UNIVERSITY OF LEIPZIG
FACULTY OF ECONOMICS AND MANAGEMENT SCIENCE

MASTER EXAM

DATE:	24.02.2023
MODULE:	**Advanced Microeconomics**
DURATION:	120 minutes
EXAMINER:	**Prof. Dr. Harald Wiese**

ENROLMENT-No.:

MASTER DEGREE COURSE:

NAME, FIRST NAME:

SIGNATURE (STUDENT):

ANNOTATIONS:
Do not use your own paper!
Write your name and your enrollment number on every single sheet.
Give reasons for your answers! Underline your solutions!
Write legibly, please! Solve all the exercises!
Sie können auf Deutsch schreiben!/You can write in English!

ALLOWED TOOLS: None

	1	2	3	4	5	6	7	8	9	10	11	12	13	$\sum$	**Grade**
Pkt:															

DATE, SIGNATURE (EXAMINER):

Problem 1 (8 points)
Consider the game (N, v) with $N = \{1,2,3\}$ and $v : 2^N \to \mathbb{R}$ where

$$v(K) = \begin{cases} 0, & K = \emptyset \\ 3, & K{=}\{2,3\} \\ 2, & K \in \{\{2\},\{3\}\} \\ 1, & \text{otherwise} \end{cases}$$

a) Calculate the Shapley payoffs for all players.

b) Determine the core.

Solution

a) Player 2 and player 3 are symmetric because $v(\emptyset \cup \{2\}) = v(\emptyset \cup \{3\})$, $v(\{2\} \cup \{1\}) = v(\{3\} \cup \{1\})$. Therefore, the Shapley payoffs Sh_2 and Sh_3 for player 2 and player 3, respectively, satisfy $Sh_2 = Sh_3$. There are six rank orders: $\rho_1 = (1,2,3)$, $\rho_2 = (1,3,2)$, $\rho_3 = (2,1,3)$, $\rho_4 = (2,3,1)$, $\rho_5 = (3,1,2)$, $\rho_6 = (3,2,1)$. The marginal contributions of player 1 are

$$\begin{aligned} MC_1(\rho_1) = MC_1(\rho_2) &= v(\{1\}) - v(\emptyset) = 1 - 0 = 1, \\ MC_1(\rho_3) = MC_1(\rho_5) &= v(\{1,2\}) - v(\{2\}) = 1 - 2 = -1, \\ MC_1(\rho_4) = MC_1(\rho_6) &= v(\{1,2,3\}) - v(\{1,2\}) = 1 - 3 = -2. \end{aligned}$$

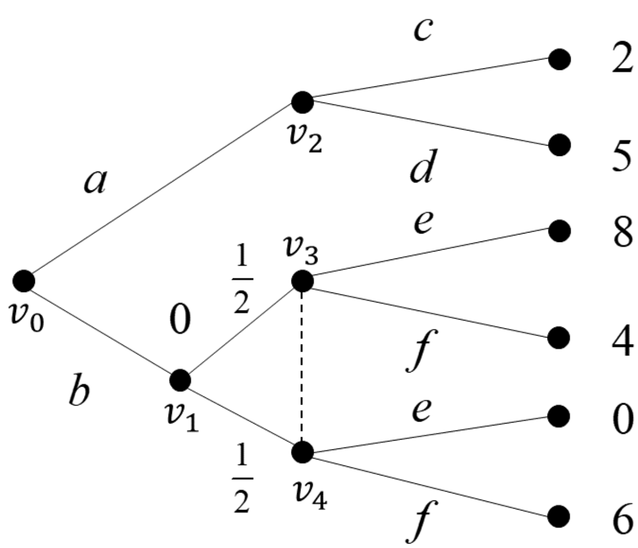
Player 1's Shapley payoff is then given by $Sh_1 = \frac{1}{6}\sum_{i=1}^{6} MC_1(\rho_i) = \frac{1}{6} \cdot (-4) = -\frac{2}{3}$. By efficiency, i.e., $Sh_1 + Sh_2 + Sh_3 = v(N)$, and by $Sh_2 = Sh_3$, we have

$$\begin{aligned} Sh_1 + Sh_2 + Sh_3 &= v(N), \\ 2Sh_2 - \frac{2}{3} &= 1 \\ 2Sh_2 &= \frac{5}{3} \\ \Rightarrow Sh_2 &= \frac{5}{6}. \end{aligned}$$

So the Shapley payoffs are $Sh = \left(-\frac{2}{3}, \frac{5}{6}, \frac{5}{6}\right)$.

b) A payoff vector $(x_1, x_2, x_3) \in \mathbb{R}^3$ from the core must be feasible, i.e., $\sum_{i=1}^{3} x_i \leq v(N) = 1$, and non-blockable by the grand coalition, i.e., $\sum_{i=1}^{3} x_i \geq v(N) = 1$. This implies $\sum_{i=1}^{3} x_i = 1$. In addition, this payoff vector must be non-blockable by coalition $\{1\}$, $\{2\}$, and $\{3\}$, which implies $x_1 \geq 1$, $x_2 \geq 2$, and $x_3 \geq 2$, respectively. We get $x_1 + x_2 + x_3 \geq 5$, which is a contraction to feasibility. Hence, the core is empty.

Problem 2 (11 points)
Consider the following decision problem with moves by nature:



a) How many subtrees does this decision tree have? Give their initial nodes!

b) Does this decision situation exhibit imperfect recall?

c) How many strategies can you find? Give two examples.

d) Determine the best pure strategies.

Solution

a) There are three subtrees starting at v_0, v_1 , and v_2.

b) No. The information sets of decision nodes v_0 and v_2 are only involving one element with only one experience, respectively. Consider the two remaining decision nodes, v_3 and v_4. The decision nodes in the information set $I(v_3) = \{v_3, v_4\} = I(v_4)$ have one and the same experience $X(v_3) = (I(v_0), b, \{v_3, v_4\}) = X(v_4)$. Thus, the decision situation does not exhibit imperfect recall.

c) There are $2^3 = 8$ strategies, for example $\lfloor a, c, e \rfloor$ and $\lfloor b, d, f \rfloor$.

d) The pure strategies yielding the highest payoff are the best pure strategies.

$$
\begin{aligned}
u(\lfloor a, c, e \rfloor) &= u(\lfloor c, c, f \rfloor) = 2 \\
u(\lfloor a, d, e \rfloor) &= u(\lfloor a, d, f \rfloor) = 5 \\
u(\lfloor b, c, e \rfloor) &= u(\lfloor b, d, e \rfloor) = \frac{1}{2} \cdot 8 + \frac{1}{2} \cdot 0 = 4 \\
u(\lfloor b, c, f \rfloor) &= u(\lfloor b, d, f \rfloor) = \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 6 = 5
\end{aligned}
$$

The highest payoff of 5 can be reached by four strategies: $\lfloor a, d, e \rfloor$, $\lfloor a, d, f \rfloor$, $\lfloor b, c, f \rfloor$, and $\lfloor b, d, f \rfloor$.

Problem 3 (4 points)
State the four duality equations of household theory!

Solution

$$
\begin{aligned}
\chi(p, V(p,m)) &= x(p,m) \\
e(p, V(p,m)) &= m \\
x(p, e(p,\bar{U})) &= \chi(p,\bar{U}) \\
V(p, e(p,\bar{U})) &= \bar{U}
\end{aligned}
$$

Problem 4 (6 points)
Consider the utility function $U(x_1, x_2) = \frac{1}{2} \ln x_1 - x_2$.

a) Determine the Hicksian demand function for both goods.

b) Determine the expenditure function.

Solution
Notation: $e(p, \bar{U})$ is the value of the expenditure function at $(p, \bar{U})$, while e is the irrational number $e = 2.718....$

a) Good 1 is a good while good 2 is a bad. So the household will consume good 1 only. We get $\chi_2(p, \bar{U}) = 0$. Plugging this into the utility function and solving for χ_1 yields $\bar{U} = \frac{1}{2} \ln \chi_1 - 0 \Leftrightarrow \ln \chi_1 = 2\bar{U} \Leftrightarrow \chi_1(p, \bar{U}) = e^{2\bar{U}}$. The Hicksian demand function for both goods is $\big(\chi_1(p, \bar{U}), \chi_2(p, \bar{U})\big) = \left(e^{2\bar{U}}, 0\right)$.

b) The expenditure function is given by $e(p, \bar{U}) = p_1\chi_1(p, \bar{U}) + p_2\chi_2(p, \bar{U}) = p_1 e^{2\bar{U}}$.

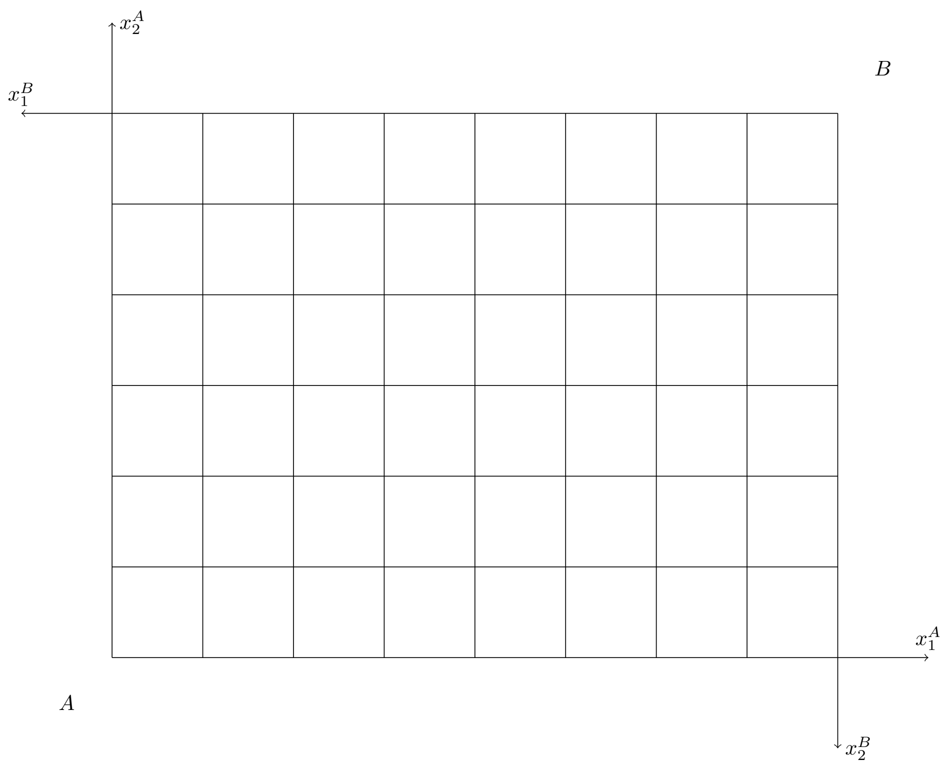
Problem 5 (12 points)
Consider an exchange economy with two agents A and B. The utility function of agent A is given by $U_A(x_1^A, x_2^A) = x_1^A - x_2^A$. The utility function of agent B is given by $U_B(x_1^B, x_2^B) = \min\left\{2x_1^B, x_2^B\right\}$. The initial endowment is given by

$$\omega = (\omega^A, \omega^B) = ((4,4),(4,2)).$$

a) Use the graphic below to illustrate the initial endowment, the indifference curves of both agents that run through the endowment, the better sets and the exchange lens, both with respect to ω.

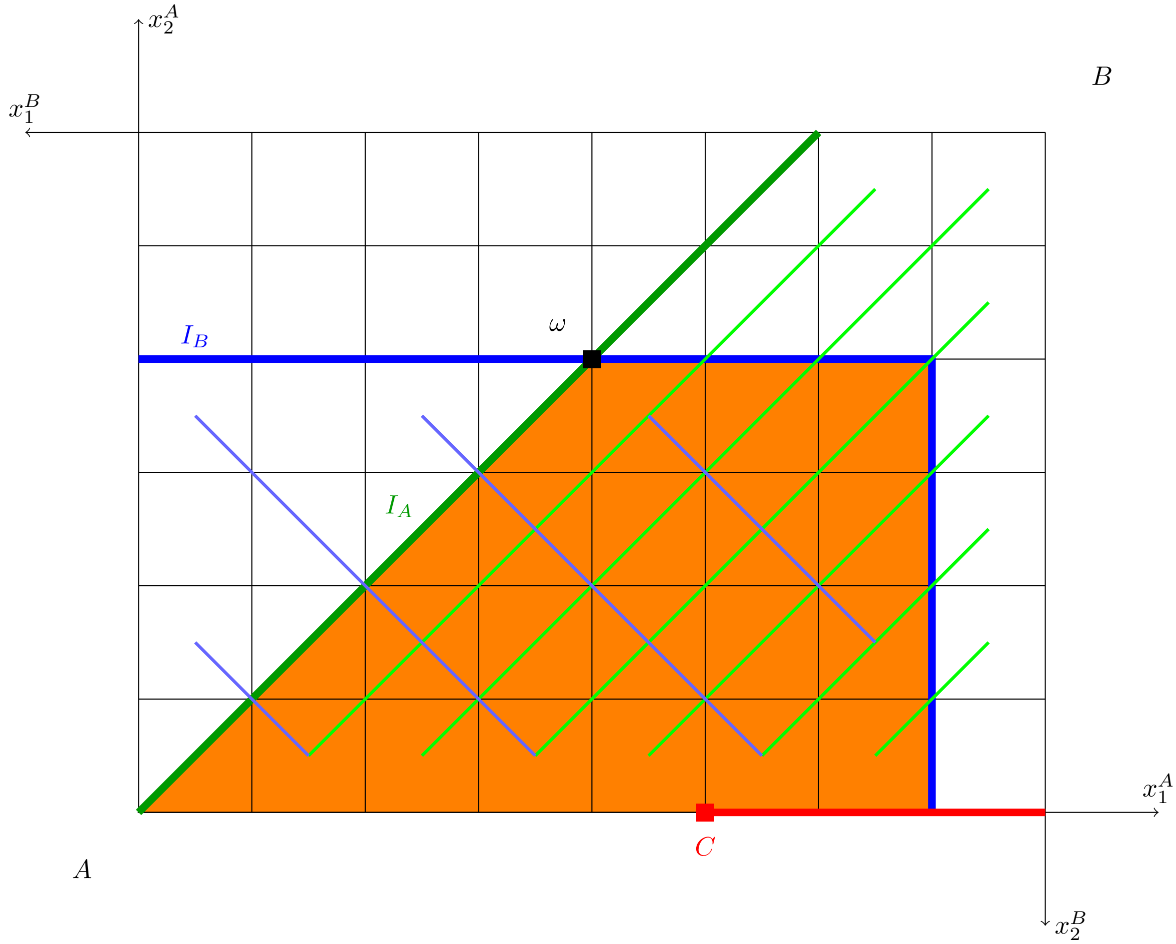
b) Is point $C = ((5,0),(3,6))$ a Pareto improvement over the initial endowment? Is it Pareto efficient?

c) Draw and explain the contract curve.



Solution

a) The initial endowment is displayed at ω. The indifference curve of agent B is L-shaped and indicated by the blue line. The indifference curve of agent A is given by the green line. The slope is positive because good 1 is a good while good 2 is a bad. The better set of agent A is below and right of the indifference curve (light green, more of good 1 and less of good 2), while the better set of agent B is below and left of the indifference curve (more of both goods). The better set of B is illustrated in light blue. The exchange lens with respect to ω is given as the orange area where at least one agent can improve without putting the other at a worse position (trapeze between (0,0), (4,4), (7,4), (7,0) including the initial endowment point).



b) Yes, C is a Pareto improvement because $U_A(\omega_1^A, \omega_1^A) = 4-4 = 0 < 5 = 5-0 = U_A(5,0)$ and $U_B(\omega_1^B, \omega_1^B) = \min\{8,2\} = 2 < 6 = \min\{6,6\} = U_B(3,6)$. It is also Pareto efficient because no agent can be made better off without making the other agent worse off: Agent A has nothing of the bad ($x_2^A = 0$). Her utility can be increased by increasing x_1^A. This would result in a decrease of good 1 for agent B which would make B worse off ($U_B < 6$). To increase the utility of agent B, he needs more of both goods which is not possible because he already has all of good 2 ($x_2^B = 6 = \omega_2^A + \omega_2^B$) available.

c) All Pareto-efficient allocations need to fulfill $x_2^A = 0$ because good 2 is a bad for consumer A and B's preferences are monotonic w.r.t. good 2. Because agent B cannot have more than 6 units of good 2, B's maximum ulility level is $U_B = 6$. In fact, only allocations fulfilling $x_2^B = 2x_1^B$ are efficient to agent B. At point C this condiction is fulfilled; C is Pareto efficient and, thus, belongs to the contract curve. Giving more (than three) units of good 1 to agent B cannot make him better than $U_B = 6$, but makes agent A worse off ($U_A < 5$). So all allocations left of point C are not Pareto optimal. All allocations to the right of point C are Pareto optimal because shifting good 1 from one agent to the other makes the agent giving up good 1 worse off. The contract curve is given by the line from $((5,0),(3,6))$ to $((8,0),(0,6))$ (indicated in red).

Problem 6 (10 points)
Consider the following two-person game with mixed strategies. Calculate both reaction functions and illustrate them graphically. Determine all equilibria in pure and properly mixed strategies.

		player 2	
		l	r
player 1	o	$(2,2)$	$(3,4)$
	u	$(0,2)$	$(2,1)$

Solution
Let σ_1 denote the probability of player 1 to play o and let σ_2 denote the probability of player 2 to play l. Player 1 always prefers o over u, o is a dominant strategy. We get

$$\sigma_1^R(\sigma_2) = 1, \forall \sigma_2$$

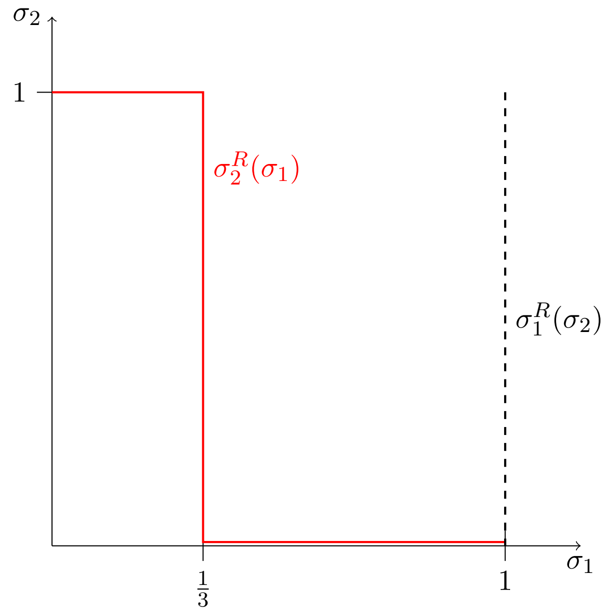
Player 2 prefers l over r if

$$\begin{aligned} 2\sigma_1 + 2(1-\sigma_1) &\geq 4\sigma_1 + (1-\sigma_1) \\ 2 &\geq 3\sigma_1 + 1 \\ \frac{1}{3} &\geq \sigma_1 \end{aligned}$$

holds. We get

$$\sigma_2^R(\sigma_1) = \begin{cases} 0, & \sigma_1 > \frac{1}{3} \\ [0,1], & \sigma_1 = \frac{1}{3} \\ 1, & \sigma_1 < \frac{1}{3}. \end{cases}$$

Hence, there is one equilibrium in pure strategies $(\sigma_1, \sigma_2) = (1,0)$ and no equilibrium in properly mixed strategies. A graphical illustration is given below.


σ_2
1
$\sigma_2^R(\sigma_1)$
$\sigma_1^R(\sigma_2)$
$\frac{1}{3}$
1
σ_1

Problem 7 (9 points)
A firm produces one good with a technology given by the production function

$$y = f(x_1, x_2) = \mathbf{max}\left(4x_1^2, x_2^2\right).$$

The factor prices are $w_1 = 6$ and $w_2 = 2$.

a) Explore whether the production function exhibits decreasing, constant, or increasing returns to scale.

b) Determine the cost function.

Solution

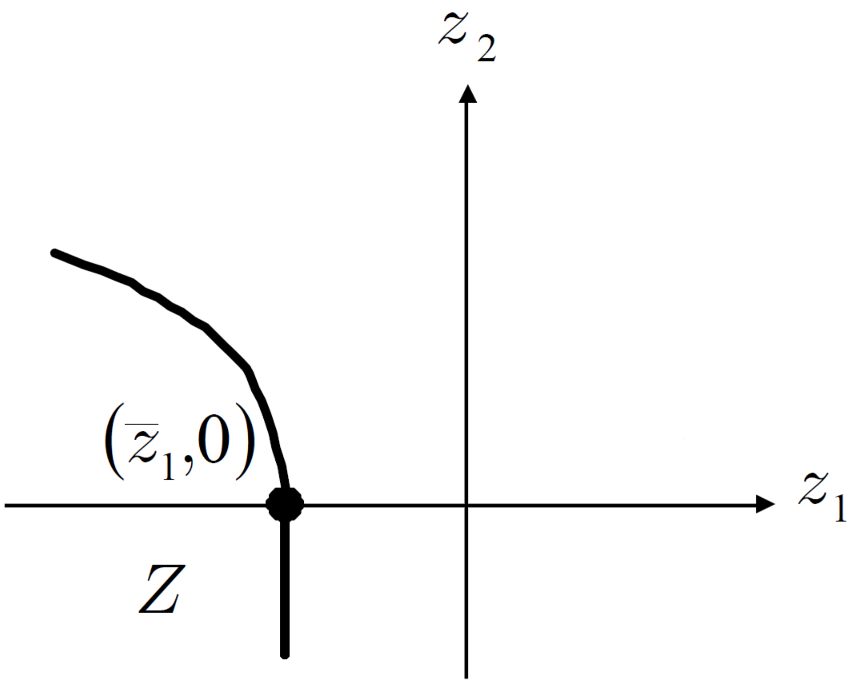
a) For all $t \geq 1$, we have

$$f(tx_1, tx_2) = \max\left(4(tx_1)^2, (tx_2)^2\right) = t^2 \max\left(4x_1^2, x_2^2\right) \geq t \max\left(4x_1^2, x_2^2\right) = tf(x_1, x_2).$$

Hence, the production exhibits increasing returns to scale.

b) The production technology is concave and either factor 1 or factor 2 is used exclusively for production. Because $y = 4x_1^2$ if factor 1 is used exclusively and $y = x_2^2$ if factor 2 is used exclusively, the expenditures to produce y units are $\frac{w_1}{2}\sqrt{y}$ if factor 1 is used exclusively and $w_2\sqrt{y}$ if factor 2 is used exclusively. Since $\frac{w_1}{2} = 3 > 2 = w_2$, factor 2 is used exclusively and we obtain the cost function $C(y) = w_2 x_2(y) = 2\sqrt{y}$.

Problem 8 (4 points)
Consider the production set depicted below.



Tick whether the production set obeys each of the following axioms:

	Yes	No
possibility of inaction		
free disposal		
nonincreasing returns to scale		
convexity		

Solution

	Yes	No
possibility of inaction		x
free disposal	x	
nonincreasing returns to scale		x
convexity	x	

Problem 9 (6 points)
Consider the following two-person game that is repeated twice. Strategies are written as quintuples $\lfloor a, a_{TT}, a_{TF}, a_{FT}, a_{FF} \rfloor$ where a is the action (T or F) at the first stage and a_{TF} the action if she chose T at the first stage and he F.

		he	
		T	F
she	T	$(8, 6)$	$(5, 5)$
	F	$(4, 4)$	$(6, 8)$

a) Is $(\lfloor T, T, F, F, F \rfloor, \lfloor T, T, F, F, F \rfloor)$ a Nash equilibrium of the twice-repeated game?

b) Is $(\lfloor T, T, F, F, F \rfloor, \lfloor T, T, F, F, F \rfloor)$ a subgame-perfect Nash equilibrium of the twice-repeated game?

Solution:

a) If $s = (s_{she}, s_{he}) = (\lfloor T, T, F, F, F \rfloor, \lfloor T, T, F, F, F \rfloor)$ is played, he gets $u_{he}(s) = 6 + 6 = 12$. If he deviates by playing $s'_{he} = \lfloor F, T, F, F, F \rfloor$, he gets $u_{he}((s_{she}, s'_{he})) = 5 + 8 = 13 > 12$. Hence, s is not a Nash equilibrium.

b) Every subgame-perfect Nash equilibrium is a Nash equilibrium. As $s = (s_{she}, s_{he}) = (\lfloor T, T, F, F, F \rfloor, \lfloor T, T, F, F, F \rfloor)$ is no Nash equilibrium (see **(a)**), s is no subgame-perfect Nash equilibrium. Alternatively, just consider the subgame that corresponds to the two-person game that is repeated twice. If he deviates by playing $s'_{he} = \lfloor F, T, F, F, F \rfloor$, he gets $u_{he}((s_{she}, s'_{he})) = 5 + 8 = 13 > 12 = u_{he}(s)$. Hence, s is no subgame-perfect Nash equilibrium.

Problem 10 (8 points)
Consider the following model of a polypsonistic labor market with education where each worker chooses his education $a \in \{0,1\}$ after the screening principal P (or principals) offers two wage rates, $w_0 \in \mathbb{R}$ for workers with education $a = 0$ and $w_1 \in \mathbb{R}$ for workers with education $a = 1$. The proportion of workers with productivity $t_h = 6$ is given by $\tau_h = \frac{1}{3}$, the proportion of workers with productivity $t_l = 3$ by $\tau_l = \frac{2}{3}$. The payoff for worker $t \in \{t_l, t_h\}$ with education $a \in \{0,1\}$ is given by

$$u_t\left(w_a, a\right) = w_a - \frac{12}{t} \cdot a,$$

while the principal's non-probabilisitic payoff for employing worker t with education a is given by

$$u_P(t, w_a) = t - w_a.$$

Determine a separating equilibrium! *Hint: The expected payoff of the principal is zero in equilibrium.*

Solution:
We search for a separating equilibrium where worker $t = t_l$ chooses $a = 0$ and worker $t = t_h$ chooses $a = 1$. Since the principal's expected payoff is zero, an obvious choice of wages is

$$\begin{aligned} w_0 &\stackrel{!}{=} E[t|a=0] = 3, \\ w_1 &\stackrel{!}{=} E[t|a=1] = 6. \end{aligned}$$

Since we have

$$\begin{aligned} (i) \quad & u_{t_l}\left(w_0, 0\right) = 3 - 0 > 2 = 6 - 4 = u_{t_l}\left(w_1, 1\right), \\ (ii) \quad & u_{t_h}\left(w_1, 1\right) = 6 - 2 = 4 > 3 - 0 = u_{t_h}\left(w_0, 0\right), \end{aligned}$$

worker $t = t_l$ is incited to choose $a = 0$, rather than $a = 1$, and worker $t = t_h$ is incited to choose $a = 1$, rather than $a = 0$. Hence, $(w_0, w_1) = (3, 6)$ and $a = 0$ for $t = t_l$ and $a = 1$ for $t = t_h$ is a separating equilibrium. **Remark:** More precisely, the workers react on the two wage rates offered by the principle. Hence, a separating equilibrium is given by the tuple $(\hat{w}, \hat{a})$ where $\hat{w} = (w_0, w_1) = (3, 6)$ and

$$\hat{a}(t, w_0', w_1') = \text{argmax}_{a \in \{0,1\}} u_t(w_a', a)$$

for all $t \in \{t_l, t_h\}$, $w_0' \in \mathbb{R}$, and $w_1' \in \mathbb{R}$.

Problem 11 (7 points)
An agent with vNM-utility function $u(x) = -\frac{1}{x}$, $x > 0$ faces the lottery $L = \left[2, 8; \frac{1}{3}, \frac{2}{3}\right]$.

a) Determine the certainty equivalent of the lottery.

b) Is the agent risk loving?

Solution

a) The certainty equivalent CE is the solution to $u(CE) = E_u(L)$. We get

$$\begin{aligned} u(CE) = -\frac{1}{CE} &\stackrel{!}{=} -\frac{1}{4} = -\frac{3}{12} = -\frac{1}{6} - \frac{1}{12} = -\frac{1}{2} \cdot \frac{1}{3} - \frac{1}{8} \cdot \frac{2}{3} = E_u(L) \\ &\Rightarrow CE = 4. \end{aligned}$$

b) We have $u'(x) = \frac{1}{x^2}$ and $u''(x) = -\frac{1}{2x^3} < 0$. Hence, the agent is not risk loving.

Problem 12 (11 points)
Consider an exchange economy with two agents, A and B. Their utility functions are given by

$$U^A(x_1^A, x_2^A) = \left(x_1^A\right)^2 x_2^A, \qquad U^B(x_1^B, x_2^B) = 2x_1^B + x_2^B,$$

their endowments by

$$w^A = (8, 2), \qquad w^B = (1, 7).$$

Show that there is no Walras equilibrium with $p_1 > 2p_2$.

Solution

We proceed in three steps. First, we show $p \cdot z(p) = 0$. Second, we show $z(p) = 0$. Third, we show that $p_1 > 2p_2$ leads to a contradiction of $z_1(p) = 0$.

First: The preferences of each agent satisfy local non-satiation because, for all consumption bundles and arbitrarily small $\varepsilon > 0$, $U^A(x_1^A + \varepsilon, x_2^A + \varepsilon) > U^A(x_1^A, x_2^A)$ and $U^B(x_1^B + \varepsilon, x_2^B + \varepsilon) > U^B(x_1^B, x_2^B)$. Hence, we can apply Walras law to get $p \cdot z(p) = 0$.

Second: In a Walras equilibrium, $z(p) \leq 0$ must hold. The preferences of B are strictly monotonic because $MU_1^B = 2 > 0$ and $MU_2^B = 1 > 0$. The strict monotonicity of B's preferences imply, for $p_i = 0$, $z_i(p) > 0$, which contradicts $z(p) \leq 0$. Hence, equilibrium prices must be positive, i.e., $p > 0$. Positive prices $p > 0$ imply that $z_i(p) < 0$ yields a contradiction to Walras law

$$p \cdot z(p) = p_i z_i(p) + p_j z_j(p) = 0 < p_j z_j(p) \leq 0$$

since $p_i z_i(p) < 0$ and $p_j z_j(p) \leq 0$ due to $z_j(p) \leq 0$. Hence, we must have $z(p) = 0$.

Third: The marginal rate of substituition of B is given by $MRS^B = \frac{MU_1^B}{MU_2^B} = 2$. For $p_1 > 2p_2$, we have $MRS^B = 2 < \frac{p_1}{p_2}$. Hence, the optimal consumption of good 1 by B must satisfy $x_1^B = 0$, which by $z_1(p) = 0$ implies that A must consume $x_1^A = w_1^A + w_1^B - x_1^B = 8 + 1 - 0 = 9$. However, each consumption bundle (x_1^A, x_2^A) with $x_1^A = 9$ and $x_2^A \geq 0$ is outside A's budget because

$$p_1 w_1^A + p_2 w_2^A = 8p_1 + 2p_2 < 8p_1 + p_1 = 9p_1 \leq 9p_1 + x_2^A p_2 = p_1 x_1^A + p_2 x_2^A.$$

This contradicts $x_1^A = 9$ and therefore $z_1(p) = 0$. Hence, there is no Walras equilibrium with $p_1 > 2p_2$.

Remark: There are alternative proofs. For example, one can determine the optimal consumption of good 2 by both agents (after proving that prices must be positive) to show a contradiction to $z_2(p) \leq 0$.

Problem 13 (14 points)
Two firms compete simultaneously in quantities. The cost function of firm 1 is common knowledge and equal to $C_1(x_1) = \frac{1}{2}x_1^2$. Firm 2 has constant marginal and average costs c which are either $c_l = 4$ or $c_h = 8$. The costs of firm 2 are known by firm 2 but not observable for firm 1. However, it is common knowledge that c_l occurs with probability $\frac{1}{2}$, while c_h occurs with probability $\frac{1}{2}$. The inverse demand function is given by

$$p(x_1 + x_2) = 24 - x_1 - x_2.$$

a) Determine the reaction functions of both firms.

b) Determine the Bayesian equilibrium.

Solution

a) Firm 2 is of two types, $c = c_l$ and $c = c_h$. The profit function of firm 2 is given by

$$\Pi_2(x_1, x_2) = (24 - x_1 - x_2)x_2 - cx_2 = (24 - c - x_1 - x_2)x_2.$$

Equating the first-order derivative to zero, yields

$$\frac{d\Pi_2(x_1, x_2)}{dx_2} = 24 - c - x_1 - 2x_2 \stackrel{!}{=} 0$$
$$\Rightarrow x_2 = \frac{24 - c}{2} - \frac{x_1}{2}.$$

Hence, the reaction function of firm 2 is given by

$$x_2^R(x_1) = \left\lfloor x_2^{l,R}(x_1), x_2^{h,R}(x_1) \right\rfloor = \left\lfloor 10 - \frac{x_1}{2}, 8 - \frac{x_1}{2} \right\rfloor.$$

Firm 1 knows that firm 2 chooses its quantity, $x_2 = x_2^l$ or $x_2 = x_2^h$, depending on its type, $c = c_l$ or $c = c_h$. The expected profit of firm 1 is given by

$$\Pi_1(x_1, x_2^l, x_2^h) = (24 - x_1 - \frac{1}{2} \cdot x_2^l - \frac{1}{2} \cdot x_2^h)x_1 - \frac{1}{2} \cdot x_1^2$$

Equating the first-order derivative to zero, yields the reaction function of firm 1:

$$\frac{d\Pi_1(x_1, x_2^l, x_2^h)}{dx_1} = 24 - 2x_1 - \frac{1}{2} \cdot x_2^l - \frac{1}{2} \cdot x_2^h - x_1 \stackrel{!}{=} 0$$
$$\Rightarrow 24 - \frac{1}{2} \cdot x_2^l - \frac{1}{2} \cdot x_2^h \stackrel{!}{=} 3x_1$$
$$\Rightarrow x_1^R(x_2^l, x_2^h) = 8 - \frac{x_2^l}{6} - \frac{x_2^h}{6}.$$

b) We determine the Bayesian equilibrium by determining the intersection of the two reaction functions:

$$\begin{aligned} x_1 &\stackrel{!}{=} x_1^R(x_2^{l,R}(x_1), x_2^{h,R}(x_1)) \\ &= 8 - \frac{1}{6}\left(10 - \frac{x_1}{2} + 8 - \frac{x_1}{2}\right) \\ &= 8 - \frac{1}{6}(18 - x_1) \\ &= 5 + \frac{x_1}{6} \\ \Rightarrow \frac{5x_1}{6} &= 5 \\ \Rightarrow x_1 &= 6. \end{aligned}$$

Inserting $x_1 = 6$ into the reaction function of firm 2 yields

$$\left\lfloor x_2^l, x_2^h \right\rfloor = \left\lfloor 10 - \frac{6}{2}, 8 - \frac{6}{2} \right\rfloor = \lfloor 7, 5 \rfloor .$$

Hence, the Bayesian equilibrium is given by $\left(x_1, \lfloor x_2^l, x_2^h \rfloor\right) = (6, \lfloor 7, 5 \rfloor)$.