\widehat{p} \overset{(1)}{\geq} 0$$

where $\widehat{p}$ is the equilibrium price vector.
- Assume, now, that $\left(\widehat{x}^i\right)_{i\in N}$ does not lie in the core. Then, there exists a coalition $S \subseteq N$ that can block $\left(\widehat{x}^i\right)_{i\in N}$, i.e., there is an allocation $\left(z^i\right)_{i\in N}$ such that
  - $z^i \succsim^i \widehat{x}^i$ for all $i \in S$, $z^j \succ^j \widehat{x}^j$ for some $j \in S$ and
  - $\sum_{i\in S} z^i \leq \sum_{i\in S} \omega^i$.

Normative theory

General equilibrium analysis - proof

- The second point, together with (1), implies

$$\widehat{p} \cdot \left(\sum_{i \in S} z^i - \sum_{i \in S} \omega^i \right) \leq 0.$$

- The first point implies

$$\begin{aligned} \widehat{p} \cdot z^i \overset{(2)}{\geq} \widehat{p} \cdot \widehat{x}^i &= \widehat{p} \cdot \omega^i \text{ for all } i \in S \text{ (by local nonsatiation) and} \\ \widehat{p} \cdot z^j \overset{(3)}{>} \widehat{p} \cdot \widehat{x}^j &= \widehat{p} \cdot \omega^j \text{ for some } j \in S \text{ (otherwise, } \widehat{x}^j \text{ not optimal} \end{aligned}$$

Normative theory

General equilibrium analysis - proof

- Summing over all these households from S yields

$$\begin{aligned}
\widehat{p} \cdot \sum_{i \in S} z^i &= \sum_{i \in S} \widehat{p} \cdot z^i \text{ (distributivity)} \\
&> \sum_{i \in S} \widehat{p} \cdot \omega^i \text{ (above inequalities (2) and (3))} \\
&= \widehat{p} \cdot \sum_{i \in S} \omega^i \text{ (distributivity).}
\end{aligned}$$

- This inequality can be rewritten as

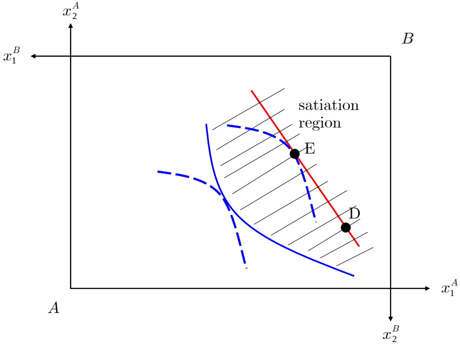
$$\widehat{p} \cdot \left(\sum_{i \in S} z^i - \sum_{i \in S} \omega^i \right) > 0,$$

contradicting the inequality noted above.

Normative theory

General equilibrium analysis

- Example where a Walras allocation does not lie in the core: Agent A's preferences violate non-satiation.
- The equilibrium point E is the point of tangency between the price line and agent B's indifference curve.
- This point is not Pareto-efficient. Agent A could forego some units of both goods without harming himself.



The second welfare theorem turns the first welfare theorem upside down:

- The first welfare theorem says: Walras allocations are Pareto efficient.
- The second welfare theorem claims: Pareto-efficient allocations can be achieved as Walras allocations.

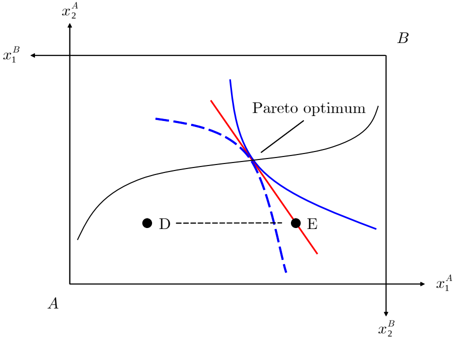
Theorem

Assume an exchange economy $\mathcal{E}$ with convex and continuous preferences for all consumers and local non-satiation for at least one household. Let $\left(\widehat{x}^i\right)_{i\in N}$ be any Pareto-efficient allocation. Then, there exists a price vector $\widehat{p}$ and an endowment $\left(\omega^i\right)_{i\in N}$ such that $\left(\widehat{x}^i\right)_{i\in N}$ is a Walras allocation for $\widehat{p}$.

Normative theory

The second welfare theorem

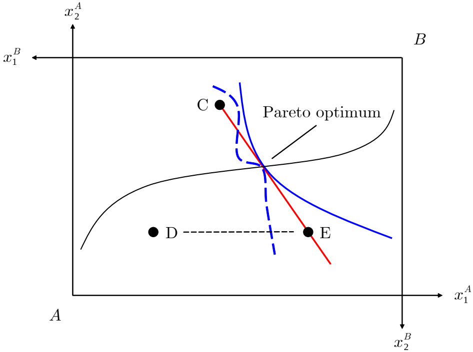
- The figure illustrates the theorem: If point E is given as endowment point, the associated Walras allocation is indeed the Pareto optimum.
- If, however, the original endowment is D instead of E, we can redistribute endowments by transfering some units of good 1 from agent B to agent A.



Normative theory

The second welfare theorem

- The figure illustrates why we assume convexity in the above theorem: Agent B does not have convex preferences.
- At the prices given by the price line, he does not demand his part of the Pareto optimum but some point C.



Further exercises: problem 1

There are two farmers Tim and Bob who harvest and trade wheat (w) and corn (c). Their endowments are $\omega^T = (\omega_c^T, \omega_w^T) = (10, 10)$ and $\omega^B = (\omega_c^B, \omega_w^B) = (30, 0)$.
Tim's preferences are represented by the utility function $U_T(c, w) = \sqrt[3]{c^2 w}$. Bob's utility is a strictly increasing function of wheat. Assume that aggregate excess demand for corn is given by

$$z_c(p_c, p_w) = \frac{-70p_c + 20p_w}{3p_c}$$

a) Show $z_c(p_c, p_w) = z_c(kp_c, kp_w)$ for all $k > 0$!
b) Determine the aggregate excess demand function for wheat!
Hint: Why can you apply Walras' law?
c) Determine the price ratio $\frac{p_c}{p_w}$ such that the corn market clears. Applying the market-clearance lemma, which prices clear the wheat market?
d) What is Tim's marginal rate of substitution $MRS = \left|\frac{dw}{dc}\right|$ between wheat and corn in equilibrium?
e) Is Bob a net supplier of corn?

Further exercises: problem 2

Assume two states of the world $g = 1, 2$ that occur with probabilities p and $1 - p$, respectively. Consider two players $i = A, B$ with vNM preferences. Assume Agent B to be risk neutral and A to be risk averse. Draw an exchange Edgeworth box where x_g^i denotes the payoff (money) enjoyed by player i if state of the world g occurs. Assume that agents like high payoffs in every state that occurs with a probability greater than zero. Agent i's endowment ω_g^i is his payoff in the case where the two agents do not interact.

(a) Imagine a bet between the two agents on the realization of the state of the world. For example, player A puts a small amount of his money on state 1. How are bets and allocations linked?

(b) What do the indifference curves look like?

(c) Reinterpret p as the price for good $g = 1$. Can you confirm the following statement: $(p, 1 - p)$ is the equilibrium price vector. In equilibrium, Agent B provides full insurance to agent A.