# Advanced Microeconomics

# Final Winter 2014/2015

27th February 2015

You have to accomplish this test within **60 minutes.**

**PRÜFUNGS-NR.:**

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

**ANFORDERUNGEN/REQUIREMENTS:**

**Lösen Sie die folgenden Aufgaben!/Solve all the exercises!**
**Schreiben Sie, bitte, leserlich!/Write legibly, please!**
**Sie können auf Deutsch schreiben!/You can write in English!**
**Begründen Sie Ihre Antworten!/Give reasons for your answers!**

| 1 | 2 | 3 | 4 | 5 | $\sum$ |
|---|---|---|---|---|---|
| | | | | | |

**Problem 1 (12 points)**
Consider the following two person game with mixed strategies. Calculate both reaction functions and illustrate them graphically. Determine all equilibria in pure and properly mixed strategies.

player 2

| player 1 | | $l$ | $r$ |
|---|---|---|---|
| | $o$ | $(8,3)$ | $(2,5)$ |
| | $u$ | $(0,2)$ | $(4,1)$ |

**Solution**
First of all we determine the payoff functions:

$$\begin{aligned} u_1(\sigma_1,\sigma_2) &= 8\sigma_1\sigma_2 + 2\sigma_1(1-\sigma_2) + 4(1-\sigma_1)(1-\sigma_2), \\ u_2(\sigma_1,\sigma_2) &= 3\sigma_1\sigma_2 + 5\sigma_1(1-\sigma_2) + 2(1-\sigma_1)\sigma_2 + (1-\sigma_1)(1-\sigma_2). \end{aligned}$$

We compute the reaction function of agent 1 by examining the derivative of the payoff function:

$$\begin{aligned} \frac{\partial u_1}{\partial \sigma_1} &= 8\sigma_2 + 2(1-\sigma_2) - 4(1-\sigma_2) \\ &= 10\sigma_2 - 2 \overset{!}{=} 0 \end{aligned}$$

and get

$$\sigma_1^R(\sigma_2) = \begin{cases} 0, & \sigma_2 < \frac{1}{5} \\ [0,1], & \sigma_2 = \frac{1}{5} \\ 1, & \sigma_2 > \frac{1}{5}. \end{cases}$$

Now, consider the payoff function of agent 2. We have

$$\begin{aligned} \frac{\partial u_2}{\partial \sigma_2} &= 3\sigma_1 - 5\sigma_1 + 2(1-\sigma_1) - (1-\sigma_1) \\ &= -3\sigma_1 + 1 \overset{!}{=} 0 \end{aligned}$$

which yields

$$\sigma_2^R(\sigma_1) = \begin{cases} 0, & \sigma_1 > \frac{1}{3} \\ [0,1], & \sigma_1 = \frac{1}{3} \\ 1, & \sigma_1 < \frac{1}{3}. \end{cases}$$

Hence, there is only one equilibrium in mixed strategies given by $\sigma_1 = \frac{1}{3}$ and $\sigma_2 = \frac{1}{5}$.

**Problem 2 (12 points)**
Consider the infinite repetition (discount factor $\delta$) of this stage game:

| | | player 2 | |
|---|---|---|---|
| | | $l$ | $r$ |
| player 1 | $o$ | $(3,3)$ | $(0,4)$ |
| | $u$ | $(4,0)$ | $(0,0)$ |

.

The payoff of player $i$ is given by

$$u_i(s) = (1-\delta)\sum_{t=0}^{\infty}\delta^t g_i(s(d_t)),$$

where $\delta \in (0,1)$, $g_i(s)$ denotes the stage-payoff of player $i$, and $d_t$ is the node resulting from action combinations $s(d_0)$ through $s(d_{t-1})$.

a) Determine the Worst-Punishment point!

b) Let $\delta$ be sufficiently large. According to the Folk-Theorems, can $(3,3)$ be an equilibrium payoff in the infinitely repeated game?

c) Consider the strategy of player 1, where first, she plays $o$, but chooses $u$ for all remaining stages, whenever player 2 chose $r$ for at least one stage before. For player 2, is it better to always play $l$ or always play $r$?

*Hint:* $\sum_{t=0}^{k}\delta^t = \frac{1-\delta^{k+1}}{1-\delta}$ *and* $\sum_{t=0}^{\infty}\delta^t = \frac{1}{1-\delta}$ *if* $|\delta| < 1$.

**Solution**

a) The Worst-Punishment point is given by

$$\begin{aligned} W &= \left(\min_{a_2}\max_{a_1} g_1(a_1,a_2), \min_{a_1}\max_{a_2} g_2(a_1,a_2)\right) \\ &= (\min\{0,4\}, \min\{0,4\}) \\ &= (0,0). \end{aligned}$$

b) According to the Folk-Theorem, if $\delta$ is sufficiently large, any payoff within the convex hull of the game which lies north-east of the Worst-Punishment point can be an equilibrium payoff. $(3,3)$ does lie in the convex hull of the game and obviously lies north-east of the Wort-Punishment point. Hence, $(3,3)$ can be an equilibrium payoff.

c) If player 2 always chooses $l$ his payoff is

$$(1-\delta)\cdot\left(\sum_{t=0}^{\infty}\delta^t\cdot 3\right) = (1-\delta)\frac{3}{1-\delta} = 3$$

If player 2 always chooses $r$ his payoff is

$$(1-\delta)\,4.$$

The first payoff is at least as high as the second payoff iff

$$\begin{aligned} 3 &\geq (1-\delta)\,4 \iff \\ 4\delta &\geq 1 \iff \\ \delta &\geq \frac{1}{4}. \end{aligned}$$

If the discount factor is small $\left(\delta < \frac{1}{4}\right)$, punishment does not hurt too much, hence, player 2 does not want to cooperate and always plays $r$.

**Problem 3 (20 points)**

Consider an exchange economy with two agents $A$ and $B$. Their preferences are given by the utility functions

$$u_A\left(x_1^A, x_2^A\right) = \min\left\{x_1^A, 2x_2^A\right\}, \quad u_B\left(x_1^B, x_2^B\right) = x_1^B + x_2^B$$

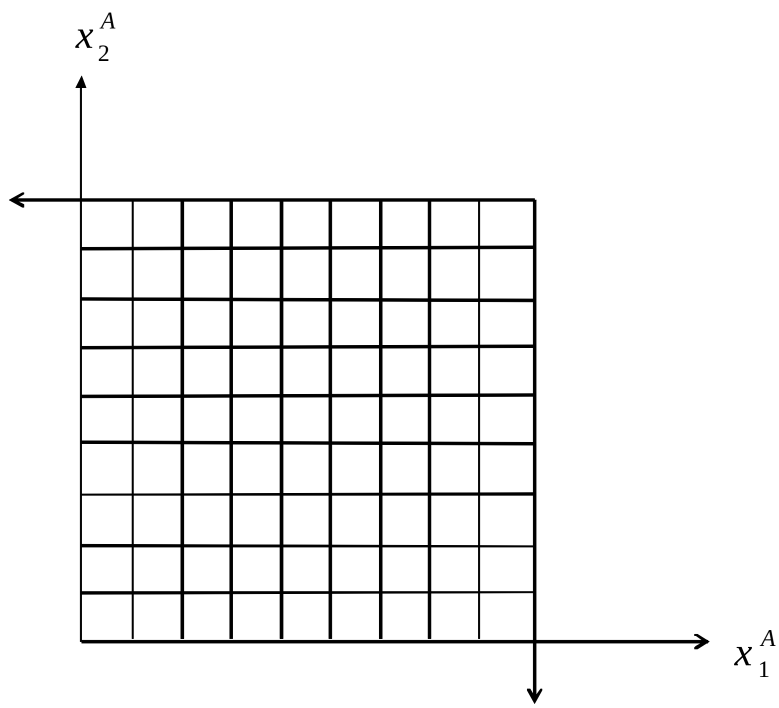
and their endowments are equal to

$$\omega^A = (2,4), \quad \omega^B = (7,5).$$

(a) Use the graphic below to sketch endowments, the indifference curves of both agents, that run through the endowments, the better set of agent A and the exchange lens.

(b) Determine the location of all Pareto-efficient points analytically, and illustrate them in the graphic below.

(c) Determine the Walrasian equilibrium analytically.



**Solution**

(b) Since agents 1's preferences describes perfect complements and agent $2's$ preferences are monotonic, player 1 does not waste resources. Hence, on the contract curve, we have

$$x_2^A\left(x_1^A\right) = \frac{1}{2}x_1^A$$

(c) For player 2 and given $p_1$ and $p_2$, the household optima are either his budget line or corner solutions. As the contract curve lies within the Edgeworth Box, we must have

$$\frac{p_1}{p_2} \overset{!}{=} MRS^B.$$

We then find

$$\frac{p_1}{p_2} \overset{!}{=} 1$$

and obtain $p_1 = p_2$. We use the budget equation

$$p_1 x_1^A + p_2 x_2^A = 2p_1 + 4p_2$$

and get, by the contract curve,

$$\frac{3}{2}p_1 x_1^A = 6p_1$$

and thus

$$x_1^A = 4,\ x_2^A = 2.$$

The Walrasian equilibrium is given by the price vector $(p_1, p_1)$ with $p_1 > 0$ and the corresponding demand system $((4,2),(5,7))$.

**Problem 4 (14 points)**
Consider a market where two firms compete simultaneously in outputs. Both firms face a quadratic cost function $C_i(x_i) = x_i^2$. While firm 1 maximizes its profit, firm 2 maximizes its revenue. The inverse demand function is given by

$$p(X) = 14 - X.$$

Determine the Cournot equilibrium!
**Solution:**
First, we calculate the reaction function of firm 1. Firm one maximizes its profit

$$\Pi_1(x_1, x_2) = (14 - x_1 - x_2)\, x_1 - x_1^2.$$

We form the derivative and get

$$\frac{\partial \Pi_1}{\partial x_1} = 14 - 4x_1 - x_2 \overset{!}{=} 0$$

$$\Longleftrightarrow \; x_1^R(x_2) = \frac{1}{4}(14 - x_2)$$

Firm 2 maximizes its revenue

$$R_2(x_1, x_2) = (14 - x_1 - x_2)\, x_2.$$

We calculate the reaction function:

$$\frac{\partial R_2}{\partial x_2} = 14 - x_1 - 2x_2 \overset{!}{=} 0$$

$$\Longleftrightarrow \; x_2^R(x_1) = \frac{1}{2}(14 - x_1).$$

The equilibrium is the intersection of the reaction functions:

$$\begin{aligned} x_1 &= \frac{1}{4}\left(14 - 7 + \frac{1}{2}x_1\right) \\ &= \frac{7}{4} + \frac{1}{8}x_1 \end{aligned}$$

$$\Longleftrightarrow \; \frac{7}{8}x_1 = \frac{7}{4}$$

$$\Longleftrightarrow \; x_1^C = 2, x_2^C = 6$$

**Problem 5 (2 points)**
Calculate the Herfindahl index for five firms supplying the quantities $x_1 = x_2 = 2, x_3 = x_4 = 4$, and $x_5 = 8$.
**Solution:**

$$\begin{aligned} X &= 20 \\ H &= \frac{4+4+16+16+64}{400} = \frac{104}{400} \\ &= \frac{26}{100} = 0.26 \end{aligned}$$