Advanced Microeconomics

Resit Winter 2009/2010

31st March 2010

You have to accomplish this test within **120 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

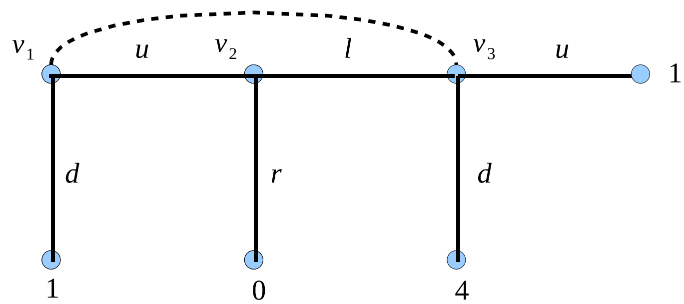
UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

1	2	3	4	5	6	7	8	9	10	$\sum$

Problem 1 (20 points)
Consider the following decision problem:



(a) Provide a list of all pure strategies the decision maker has!

(b) Determine the information partition of the decision maker!

(c) Determine the experience of the decision maker at the nodes v_1 and v_3! Is this a decision situation with perfect recall?

(d) Are there any proper subtrees?

(e) Determine all optimal mixed strategies! What payoff does the decision maker obtain using an optimal mixed strategy?

(f) Determine all optimal behavioral strategies! What payoff does the decision maker obtain using an optimal behavioral strategy? Comment briefly on the relationship between your answers to (c), (e), and (f)!

Problem 2 (10 points)
Let R be a relation on a set X.

(a) Define formally that R is transitive!

(b) Consider the following property called 'asymmetry' (*Attention: asymmetry is not identical to antisymmetry)*: For all $x, y \in X$: If xRy, then not yRx. Is a weak preference relation asymmetric? What about a strict preference relation?

(c) Show that any transitive and asymmetric relation R satisfies the following property: There is no triple $x, y, z \in X$ such that xRy and yRz and zRx!

Problem 3 (15 points)
Two firms compete on a market with inverse demand function

$$p(Q) = \begin{cases} a - b \cdot Q, & Q \in \left[0, \frac{a}{b}\right], \\ 0, & Q > \frac{a}{b}, \end{cases}$$

where $a, b > 0$. The firms have identical marginal and unit costs c satisfying $0 < c < a$. The firms simultaneously choose their respective quantities, q_1 and q_2. Of course, $Q = q_1 + q_2$.

Firm 1 is a regular firm trying to maximize its profits. Firm 2 is run by a government agency; the politicians are obsessed with concentration. Consequently, firm 2 is ordered to minimize the Herfindahl index which is given by $H = \left(\frac{q_1}{q_1+q_2}\right)^2 + \left(\frac{q_2}{q_1+q_2}\right)^2$ for this particular market.

(a) Derive the best response function of firm 1!

(b) Derive the best response function of firm 2 assuming $q_1 > 0$!

(c) Determine the Nash equilibrium in this game assuming $q_1 > 0$!

Problem 4 (15 points)
John's Von Neumann and Morgenstern utility function satisfies $u(0) = 0$ and $u(1) = 1$. In addition, John is risk seeking.

(a) Show that the inequality $u(x) \leq x$ holds for all $x \in [0, 1]$. *Hint: Consider the lottery* $L_x = [0, 1; 1 - x, x]$ *and apply the fact that John is risk seeking!*

(b) John has to choose between the following two lotteries:

$$\begin{aligned} L_A &= \left[0, 0.1, 0.9, 1; \frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}\right] \text{ and} \\ L_B &= \left[0, 0.1, 0.9, 1; \frac{5}{10}, \frac{2}{10}, \frac{1}{10}, \frac{2}{10}\right]. \end{aligned}$$

Can we infer something about his preferences over these two lotteries? *Hint: Compare the expected utility of the two lotteries, simplify until you can use the inequality* $u(x) \leq x$*!*

Problem 5 (10 points)
Mummy gives Peter a knife to part the cake into two. Then Sandra chooses one of them. The cake's size is 1. The share that goes to Sandra is denoted by s and Peter's share is denoted by p. Peter's utility function is

$$u(p, s) = p,$$

Sandra's is

$$u(p, s) = 2 \cdot s.$$

Determine the players' strategies. Find all subgame perfect equilibria.

Problem 6 (20 points)
Consider an exchange economy where actor A has the utility function

$$u^A\left(x_1^A, x_2^A\right) = x_1^A \cdot x_2^A$$

and actor B has the utility function

$$u^B\left(x_1^B, x_2^B\right) = \left(x_1^B\right)^{\frac{1}{4}} \cdot \left(x_2^B\right)^{\frac{3}{4}}.$$

Endowments are given by $\omega^A = (4, 8)$ and $\omega^B = (8, 4)$.

(a) Draw the Edgeworth box depicting this situation! Your drawing should contain the following objects:

- endowments of both actors,
- the indifference curve of each actor which contains his endowment,
- the set of all bundles $\left(x_1^A, x_2^A\right)$ satisfying $u^A\left(x_1^A, x_2^A\right) \geq u^A\left(\omega_1^A, \omega_2^A\right)$!

(b) Determine the contract curve and insert two of its points into the drawing.

(c) Determine the set of all Walras equilibria!

Problem 7 (5 points)
Sketch a few isoquants that reflect decreasing returns to scale.

Problem 8 (5 points)
What is the difference between the Hicksian and Marshallian demand?

Problem 9 (10 points)
Explain the differences between the adverse-selection model and the moral-hazard model. Sketch a 'real-world problem' where aspects of both models are difficult to tell apart by the principal.

Problem 10 (10 points)

Players 1 and 2 each choose a number from the set $\{1, ..., K\}$. If the players choose the same number then player 2 has to pay 1 Euro to player 1; otherwise no payment is made. Each player maximizes his expected monetary payoff. Show: The unique (mixed strategy) Nash equilibrium is

$$\left(\left(\frac{1}{K}, ..., \frac{1}{K}\right), \left(\frac{1}{K}, ..., \frac{1}{K}\right)\right).$$