Advanced Microeconomics

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What I like about economic theory and microeconomics

- Not just talk, but theory with formal models (—> sociology)
- The core of neoclassical economics: Walras' model of perfect competition (chapter XIX)
  - interdependence of markets and
  - prices to bring markets into equilibrium
- and also and beyond:
  - decisions under uncertainty (chapters III and IV)
  - detailed analysis of consumer behavior (chapters VI and VII)
  - interactive decisions = game theory (chapters X to XIII)
  - simple concepts like Pareto efficiency with wide ramifications (chapter XIV)
  - auction theory (chapters XVII and XVIII)
  - adverse selection (chapter XXII) and hidden action (XXIII)
- However: hard work ahead

Technical stuff

- 3 class meetings
  - Monday 09:15 - 10:45
  - Wednesday 09:15 - 10:45
  - Friday 09:15 - 10:45
- mixing lectures and exercises
- written exam 120 min. at two dates
  - midterm in December (60 min.)
  - final in February (60 min.)

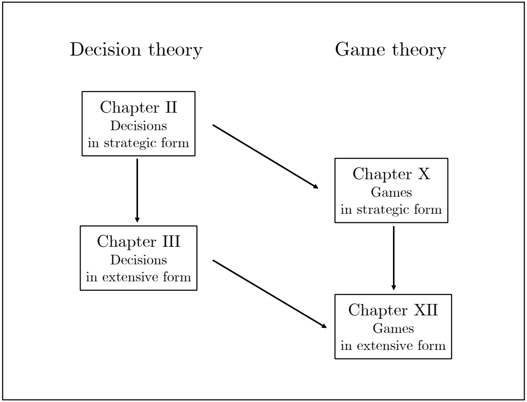
Literature

- Harald Wiese: Advanced microeconomics (which is more detailed than the lecture)
- Andreu Mas-Colell, Michael D. Whinston, Jerry R. Green: Microeconomic Theory
- Hugh Gravell, Ray Rees: Microeconomics
- Geoffrey Jehle, Philip Reny: Advanced Microeconomic Theory
- Samuel Bowles: Microeconomics – Behavior, Institutions, and Evolution
- Thomas Hönscheid: Schwester Helga

Course overview

Part A. Basic decision and preference theory:

- Decisions in strategic form
- Decisions in extensive form
- Ordinal preference theory
- Decisions under risk



Course overview

Part B. Household theory and theory of the firm:

- The household optimum
- Comparative statics and duality theory
- Production theory
- Cost minimization and profit maximization

Course overview

Part C. Games and industrial organization:

- Games in strategic form
- Price and quantity competition
- Games in extensive form
- Repeated games

Course overview

Part D. Bargaining theory and Pareto optimality:

- Pareto optimality in microeconomics
- Cooperative game theory

Course overview

Part E. Bayesian games and mechanism design:

- Static Bayesian games
- The revelation principle and mechanism design

Course overview

Part F. Perfect competition and competition policy:

- General equilibrium theory I: the main results
- General equilibrium theory II: criticism and applications
- Introduction to competition policy and regulation

Course overview

Part G. Contracts and principal-agent theories:

- Adverse selection
- Hidden action

Part A. Basic decision and preference theory

1. **Decisions in strategic (static) form**
2. Decisions in extensive (dynamic) form
3. Ordinal preference theory
4. Decisions under risk

Decisions in strategic form

Overview

1. **Introduction**
2. Sets, functions, and real numbers
3. Dominance and best responses
4. Mixed strategies and beliefs
5. Rationalizability

Introduction

Decision in strategic form:

- helps to ease into game theory;
- concerns one-time (once-and-for-all) decisions where payoffs depend on:
  - strategy;
  - state of the world.

Introduction

Example

		state of the world	
		bad weather	good weather
strategy	production of umbrellas	100	81
	production of sunshades	64	121

Definition

A decision situation in strategic form is a triple

$$\Delta = (S, W, u),$$

with

- S (decision maker's strategy set),
- W (set of states of the world), and
- $u : S \times W \to \mathbb{R}$ (payoff function).

$\Delta = (S, u : S \to \mathbb{R})$ is called a decision situation in strategic form without uncertainty.

In this example:

- $S = \{\text{umbrella, sunshade}\}$;
- $W = \{\text{bad weather, good weather}\}$;
- payoff function u given by

$$\begin{aligned}
u\left(\text{umbrella, bad weather}\right) &= 100, \\
u\left(\text{umbrella, good weather}\right) &= 81, \\
u\left(\text{sunshade, bad weather}\right) &= 64, \\
u\left(\text{sunshade, good weather}\right) &= 121.
\end{aligned}$$

Note that:

- decision maker can choose *only one* strategy from S;
- *only one* state of the world from W can actually happen.

Definition

Decision situation without uncertainty:

$$u(s, w_1) = u(s, w_2)$$

for all $s \in S$ and all $w_1, w_2 \in W$.
Trivial case: $|W| = 1$.

Assumptions:

- 2 boxes:
  - 1000 € in box 1;
  - 0 € or 1 mio. € in box 2.
- strategies:
  - open only box 2
  - open both boxes
- Higher Being put 1 mio. € in box 2 if she foresees that you are not greedy (that you open only box 2).
- Higher Being's predictions are often correct (She knows the books you read), but may be wrong.

Problem

What would you do?

	prediction: box 2, only	prediction: both boxes
you open box 2, only	1 000 000 Euros	0 Euro
you open both boxes	1 001 000 Euros	1 000 Euro

	prediction is correct	prediction is wrong
you open box 2, only	1 000 000 Euros	0 Euro
you open both boxes	1 000 Euro	1 001 000 Euros

We come back to Newcomb's problem later ...

Definition

$\Delta = (S, \pi)$, where:

- $S = [0, \infty)$ (set of output decisions),
- $\pi : S \to \mathbb{R}$ (payoff function) defined by $\pi(s) = p(s)s - C(s)$ with
  - $p : S \to [0, \infty)$ (inverse demand function);
  - $C : S \to [0, \infty)$ (cost function).

1. Introduction
2. **Sets, functions, and real numbers**
3. Dominance and best responses
4. Mixed strategies and beliefs
5. Rationalizability

Sets

Set, element, and subset

Definition (set and elements)

Set – any collection of "elements" that can be distinguished from each other. Set can be empty: $\varnothing$.

Definition (subset)

Let M be a nonempty set. A set N is called a subset of M ($N \subseteq M$) if every element from N is contained in M. $\{\}$ are used to indicate sets. $M_1 = M_2$ iff $M_1 \subseteq M_2$ and $M_2 \subseteq M_1$.

Problem

True?

- $\{1,2\} = \{2,1\} = \{1,2,2\}$
- $\{1,2,3\} \subseteq \{1,2\}$

Definition

Let M be a nonempty set. A tuple on M is an **ordered** list of elements from M. Elements can appear several times. A tuple consisting of n entries is called n-tuple. $()$ are used to denote tuples.
$(a_1, ..., a_n) = (b_1, ..., b_m)$ if $n = m$ and $a_i = b_i$ for all $i = 1, ..., n$.

Problem

True?

- $(1, 2, 3) = (2, 1, 3)$
- $(1, 2, 2) = (1, 2)$

Definition

Let M_1 and M_2 be nonempty sets. The Cartesian product of M_1 and M_2 ($M_1 \times M_2$) is defined by

$$M_1 \times M_2 := \{(m_1, m_2) : m_1 \in M_1, m_2 \in M_2\}.$$

Example

In a decision situation in strategic form, $S \times W$ is the set of tuples (s, w).

Problem

Let $M := \{1, 2, 3\}$ and $N := \{2, 3\}$. Find $M \times N$ and depict this set in a two-dimensional figure where M is associated with the abscissa (x-axis) and N with the ordinate (y-axis).

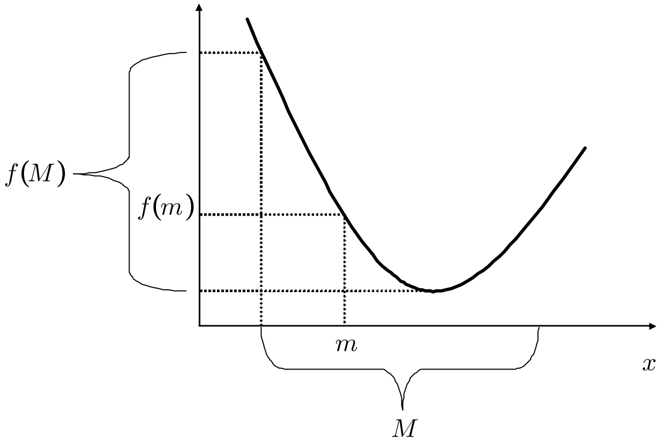
Definition

Let M and N be nonempty sets.

- A function $f : M \to N$ associates with every $m \in M$ an element from N, denoted by $f(m)$ and called the value of f at m.
- Sets
  - M – domain (of f)
  - N – range (of f)
  - $f(M) := \{f(m) : m \in M\} = \bigcup_{m \in M} \{f(m)\}$ – image (of f).
- Injective function – if $f(m) = f(m')$ implies $m = m'$ for all $m, m' \in M$.
- Surjective function – if $f(M) = N$ holds.
- Bijective function – both injective and surjective.

Functions

value versus image



$$f(M) := \{f(m) : m \in M\} = \bigcup_{m \in M} \{f(m)\}$$

Functions

Exercise

Problem

Let $M := \{1, 2, 3\}$ *and* $N := \{a, b, c\}$.
Define $f : M \to N$ *by*

- $f(1) = a$,
- $f(2) = a$ *and*
- $f(3) = c$.

Is f *surjective or injective?*

Functions

Two different sorts of arrows

1. $f : M \rightarrow N$ (domain is left; range is right).
2. $m \mapsto f(m)$ on the level of individual elements of M and N.

Example

A quadratic function may be written as

$$\begin{aligned} f \quad : \quad & \mathbb{R} \rightarrow \mathbb{R}, \\ & x \mapsto x^2. \end{aligned}$$

or in a shorter form: $f : x \mapsto x^2$
or very short (and in an incorrect manner): $f(x) = x^2$

Functions

Inverse function

Definition

Let $f : M \to N$ be an injective function. The function $f^{-1} : f(M) \to M$ defined by

$$f^{-1}(n) = m \Leftrightarrow f(m) = n$$

is called f's inverse function.

Problem

Let $M := \{1, 2\}$ *and* $N := \{6, 7, 8, 9\}$. *Define* $f : M \to N$ *by* $f(m) = 10 - 2m$. *Define f's inverse function.*

Cardinality

Definition

$f : M \rightarrow N$ a bijective function. Then, M and N are said to have the same cardinality (denoted by $|M| = |N|$).
If a bijective function $f : M \rightarrow \{1, 2, ..., n\}$ exists, M is finite and contains n elements. Otherwise M is infinite.

Problem

Let $M := \{1, 2, 3\}$ *and* $N := \{a, b, c\}$. *Show* $|M| = |N|$.

- Real numbers:

sets	symbol	elements
natural numbers	$\mathbb{N}$	$\{1, 2, 3, ...\}$
integers	$\mathbb{Z}$	$\{..., -2, -1, 0, 1, 2, ...\}$
rational numbers	$\mathbb{Q}$	$\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\right\}$
irrational real numbers	$\mathbb{R}\backslash\mathbb{Q}$	$\sqrt{2} = 1.414\,2...$, $e = 2.7183...$, etc.

- Note $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$.

Problem

$\frac{1}{8}$ and $\frac{4}{7}$ are rational numbers. Write these numbers as $0.1...$ and $0.5...$ and show that a repeating pattern emerges.

Theorem (cardinality)

The sets $\mathbb{N}, \mathbb{Z}$, *and* $\mathbb{Q}$ *are countably infinite, i.e., we have*

$$|\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}| .$$

However:

$$|\mathbb{Q}| < |\mathbb{R}|$$

and even

$$|\mathbb{Q}| < |\{x \in \mathbb{R} : a \leq x \leq b\}|$$

for any numbers a, b *with* $a < b$.

$$
\begin{aligned}
[a, b] &:= \{x \in \mathbb{R} : a \leq x \leq b\}, \\
[a, b) &:= \{x \in \mathbb{R} : a \leq x < b\}, \\
(a, b] &:= \{x \in \mathbb{R} : a < x \leq b\}, \\
(a, b) &:= \{x \in \mathbb{R} : a < x < b\}, \\
[a, \infty) &:= \{x \in \mathbb{R} : a \leq x\} \text{ and} \\
(-\infty, b] &:= \{x \in \mathbb{R} : x \leq b\}.
\end{aligned}
$$

Problem

Given the above definition for intervals, can you find an alternative expression for $\mathbb{R}$?

Convex combination

Definition

Let x and y be elements of $\mathbb{R}$. $\Rightarrow$

$$kx + (1-k)\,y, k \in [0,1]$$

– the convex combination (the linear combination) of x and y.

Problem

$0 \cdot x + (1-0)\,y = ?$

Problem

Where is $\frac{1}{4} \cdot 1 + \frac{3}{4} \cdot 2$, *closer to* 1 *or closer to* 2?

$$[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\} = \{ka + (1-k)\,b : k \in [0,1]\}$$

Weak and strict dominance

Definition

$\Delta = (S, W, u)$ – decision situation in strategic form.

- Strategy $s \in S$ (weakly) dominates strategy $s' \in S$ iff
  - $u(s, w) \geq u(s', w)$ holds for all $w \in W$ and
  - $u(s, w) > u(s', w)$ is true for at least one $w \in W$.
- Strategy $s \in S$ strictly dominates strategy $s' \in S$ iff $u(s, w) > u(s', w)$ holds for all $w \in W$.
- Dominant strategy – a strategy that dominates every other strategy (weakly or strictly).
- s' is called (weakly) dominated or strictly dominated.

Weak and strict dominance

Newcomb's problem, version 1

	prediction: box 2, only	prediction: both boxes
you open box 2, only	1 000 000 Euros	0 Euro
you open both boxes	1 001 000 Euros $\boxed{\text{R}}$	1 000 Euro $\boxed{\text{R}}$

Which strategy is best

- for the first state of the world,
- for the second state of the world?

Weak and strict dominance

Newcomb's problem, version 2

	prediction is correct	prediction is wrong
you open box 2, only	1 000 000 Euros $\boxed{R}$	0 Euro
you open both boxes	1 000 Euro	1 001 000 Euros $\boxed{R}$

Which strategy is best

- for the first state of the world,
- for the second state of the world?

Power set

Definition

Let M be any set. The set of all subsets of M is called the power set of M and is denoted by 2^M.

Examples

$M := \{1, 2, 3\}$ has the power set

$$2^M = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

Note that the empty set $\emptyset$ also belongs to the power set of M !

Problem

Can you derive a general rule to calculate the number of elements of any power set?

arg max

Example

Consider a firm that tries to maximize its profit Π by choosing the output x optimally. The output x is taken from a set X (e.g. the interval $[0, \infty)$) and the profit is a real (Euro) number.

$$\begin{aligned} \Pi(x) &\in \mathbb{R}: \text{ profit resulting from the output } x, \\ \max_x \Pi(x) &\in \mathbb{R}: \text{ maximal profit by choosing } x \text{ optimally}, \\ \underset{x}{\operatorname{argmax}}\, \Pi(x) &\subseteq X: \text{ set of outputs that lead to the maximal profit} \end{aligned}$$

Of course,

$$\max_x \Pi(x) = \Pi(x^*) \text{ for all } x^* \text{ from } \underset{x}{\operatorname{argmax}}\, \Pi(x).$$

arg max

The following cases are possible:

- $\mathrm{argmax}_x \; \Pi(x)$ contains several elements.
- $\mathrm{argmax}_x \Pi(x)$ contains just one element.
- $\mathrm{argmax}_x \; \Pi(x) = \emptyset$.

Best response

Definition

$\Delta = (S, W, u)$.
The function $s^R : W \to 2^S$ – a best-response function (a best response, a best answer) if s^R is given by

$$s^R(w) := \arg\max_{s \in S} u(s, w).$$

Problem

Use best-response functions to characterize s as a dominant strategy.
Hint: „characterization" means that you are to find a statement that is equivalent to the definition.

Probability distribution

Definition

Probability distribution on M : $\quad prob : 2^M \to [0,1]$ with

- $prob\,(\emptyset) = 0$,
- $prob\,(A \cup B) = prob\,(A) + prob\,(B)$ for all $A, B \in 2^M$ obeying $A \cap B = \emptyset$ and
- $prob\,(M) = 1$ (summing condition).

Subsets of M are also called **events**.

Problem

Probability for the events

- A : *"the number of pips (spots) is 2",*
- B : *"the number of pips is odd", and*
- $A \cup B$

Mixed strategy

Definition

Let S be a finite strategy set. A mixed strategy σ is a probability distribution on S :

$$\sigma(s) \geq 0 \text{ for all } s \in S \text{ and } \sum_{s \in S} \sigma(s) = 1 \text{ (summing condition)}$$

- Σ (set of mixed strategies) —> Σ also stands for "sum", as above
- $\sigma(s) = 1$ for one $s \in S$ —> identify s with σ
- $\sigma(s) > 0$ and $\sigma(s') > 0$ for $s \neq s'$ —> σ properly mixed strategy
- Notation: $\left(\sigma(s_1), \sigma(s_2), ..., \sigma\left(s_{|S|}\right)\right)$.

Mixed strategy

Decision situation in strategic form with mixed strategies

Definition

$\Delta = (S, W, u)$

Example

Choosing umbrella with probability $\frac{1}{3}$ and sunshade with probability $\frac{2}{3}$ is a mixed strategy.

Belief

Definition

Let W be a set of states of the world.

- Ω (set of probability distributions on W);
- $\omega \in \Omega$ (belief);
- Notation: $\left(\omega\left(w_1\right), ..., \omega\left(w_{|W|}\right)\right)$.

Example

Bad weather with probability $\frac{1}{4}$ and good weather with probability $\frac{3}{4}$

Extending the payoff function

So far,

$$u : S \times W \to \mathbb{R}$$

Two extensions:

- probability distribution ω on W
  rather than a specific state of the world $w \in W$
  —$>$ lottery
- mixed strategy σ on S
  rather than a specific strategy $s \in S$

Extending the payoff function

beliefs and lotteries

		state of the world	
		bad weather, $\frac{1}{4}$	good weather, $\frac{3}{4}$
strategy	production of umbrellas	100	81
	production of sunshades	64	121

For example,

$$L_{\text{umbrella}} = \left[100, 81; \frac{1}{4}, \frac{3}{4}\right]$$

Lottery and expected value

Definition

A tuple

$$L = [x; p] := [x_1, ..., x_\ell; p_1, ..., p_\ell]$$

is called a lottery where $x_j \in \mathbb{R}$ is the payoff accruing with probability p_j

- $\mathcal{L}$ (set of simple lotteries).
- $\ell = 1$
    - L is called a trivial lottery
    - identify $L = [x; 1]$ with x.

Definition

Assume a simple $L = [x_1, ..., x_\ell; p_1, ..., p_\ell]$. Its expected value is denoted by $E(L)$ and given by

$$E(L) = \sum_{j=1}^{\ell} p_j x_j.$$

Extending the payoff function

mixed strategies

Definition

The payoff under σ and w is defined by

$$u(\sigma, w) := \sum_{s \in S} \sigma(s) \, u(s, w)$$

Problem

Calculate the expected payoff in the umbrella-sunshade decision situation if the firm chooses umbrella with probability $\frac{1}{3}$ and sunshade with $\frac{2}{3}$. Differentiate between $w =$"bad weather" and $w =$"good weather".

- Thus, the payoff for a mixed strategy is the mean of the payoffs for the pure strategies.
- How are best pure and best mixed strategies related?

Best pure and best mixed strategies

Lemma

Best pure and best mixed strategies are related by the following two claims:

- *Any mixed strategy that puts positive probabilities on best pure strategies, only, is a best strategy.*
- *If a mixed strategy is a best strategy, every pure strategy with positive probability is a best strategy.*

Mixing strategies and states of the world

Exercise

Problem

Consider again the umbrella-sunshade decision situation in which

- *the firm chooses umbrella with* $\frac{1}{3}$ *and sunshade with probability* $\frac{2}{3}$ *and*
- *the weather is bad with probability* $\frac{1}{4}$ *and good with probability* $\frac{3}{4}$.

Calculate $u\left(\left(\frac{1}{3}, \frac{2}{3}\right), \left(\frac{1}{4}, \frac{3}{4}\right)\right)$!

Extending the payoff function

summary

The payoff function $u : S \times W \to \mathbb{R}$ has been extended:

- beliefs:

$$\begin{aligned} u &: S \times \Omega \to \mathbb{R} \\ & (s, \omega) \mapsto u(s, \omega) = \sum_{w \in W} \omega(w)\, u(s, w) = E(L_s) \end{aligned}$$

- mixed strategies

$$\begin{aligned} u &: \Sigma \times W \to \mathbb{R} \\ & (\sigma, w) \mapsto u(\sigma, w) = \sum_{s \in S} \sigma(s)\, u(s, w) \end{aligned}$$

- both beliefs and mixed strategies

$$\begin{aligned} u &: \Sigma \times \Omega \to \mathbb{R} \\ & (\sigma, \omega) \mapsto u(\sigma, \omega) = \sum_{s \in S} \sum_{w \in W} \sigma(s)\, \omega(w)\, u(s, w) \end{aligned}$$

Best-response functions

Definition

Given $\Delta = (S, W, u)$, there are four best-response functions:

$$\begin{aligned}
s^{R,W} &: \quad W \to 2^S, \text{ given by } s^{R,W}(w) := \arg\max_{s \in S} u(s, w), \\
\sigma^{R,W} &: \quad W \to 2^\Sigma, \text{ given by } \sigma^{R,W}(w) := \arg\max_{\sigma \in \Sigma} u(\sigma, w), \\
s^{R,\Omega} &: \quad \Omega \to 2^S, \text{ given by } s^{R,\Omega}(\omega) := \arg\max_{s \in S} u(s, \omega), \text{ and} \\
\sigma^{R,\Omega} &: \quad \Omega \to 2^\Sigma, \text{ given by } \sigma^{R,\Omega}(\omega) := \arg\max_{\sigma \in \Sigma} u(\sigma, \omega)
\end{aligned}$$

s^R or σ^R instead of $s^{R,W}$, etc. (if there is no danger of confusion)

Problem

Complete the sentence: $\sigma \in \sigma^{R,W}(w)$ *implies* $\sigma(s) = 0$ *for all ... .*

Theorem

$\Delta = (S, W, u)$. *We have*

- $\sigma \in \Sigma$ *and* $\sum_{s \in s^{R,\Omega}(\omega)} \sigma(s) = 1$ *imply* $\sigma \in \sigma^{R,\Omega}(\omega)$ *and*
- $\sigma \in \sigma^{R,\Omega}(\omega)$ *implies* $s \in s^{R,\Omega}(\omega)$ *for all* $s \in S$ *with* $\sigma(s) > 0$.

These implications continue to hold for W *and* w *rather than* Ω *and* ω.

Best-response functions

Example

	w_1	w_2
s_1	4	1
s_2	1	2

Let $\omega := \omega(w_1)$ be the probability of w_1. We have $s_1 \in s^{R,\Omega}(\omega)$ in the case of

$$\omega \cdot 4 + (1-\omega) \cdot 1 \geq \omega \cdot 1 + (1-\omega) \cdot 2,$$

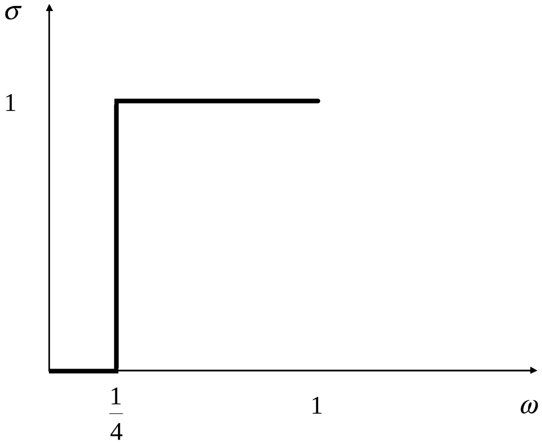
i.e., if $\omega \geq \frac{1}{4}$ holds.

Best-response functions

- Best-response function is given by $\sigma^{R,\Omega} : \Omega \to 2^{\Sigma}$.
- For $\omega \neq \frac{1}{4}$, there is exactly one best strategy,
  - $\sigma = 0$ ($\sigma = (0,1) = s_2$) or
  - $\sigma = 1$ ($\sigma = (1,0) = s_1$).
- $\omega = \frac{1}{4}$ implies that every pure strategy (—> every mixed strategy) is best.

$$\sigma^{R,\Omega}(\omega) = \begin{cases} 1, & \omega > \frac{1}{4} \\ [0,1], & \omega = \frac{1}{4} \\ 0, & \omega < \frac{1}{4} \end{cases}$$

Best-response functions



Best-response functions

Exercise

Problem

Sketch the best-response function $\sigma^{R,\Omega}$ *for*

	w_1	w_2
s_1	1	3
s_2	2	1

Decisions in strategic form:
Rationalizability

Rationalizable strategies

Example

	w_1	w_2
s_1	4	4
s_2	1	5
s_3	5	1

- Can a rational decision maker choose s_1 although it is not a best response to w_1 and not a best response to w_2?
- Yes: the rational decision maker may entertain the belief ω on W with $\omega(w_1) = \omega(w_2) = \frac{1}{2}$.
- Given this belief, s_1 is a perfectly reasonable strategy.

Problem

Show $s_1 \in s^{R,\Omega}\left(\left(\frac{1}{2}, \frac{1}{2}\right)\right)$*!*

Rationalizable strategies

Definition

$\Delta = (S, W, u)$.

- A mixed strategy $\sigma \in \Sigma$ is called rationalizable with respect to W if a $w \in W$ exists such that $\sigma \in \sigma^{R,W}(w)$.
- Strategy $\sigma \in \Sigma$ is called rationalizable with respect to Ω if a belief $\omega \in \Omega$ exists such that $\sigma \in \sigma^{R,\Omega}(\omega)$.

Nobel price 2014

In 2014, the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel was awarded to the French economist
Jean Tirole (Toulouse)

for his analysis of market power and regulation.

Further exercises: Problem 1

(a) If strategy $s \in S$ strictly dominates strategy $s' \in S$ and strategy s' strictly dominates strategy $s'' \in S$, is it always true that strategy s strictly dominates strategy s''?

(b) If strategy $s \in S$ weakly dominates strategy $s' \in S$ and strategy s' weakly dominates strategy $s'' \in S$, is it always true that strategy s weakly dominates strategy s''?

Further exercises: Problem 2 (without figure)

Consider the problem of a monopolist faced with the inverse demand function $p(q) = a - b \cdot q$, in which a can either be high, a^h, or low, a^l. The monopolist produces with constant marginal and average cost c. Assume that $a^h > a^l > c$ and $b > 0$. Think of the monopolist as setting the quantity, q, and not the price, p.

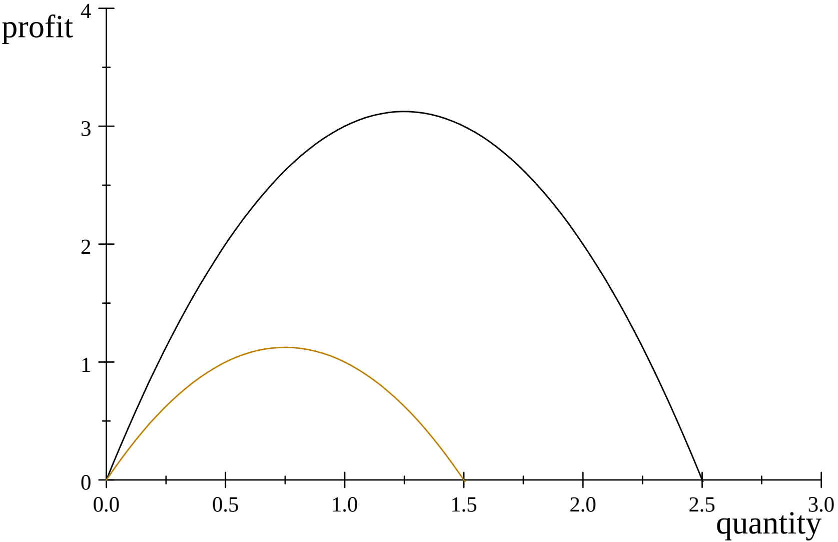
(a) Formulate this monopolist's problem as a decision problem in strategic form. Determine $s^{R,W}$!

(b) Assume $a^h = 6, a^l = 4, b = 2, c = 1$ so that you obtain the plot given in the figure. Show that any strategy $q \notin \left[\frac{a^l - c}{2 \cdot b}, \frac{a^h - c}{2 \cdot b}\right]$ is dominated by either $s^{R,W}\left(a^h\right)$ or $s^{R,W}\left(a^l\right)$. Show also that no strategy $q \in \left[\frac{a^l - c}{2 \cdot b}, \frac{a^h - c}{2 \cdot b}\right]$ dominates any other strategy $q' \in \left[\frac{a^l - c}{2 \cdot b}, \frac{a^h - c}{2 \cdot b}\right]$.

(c) Determine all rationalizable strategies with respect to W.

(d) Difficult: Determine all rationalizable strategies with respect to Ω. Hint: Show that the optimal output is a convex combination of $s^{R,W}\left(a^h\right)$ and $s^{R,W}\left(a^l\right)$.

Further exercises: Problem 2 (figure)



Further exercises: Problem 3

Prove the following assertions or give a counter-example!

(a) If $\sigma \in \Sigma$ is rationalizable with respect to W, then σ is rationalizable with respect to Ω.

(b) If $s \in S$ is a weakly dominant strategy, then it is rationalizable with respect to W.

(c) If $s \in S$ is rationalizable with respect to W, then s is a weakly dominant strategy.