Advanced Microeconomics

Pretest Winter 2009/2010

You are assumed to accomplish this test within **120 minutes.**

PRÜFUNGS-NR.:

STUDIENGANG:

NAME, VORNAME:

UNTERSCHRIFT DES STUDENTEN:

ANFORDERUNGEN/REQUIREMENTS:

Lösen Sie die folgenden Aufgaben!/Solve all the exercises!
Schreiben Sie, bitte, leserlich!/Write legibly, please!
Sie können auf Deutsch schreiben!/You can write in English!
Begründen Sie Ihre Antworten!/Give reasons for your answers!

Problem 1 (10 points)
Consider the relation 'nearly as fast as' denoted by $\sim$, and defined on some set of sprinters:
Sprinter A is 'nearly as fast as' sprinter B if the time they need to accomplish a $100m$ track differs by at most 0.01 seconds, i.e., $A \sim B$ if $\|t_A - t_B\| \leq 0.01$ (t_i denotes sprinter i's record time to run $100m$).
Define a specific set of sprinters with their record times (not below 9 seconds!) such that this relation is neither complete nor transitive! How about symmetry?

Problem 2 (10 points)
Recall the second welfare theorem:

Theorem. *Assume an exchange economy $\mathcal{E}$ with convex and continuous preferences for all consumers and local non-satiation for at least one household. Let $\left(\widehat{x}^i\right)_{i \in N}$ be any Pareto-efficient allocation. Then, there exists a price vector $\widehat{p}$ and an endowment $\left(\omega^i\right)_{i \in N}$ such that $\left(\widehat{x}^i\right)_{i \in N}$ is a Walras allocation for $\widehat{p}$.*

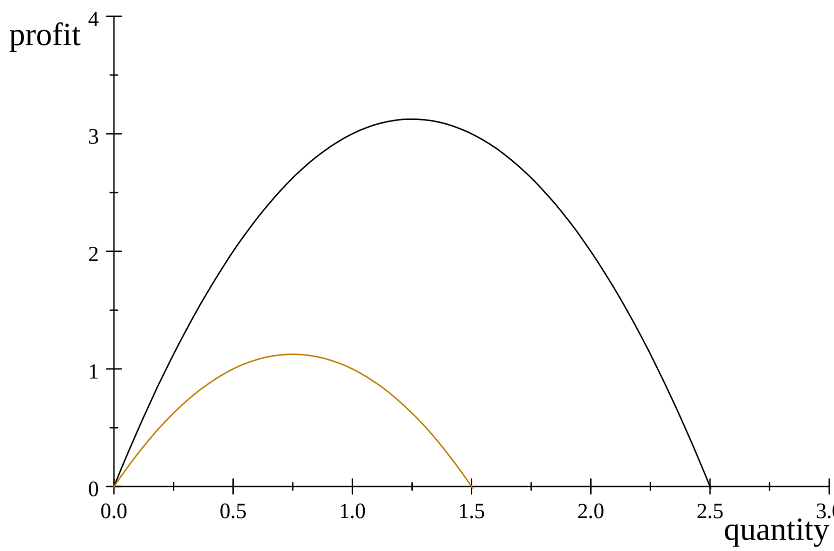
Graphically sketch a situation where preferences are not convex and hence no combination of $\left(\omega^i\right)_{i \in N}$ and $\widehat{p}$ supports the Pareto-efficient allocation $\left(\widehat{x}^i\right)_{i \in N}$.

Problem 3 (15 points)
Consider the problem of a monopolist faced with the inverse demand function $p(q) = a - b \cdot q$, in which a can either be high, a^h, or low, a^l. The monopolist produces with constant marginal and average cost c. Assume $a^h > a^l > c$ and $b > 0$. Think of the monopolist as setting the quantity, q, and not the price, p.

(a) Formulate this monopolist's problem as a decision problem in strategic form. Determine $s^{R,W}$!

(b) Assume $a^h = 6, a^l = 4, b = 2, c = 1$ so that you obtain the plot given in the figure. Show that any strategy $q \notin \left[\frac{a^l - c}{2 \cdot b}, \frac{a^h - c}{2 \cdot b}\right]$ is dominated by either $s^{R,W}\left(a^h\right)$ or $s^{R,W}\left(a^l\right)$. Show also that no strategy $q \in \left[\frac{a^l - c}{2 \cdot b}, \frac{a^h - c}{2 \cdot b}\right]$ dominates any other strategy $q' \in \left[\frac{a^l - c}{2 \cdot b}, \frac{a^h - c}{2 \cdot b}\right]$.

(c) Determine all rationalizable strategies with respect to W.



Problem 4 (10 points)
Show: A function f that is concave is also quasi-concave.
Remember the

Definition. *$f : R^\ell \to R$ is called quasi-concave if*

$$f(kx + (1 - k) y) \geq \min(f(x), f(y))$$

holds for all x, $y \in R^\ell$ and all $k \in [0, 1]$.

Problem 5 (15 points)
Consider the following utility function $U(x_1, x_2) = \min\{x_1, 2 \cdot x_2\}$. Determine the Marshallian and Hicksian demand functions, the expenditure function, and the indirect utility function!

Problem 6 (20 points)
In some exchange economy agent A has the utility function

$$U^A\left(x_1^A, x_2^A\right) = \min\left\{x_1^A, 2 \cdot x_2^A\right\}$$

and agent B has the utility function

$$U^B\left(x_1^B, x_2^B\right) = x_1^B + x_2^B.$$

Initial endowments are given by $\omega^A = (4, 8)$ and $\omega^B = (7, 1)$.

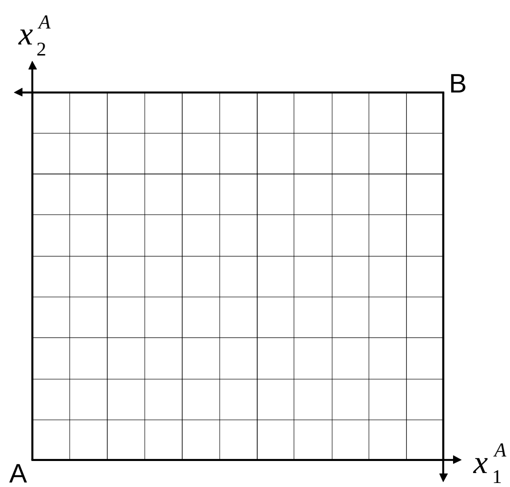
(a) Draw a suitable Edgeworth box in the diagram below! Do not forget

- the initial endowments of both actors,
- two indifference curves, one for each agent, which passes through the initial endowments (*Hint: be very careful!*),
- all bundles $\left(x_1^A, x_2^A\right)$ such that $U^A\left(x_1^A, x_2^A\right) \geq U^A\left(\omega_1^A, \omega_2^A\right)$)!

(b) Determine the greatest utility payoff that agent B can achieve through voluntary exchange with agent A!

(c) Draw the contract curve!

(d) Can you find arguments for why equilibrium prices p_1, p_2 must obey $p_1 = p_2 > 0$? In that case, what are the good bundles chosen in a Walras equilibrium? Why can we not have an equilibrium in case of $p_1 < p_2$?



Problem 7 (15 points)
Find all the equilibria in pure and properly mixed strategies:

player 2

player 1		s_2^1	s_2^2
	s_1^1	$4,3$	$2,2$
	s_1^2	$1,1$	$3,4$

Please draw the best responses!

Problem 8 (10 points)
Explain the problem of adverse selection and discuss solutions!

Problem 9 (5 points)
Determine the certainty equivalent for the lottery $L = \left(1, 4, 9; \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$ where the vNM utility function is given by $u(x) = \sqrt{x}$.

Problem 10 (10 points)

(a) Which of the following figures are decision trees or game trees?

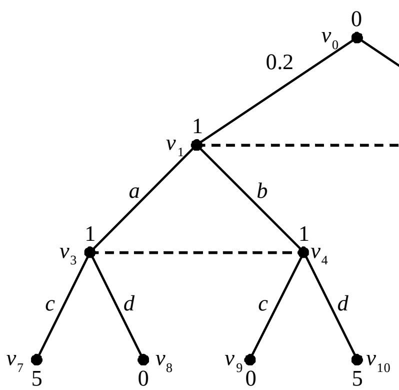
(b) Which of the games identified in (a) exhibit

- perfect/imperfect information and/or
- perfect/imperfect recall.

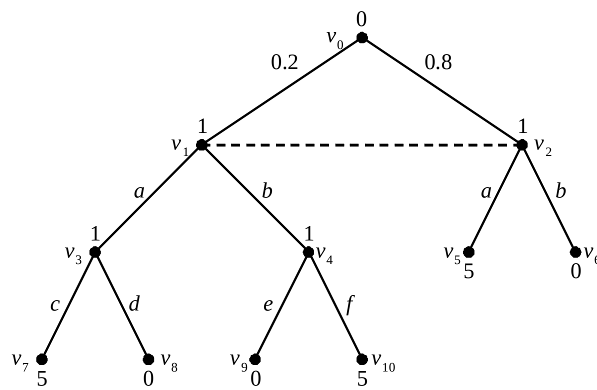
Give a short explanation of your answer*s*!

(c) List player 1's strategies in what is depicted in figure 1!

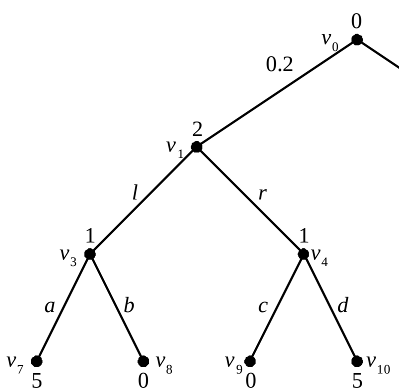
[1]



[2]



[3]



[4]