

Advanced Microeconomics

The revelation principle and mechanism design

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1. Introduction
2. First-price auction

The revelation principle and mechanism design

Introduction

- ▶ By mechanism design, we understand the problem of setting up a Bayesian game so as to benefit the principal.
- ▶ Example: first-price auction versus second-price auction
- ▶ Problem: players do not, in general, “tell the truth” (e.g., bid according to willingness to pay)
- ▶ Revelation principle: we can restrict attention to mechanisms where players “tell the truth” or “reveal their own type”
- ▶ Thus, the revelation principle helps to find the best mechanism by restricting the set of candidate mechanisms.

The revelation principle and mechanism design

Nobel Prize

In 2007, the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel was awarded to the US economists

1/3 Leonid Hurwicz (University of Minnesota),

1/3 Eric S. Maskin (Institute for Advanced Study, Princeton), and

1/3 Roger B. Myerson (University of Chicago)

for having laid the foundations of mechanism design theory.

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Example: first-price auction and half-price auction

- ▶ In the first-price auction, every bidder acts strategically because a bid does not only influence the chance of getting the object but also the price the bidder has to pay in case of winning the auction.
- ▶ We found the equilibrium strategy combination $s^* = (s_1^*, s_2^*)$ defined by

$$s_1^* : [0, 1] \rightarrow \mathbb{R}_+, \quad t_1 \mapsto s_1^*(t_1) = \frac{t_1}{2} \text{ and}$$
$$s_2^* : [0, 1] \rightarrow \mathbb{R}_+, \quad t_2 \mapsto s_2^*(t_2) = \frac{t_2}{2}.$$

- ▶ In that combination, every player uses the half-bid strategy.

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Example: first-price auction and half-price auction

Half-price auction:

- ▶ the object goes to the highest bidder but
- ▶ the payment is set at half the announced willingness to pay

Construction (using the first-price auction and the half-bid strategies):

- ▶ The half-bid strategies have every bidder halve his type, $a_i = \frac{t_i}{2}$. Under the first-price auction, the successful bidder pays $a_i = \frac{t_i}{2}$.
- ▶ The half-price auction tries to achieve the same outcome for players who tell the truth. Thus, the factor $\frac{1}{2}$
 - ▶ stemming from the players' strategic behavior
 - ▶ is brought into the half-price auction by requiring that the successful bidder pays $\frac{a_i}{2}$, only.

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Example: first-price auction and half-price auction

- ▶ Assume truthful bidding in the half-price auction:

$$id_i : [0, 1] \rightarrow [0, 1], t_i \mapsto id_i(t_i) = t_i.$$

- ▶ Bidder with highest willingness to pay obtains the object:

$$\underbrace{t_1 \leq t_2} \quad \Leftrightarrow \quad \underbrace{\frac{t_1}{2} \leq \frac{t_2}{2}}.$$

comparison of the player's
truthful announcements
under the half-price auction

comparison of the player's
strategic announcements
under the first-price auction

- ▶ The successful bidder i pays $t_i/2$:
 - ▶ in the first-price auction, by understating the willingness to pay.
 - ▶ in the half-price auction, by truthful announcement.
- ▶ Is (id_1, id_2) a Bayesian equilibrium of the half-price auction?

Yes ...

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Example: first-price auction and half-price auction

- ▶ s^* is an equilibrium of the first-price auction.
- ▶ Assume $s_1 = id_1$ were not a best response to $s_2 = id_2$ in the half-price auction. Then there is a type t_1 and an action (announced type) $a_1 \neq t_1$ such that

$$\underbrace{\left(t_1 - \frac{a_1}{2}\right) \tau(\{t_2 : a_1 > t_2\})}_{\text{untruthful bid}} > \underbrace{\left(t_1 - \frac{t_1}{2}\right) \tau(\{t_2 : t_1 > t_2\})}_{\text{truthful bid}}$$

$$\left(t_1 - \frac{a_1}{2}\right) \tau\left(\left\{t_2 : \frac{a_1}{2} > \frac{t_2}{2}\right\}\right) > \left(t_1 - \frac{t_1}{2}\right) \tau\left(\left\{t_2 : \frac{t_1}{2} > \frac{t_2}{2}\right\}\right)$$

- ▶ Contradiction to s_1^* being a best response to s_2^* in the first-price auction.
- ▶ (id_1, id_2) is a Nash equilibrium in the half-price auction.

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Example: first-price auction and half-price auction

In line with the revelation principle

*we can restrict attention to mechanisms where players
“tell the truth” or “reveal their own type”*

the results from the first-price auction

- ▶ who gets the object
- ▶ what price does the auctioneer get

are also obtained by the half-price auction