Advanced Microeconomics

Final Winter 2012/2013

22nd February 2013

You have to accomplish this test within **60 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	7	$\sum$

Problem 1 (8 points)
Consider the game (N, v) given by $N = \{1,2,3,4,5\}$ and $v : 2^N \to \mathbb{R}$, defined by

$$v(K) = \begin{cases} 1, & \{1,2,4\} \subseteq K \\ 0, & \text{otherwise} \end{cases} .$$

Calculate the Shapley payoffs for all players!
Hint: Use the properties of the Shapley payoffs!

Problem 2 (10 points)
There are two agents A and B who produce and trade apples (a) and bananas (b). Their endowments are $\omega^A = \left(\omega_a^A, \omega_b^A\right) = (20, 0)$ and $\omega^B = \left(\omega_a^B, \omega_b^B\right) = (10, 50)$. The utility function of agent A is a strictly increasing function of apples, the utility function of agent B is a strictly increasing function of bananas.

Assume that aggregate excess demand for apples is given by

$$z_a = \frac{p_a^2 - 4p_a p_b}{p_b^2},$$

where p_a describes the price for apples and p_b the price for bananas.

(a) Does $p \cdot z(p) = 0$ hold?

(b) Determine the aggregate excess demand function for bananas!

(c) Determine the price ratio such that the apple market clears! Determine the price ratio for the clearance of the banana market in two different ways!

Hint: Use the following theorems!

Walras' Law: Every consumer demands a bundle of goods obeying $p \cdot x^i \leq p \cdot \omega^i$ where local non-satiation implies equality. For all consumers together, we have $p \cdot z(p) \leq 0$ and, assuming local non-satiation, $p \cdot z(p) = 0$.

Market Clearance Lemma: In case of local nonsatiation, if all markets but one are cleared, the last one also clears or its price is zero.

Free goods Lemma: Assume local non-satiation and weak monotonicity for all households. If $\left[\widehat{p}, \left(\widehat{x}^i\right)_{i\in 1,..,n}\right]$ is a Walras equilibrium and the excess demand for a good is negative, this good must be free.

Problem 3 (8 points)
On the labour market, an agent may be of low or high type. The personnel manager (Mrs Smith) can offer contracts (w, e), where w stands for the wage and e for the required effort. She cannot observe the quality, but she knows the utility function of the low-quality type

$$u_{low}(w, e) = 2w - e$$

and of the high-quality type

$$u_{high}(w, e) = w - 2e.$$

The agent's utility is 0 if he rejects both contracts.

The personnel manager offers menues of two contracts. Describe the agent's decision in each case!

(a) $(w_1, e_1) = (5, 4)$ and $(w_2, e_2) = (3, 1)$

(b) $(w_1, e_1) = (3, 2)$ and $(w_2, e_2) = (4, 3)$

(c) $(w_1, e_1) = (6, 1)$ and $(w_2, e_2) = (4, 2)$

Problem 4 (15 points)
Two firms 1 and 2 compete in quantities. Firm 1 is the leader and firm 2 the follower. Firm 2 can observe the quantity offered by firm 1. $p(X) = 12 - 2X$ is the inverse demand function, where $X = x_1 + x_2$. $C(x_1) = \frac{1}{4}x_1^2$ is firm 1's cost function and $C(x_2) = x_2^2$ is firm 2's cost function. Assume that firm 1's agent set is $\{0, 4, 6\}$, while firm 2's action set is $\mathbb{R}_+$.

a) How many subgames do we have? Write down one strategy of firm 2.

b) Calculate the subgame perfect equilibria!

c) Find firm 1's limit quantity x_1^L! Is deterrence beneficial for firm 1?

Problem 5 (8 points)

Two firms, A and B, produce the same good with different technologies inducing high or low cost per unit, respectively. Both have the following ex ante (common) knowledge on the probability distribution of their unit cost

		Firm A: c_A^L	Firm A: c_A^H
Firm B	c_B^L	$\frac{1}{4}$	$\frac{1}{2}$
	c_B^H	$\frac{1}{8}$	$\frac{1}{8}$

(a) Determine firm A's belief for c_B^L if firm A has high costs!

(b) Assume that the firms are price setters. Define strategies for firm A! Define the Bayesian equilibrium in this setting!

Problem 6 (9 points)
The following game is repeated twice. The payoff is the sum of the payoffs of each stage:

		Player 2: C	Player 2: D
Player 1	A	$3, 3$	$0, 4$
	B	$4, 0$	$1, 1$

(a) How many strategies does player 1 have? Write down one of them!

(b) Determine all subgame perfect equilibria!

Problem 7 (2 points)
Calculate the Herfindahl index for three firms supplying the quantities $q_1 = 100$, $q_2 = 200$ and $q_3 = 300$!