

Advanced Microeconomics

The revelation principle and mechanism design

Harald Wiese

University of Leipzig

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- 1 Introduction
- 2 First-price auction

The revelation principle and mechanism design

Introduction

- By mechanism design, we understand the problem of setting up a Bayesian game so as to benefit the principal.
- Example: first-price auction versus second-price auction
- Problem: players do not, in general, “tell the truth” (e.g., bid according to willingness to pay)
- Revelation principle: we can restrict attention to mechanisms where players “tell the truth” or “reveal their own type”
- Thus, the revelation principle helps to find the best mechanism by restricting the set of candidate mechanisms.

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Nobel Prize

In 2007, the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel was awarded to the US economists

1/3 Leonid Hurwicz (University of Minnesota),

1/3 Eric S. Maskin (Institute for Advanced Study, Princeton), and

1/3 Roger B. Myerson (University of Chicago)

for having laid the foundations of mechanism design theory.

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Example: first-price auction and half-price auction

- In the first-price auction, every bidder acts strategically because a bid does not only influence the chance of getting the object but also the price the bidder has to pay in case of winning the auction.
- We found the equilibrium strategy combination $s^* = (s_1^*, s_2^*)$ defined by

$$s_1^* : [0, 1] \rightarrow \mathbb{R}_+, \quad t_1 \mapsto s_1^*(t_1) = \frac{t_1}{2} \text{ and}$$
$$s_2^* : [0, 1] \rightarrow \mathbb{R}_+, \quad t_2 \mapsto s_2^*(t_2) = \frac{t_2}{2}.$$

- In that combination, every player uses the half-bid strategy.

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Example: first-price auction and half-price auction

Half-price auction:

- the object goes to the highest bidder but
- the payment is set at half the announced willingness to pay

Construction (using the first-price auction and the half-bid strategies):

- The half-bid strategies have every bidder halve his type, $a_i = \frac{t_i}{2}$.
Under the first-price auction, the successful bidder pays $a_i = \frac{t_i}{2}$.
- The half-price auction tries to achieve the same outcome for players who tell the truth. Thus, the factor $\frac{1}{2}$
 - stemming from the players' strategic behavior
 - is brought into the half-price auction by requiring that the successful bidder pays $\frac{a_i}{2}$, only.

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Example: first-price auction and half-price auction

- Assume truthful bidding in the half-price auction:

$$id_i : [0, 1] \rightarrow [0, 1], t_i \mapsto id_i(t_i) = t_i.$$

- Bidder with highest willingness to pay obtains the object:

$$\underbrace{t_1 \leq t_2}$$

comparison of the player's
truthful announcements
under the half-price auction

\Leftrightarrow

$$\underbrace{\frac{t_1}{2} \leq \frac{t_2}{2}}$$

comparison of the player's
strategic announcements
under the first-price auction

- The successful bidder i pays $t_i/2$:
 - in the first-price auction, by understating the willingness to pay.
 - in the half-price auction, by truthful announcement.
- Is (id_1, id_2) a Bayesian equilibrium of the half-price auction? Yes ...

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Example: first-price auction and half-price auction

- s^* is an equilibrium of the first-price auction.
- Assume $s_1 = id_1$ were not a best response to $s_2 = id_2$ in the half-price auction. Then there is a type t_1 and an action (announced type) $a_1 \neq t_1$ such that

$$\underbrace{\left(t_1 - \frac{a_1}{2}\right) \tau(\{t_2 : a_1 > t_2\})}_{\text{untruthful bid}} > \underbrace{\left(t_1 - \frac{t_1}{2}\right) \tau(\{t_2 : t_1 > t_2\})}_{\text{truthful bid}} \text{ or}$$

$$\left(t_1 - \frac{a_1}{2}\right) \tau\left(\left\{t_2 : \frac{a_1}{2} > \frac{t_2}{2}\right\}\right) > \left(t_1 - \frac{t_1}{2}\right) \tau\left(\left\{t_2 : \frac{t_1}{2} > \frac{t_2}{2}\right\}\right)$$

- Contradiction to s_1^* being a best response to s_2^* in the first-price auction.
- (id_1, id_2) is a Nash equilibrium in the half-price auction.

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Example: first-price auction and half-price auction

In line with the revelation principle

we can restrict attention to mechanisms where players “tell the truth” or “reveal their own type”

the results from the first-price auction

- who gets the object
- what price does the auctioneer get

are also obtained by the half-price auction