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– very preliminary draft –

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Abstract: The 2007-2008 financial crisis highlighted that a turmoil in the financial sector including bursting asset price bubbles can cause pronounced and persistent fluctuations in real economic activity. This justifies the consideration of evolving and bursting asset price bubbles as another source of fluctuations in business cycle models. In this paper rational asset price bubbles are incorporated into a life-cycle RBC model as first developed by Ríos-Rull (1996). The calibration of the model to the post-war US economy and the numerical solution show that the model is able to depict plausible bubble-driven business cycles. In particular, the model generates i) a higher and empirically more plausible volatility of consumption at the cost of ii) a lower and empirically less plausible contemporaneous correlation of consumption with output than the life-cycle RBC model without bubbles.

Keywords: Computable General Equilibrium, Bubble, Asset Price, Real Activity.
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1. Introduction

The 2007-2008 financial crisis highlighted that a turmoil in financial markets and bursting asset price bubbles can cause pronounced and persistent fluctuations in real economic activity. This motivated renewed interest in considering financial markets within DSGE models as an impulse and a propagation mechanism for aggregate fluctuations. Although financial frictions were incorporated in RBC models over 25 years ago by Bernanke and Gertler (1989) and are becoming more prevalent in recent contributions on DSGE models (see, for instance, Brunnermeier and Sannikov (2014), Boz and Mendoza (2014), or Covas and Den Haan (2012)) asset price bubbles are rarely considered in DSGE models. Building on the contribution by Martin and Ventura (2012) I therefore set up and numerically solve a life-cycle RBC model with asset price bubbles in this paper. The calibration of the model to the post-war US economy and the numerical solution show that the model is able to depict plausible bubble-driven business cycles. In particular, the model generates i) a higher and empirically more plausible volatility of consumption at the cost of ii) a lower and empirically less plausible contemporaneous correlation of consumption with output than the life-cycle RBC model without bubbles.

This paper is related to the following three strands of literature. First, since the seminal contribution by Tirole (1985) rational asset price bubbles have been incorporated in general equilibrium models. In the Tirole (1985) model bubbles always crowd-out capital. Empirically it is not plausible that the capital stock and output decline during episodes of existing asset price bubbles, and increase when bubbles burst. The correlation should be the reverse. Therefore, recent models extend the Tirole (1985) model – mostly by financial frictions – in order to derive equilibria where bubbles create a crowding-in of capital such that capital and output increase during periods of existing bubbles (see, among others, Galí (2014), Martin and Ventura (2012), and Farhi and Tirole (2012)). Most of these contributions consider *two-period* overlapping generation (OLG) models and derive explicit conditions for the existence of bubbles and the characteristics of bubbles in general equilibrium. I contribute to this literature by considering rational bubbles within a *large-scale* OLG model.

Second, models that incorporate aggregate uncertainty and analyze business cycles exist also within the the literature on large-scale OLG models as initiated by Auerbach and Kotlikoff (1987). The first to develop a life-cycle RBC model was Ríos-Rull (1996), and more recent contribution are, among others, Heer and Maußner (2012) and Iacoviello and Pavan (2013). The model presented in this paper can be seen as an extension of the Ríos-Rull (1996) model by financial frictions and rational asset price bubbles.

Third, within the infinitely-lived agent DSGE literature a few contributions do also consider asset price bubbles. Bernanke and Gertler (1999) consider *irrational* asset price bubbles within an RBC model with financial frictions. As the authors state, they "use the term "bubble" [here] loosely to denote temporary deviations of asset prices from fundamental values" (p. 19). Two further contributions within this literature are Miao *et al.* (2016) and Luik and Wesselbaum (2014). The numerical solution in both studies is based on local perturbation methods. By log-linearizing the dynamic systems of equations only small shocks can be considered. It is hard to justify, however, that bursting bubbles presents a small shock and that local solution techniques are an accurate approximation. I therefore apply a global solution technique for solving the model in this paper.

To the best of my knowledge, this paper is the first to set up and numerically solve a life-cycle RBC model featuring *rational* bubbles. This is important for at least two reasons. First, bubbles in two-period OLG models exist at least for some 30 years, which is implausibly long. In order to depict a short-term phenomenon like asset price bubbles, one period in the model should correspond at least to one year in calendar time. Second, and more importantly, by implementing rational bubbles in a large-scale OLG model the effect of evolving and bursting bubbles as an impulse for aggregate fluctuations can be analyzed and compared to business cycles resulting from TFP shocks.

The paper is structured as follows. In section 2 the model is presented and an equilibrium is defined. In the following section 3 the calibration is explained and the model is solved numerically. Section 4 extends the model to elastic labor supply and compares the results with the model without elastic labor supply. The last section concludes.

2. The Model

2.1. Set Up

Demographics. Time is discrete and the economy exists up to infinity. The economy consists of individuals of different age groups. Each period some individuals enter and some individuals leave the economy. The age of an individual is denoted by the subscript s. Young individuals enter the economy at a real life age of 21 years which is denoted by $s = 1$. After entering the economy individuals supply inelastically one unit of labor per period for $T \geq 1$ periods. Then individuals live for further $T^R \geq 1$ periods in retirement before dying and leaving the economy. Hence, the life-span for every individual in this economy is $T + T^R$ periods. In the classic OLG-model developed by Allais (1947), Samuelson (1958), and Diamond (1965) $T = T^R = 1$. In the vein of Auerbach and Kotlikoff (1987) I will calibrate the parameters T and T^R such that one period in the model corresponds to one year.

Total population is assumed to grow at a constant rate $n \geq 0$. Let N_t^s denote the size of the cohort consisting of individuals of age s in period t. Total population is then given by $N_t = \sum_{s=1}^{T+T^R} N_t^s$. Without loss of generality total population in period $t = 0$ is set equal to one. It follows that total population is given by

$$N_t = (1+n)^t, \tag{1}$$

and the cohort size is given by

$$N_t^s = \begin{cases} (1+n)^{t-s+1} N_0^1, & \text{if} \qquad 0 < s \leq T + T^R \\ 0, & \text{else} \end{cases} \tag{2}$$

where $N_0^1 = \dfrac{1}{\sum_{s=1}^{T+T^R} (1+n)^{1-s}}$ is the size of the cohort consisting of individuals that enter the economy in period $t = 0$.

Utility. After entering the economy individuals supply inelastically one unit of labor in every period for T periods, earning labor income w_t. For $s > T$ retirement is mandatory, households do not work and live from wealth accumulated in earlier periods of their lives.

Preferences of an individual of age $s = 1$ at period t are described by the intertemporal von Neumann-Morgenstern expected utility function

$$\mathbb{E}_t U_t^s = \mathbb{E}_t \sum_{s=1}^{T+T^R} \beta^{s-1} u(c_{t+s-1}^s), \tag{3}$$

where $0 < \beta$ is the subjective discount factor, $u(c)$ is instantaneous utility, and c_t^s denotes consumption of an individual of age s in period t. Instantaneous utility is given by the CRRA utility function

$$u(c_{t+s-1}^s) = \frac{\left(c_{t+s-1}^s\right)^{1-\epsilon} - 1}{1-\epsilon}, \tag{4}$$

where $\epsilon > 0$ is a constant preference parameter that equals the inverse of the intertemporal elasticity of substitution as well as the Arrow-Pratt measure of relative risk-aversion.

Firms. Firms of mass one employ two production factors – capital K_t and labor L_t – to produce a homogeneous final output good according to the Cobb-Douglas technology

$$Y_t = A_t K_t^{\alpha} \left(E_t L_t\right)^{1-\alpha}, \tag{5}$$

where $0 < \alpha < 1$ is the capital income share. Technological progress is labor augmenting, deterministic and expressed as $E_{t+1} = (1+g)E_t$. The variable A_t depicts exogenous and stochastic total factor productivity (TFP) shocks evolving according to an AR(1) process:

$$\ln A_{t+1} = \rho \ln A_t + z_{t+1}^a, \tag{6}$$

where $z_t^a \sim N(0, \sigma^a)$ is the innovation term and $0 \leq \rho < 1$ measures the persistence of TFP-shocks.

Competitive profit maximization yields wages w_t and interest rates r_t

$$w_t = (1-\alpha)\frac{Y_t}{L_t}, \tag{7}$$

$$r_t = \alpha \frac{Y_t}{K_t} - \delta, \tag{8}$$

where δ is the capital depreciation rate.

Financial friction. Besides differences in age individuals further differ with respect to their investment efficiency. A small share of the population is comparably productive at investing in capital used for production. Think of these productive individuals as the entrepreneurs in the economy or individuals with higher financial literacy. The population share of productive individuals is constant and given by $0 \leq \eta \leq 1$. These productive individuals are able to transform one unit of the final output good one-to-one into productive capital used in production. The remainder of the population, $1 - \eta$, is unproductive at investing and has to incur costs of $1 - \sigma$ when investing one unit of the final output good, where $0 \leq \sigma \leq 1$. The gross return to investment in capital faced by productive individuals is then equal to $1 + r_t$ while it is $\sigma(1 + r_t)$ for unproductive individuals. Individuals are born as either productive or unproductive investors and stay productive or unproductive investors throughout their whole life.

If financial markets were frictionless, unproductive individuals would lend all their wealth to productive individuals such that both groups would be better off, the allocation of investment would be efficient, and the capital stock in the economy would be larger than in an economy without lending between unproductive and productive individuals. However, it is assumed – as in Martin and Ventura (2012) – that unproductive individuals lending resources to productive individuals face a default probability greater than $1 - \sigma$ such that it is never profitable for unproductive individuals to lend resources to productive individuals. The result is a financial friction: no borrowing or lending takes place between productive and unproductive individuals, investments are inefficiently allocated, and a wedge is drawn between the returns to capital faced by productive and unproductive individuals.

Bubbles. Individuals can save labor income in order to smooth consumption and finance consumption in retirement when no labor income is earned. Therefore individuals can either supply capital $a_t^{j,s}$ for the production of a final output good, or purchase a zero-dividend asset $b_t^{j,s}$, where $a_t^{j,s}$ ($b_t^{j,s}$) denotes the capital stock (zero-dividend assets) held by an individual of productivity type $j \in \{U, P\}$ and age s at the beginning of period t.[1] Note, $b_t^{j,s}$ denotes the value of the zero-dividend asset in terms of the final output

[1] Tirole (1985) first introduced rational asset price bubble in a simple two-period OLG model.

good. An asset price can be decomposed into a fundamental and a bubble component, where the fundamental component is the sum of the discounted stream of expected future dividends or rents.[2] The zero-dividend asset's fundamental is by definition zero such that its price contains a bubble when b_t is greater than zero. Following Tirole (1985) I refer to the zero-dividend asset just as "bubble" in the remainder.

The aggregate bubble B_t is obtained by summing up all the bubbles held by individuals in the economy

$$B_t = \sum_{s=1}^{T+T^R} N_t^s \left[\eta b_t^{P,s} + (1-\eta) b_t^{U,s}\right], \tag{9}$$

where $b_{t+1}^{j,s+1}$ is the quantity of bubbles purchased by an individual of productivity type $j \in \{U, P\}$ and of age s at the end of period t. The size of the bubbles held by all productive (unproductive) individuals of age s in period t is $\eta N_t^s b_t^{P,s}$ $((1-\eta) N_t^s b_t^{P,s})$. Summing up the bubbles of all cohorts and productivity types gives B_t as described by (9).

In a given period t bubbles can be initiated and existing bubbles can burst.[3] Following Martin and Ventura (2012) the creation and destruction of bubbles is governed by investors' sentiment $z_t^b \in \{0, 1\}$. When investors are optimistic $z_t^b = 1$, they believe that the zero-dividend asset has a positive value, new bubbles can be initiated, and $b_t \geq 0$. When investors are pessimistic $z_t^b = 0$, they want to sell all their zero-dividend assets, they will not purchase further zero-dividend assets, and $b_t = 0$. Investors' sentiment z_t^b follows a simple Markov-process with the transition matrix $\mathbf{P}$, as depicted in Table 1. If investors are optimistic in period t, $z_t^b = 1$, the probability that investors are also optimistic in the next period $t+1$ is given by the constant parameter $0 \leq o \leq 1$. The probability of investors becoming pessimistic after being optimistic is hence $1-o$. Similarly, if investors are pessimistic in period t, $z_t^b = 0$, the probability that investors are also pessimistic in the next period $t+1$ is given by the constant parameter $0 \leq p \leq 1$. The probability of switching from pessimism to optimism is then $1-p$.

When investors are optimistic, $z_t = 1$, working productive individuals create new bubbles $\hat{b}_t^N$ and sell them to other individuals in the same period. For the sake of simplicity

[2] See, for instance, Blanchard and Watson (1983, 2f.), or, more recently, Brunnermeier (2008, 3).
[3] Weil (1987) first added a stochastic probability of bursting bubbles to the Tirole (1985) model.

Table 1: Transition matrix $\mathbf{P}$

	$z_{t+1}^b = 1$	$z_{t+1}^b = 0$
$z_t^b = 1$	o	$1 - o$
$z_t^b = 0$	$1 - p$	p

it is assumed that each working productive individual creates the same amount of new bubbles. The creation of new bubbles is exogenous and given by

$$\hat{b}_t^N = z_t^b \theta E_t, \tag{10}$$

where $\theta \geq 0$ is the size of newly created bubbles when technological growth is absent ($E_t = 1$) and investors are optimistic ($z_t^b = 1$). The parameter θ has to be multiplied by labor productivity E_t in order for a balanced-growth path to exist. The aggregate value of newly created bubbles is then

$$B_t^N := \hat{b}_t^N \sum_{s=1}^{T} \eta N_t^s = \frac{\eta}{1+\phi} N_t \hat{b}_t^N, \tag{11}$$

where $\phi := \frac{\sum_{s=T+1}^{T+T^R} N_t^s}{\sum_{s=1}^{T} N_t^s}$ is the constant old-age dependency ratio, $\frac{1}{1+\phi}$ is the constant population share of all working individuals, and $\frac{\eta}{1+\phi} N_t$ is the size of all productive working individuals.

The ex-post realized return to bubbles is given by

$$1 + q_t = \begin{cases} 1 + q_t^o = \dfrac{B_{t+1}}{B_t + B_t^N} & \text{if} \qquad z_t^b = 1, \\ 0 & \text{if} \qquad z_t^b = 0, \end{cases} \tag{12}$$

or equivalently as $(1 + q_t) = z_t^b(1 + q_t^o)$, where q_t^o is the return to bubbles under the realization $z_t^b = 1$.

The timing of events is described in detail in appendix A.1.

Household optimization. For $j \in \{P, U\}$ the households' budget constraints read

$$\begin{aligned}
c_t^{j,s} + a_{t+1}^{j,s+1} + b_{t+1}^{j,s+1} &= w_t^j + R_t^j a_t^{j,s} + z_t^b (1+q_t^o) b_t^{j,s} && s = 1, ..., T \\
c_t^{j,s} + a_{t+1}^{j,s+1} + b_{t+1}^{j,s+1} &= R_t^j a_t^{j,s} + z_t^b (1+q_t^o) b_t^{j,s} && s = T+1, ..., T+T^R \\
0 = a_t^{j,1} = b_t^{j,1} &= a_t^{j,T+T^R+1} = b_t^{j,T+T^R+1}. && (13)
\end{aligned}$$

The first equation depicts the budget constraints of all working individuals and the second equation the budget constraint of all retired individuals. Productive working individuals earn not only labor income, but also income from creating new bubbles, i.e. $w_t^P = w_t + z_t^b(1+q_t^o)\hat{b}_t^N$, while unproductive individuals earn only labor income, i.e. $w_t^U = w_t$. Due to the financial friction, productive individuals face higher returns to capital than unproductive individuals such that $R_t^P = 1 + r_t < R_t^U = \sigma(1+r_t)$. The third line of the budget constraints states that individuals enter the economy with zero wealth and that individuals leave no debt after death.

The households optimization problem then reads

$$\max_{\{c_{t+s-1}^{j,s},\, a_{t+s}^{j,s+1},\, b_{t+s}^{j,s+1}\}_{s=1}^{T+T^R}} \mathbb{E}_t \sum_{s=1}^{T+T^R} \beta^{s-1} u(c_{t+s-1}^{j,s}),$$
$$\text{s.t.} \qquad (13) \qquad \text{and} \qquad b_{t+1}^{j,s+1} \geq 0.$$

The non-negativity constraint on bubbles is imposed because bubbles are assumed to be freely disposable. It is possible, however, that households run temporary into debt, i.e. $a_t^{j,s} < 0$.

The first order conditions consist of two stochastic Euler equations – the first is derived from the first order condition for optimal capital and the second from the first order condition for optimal bubbles – a complementary slackness condition, and the budget constraints:

$$\begin{aligned}
\left(c_t^{j,s}\right)^{-\epsilon} =& \beta \mathbb{E}_t \left\{ R_{t+1}^j \left(c_{t+1}^{j,s+1}\right)^{-\epsilon} \right\} && s = 1, ..., T+T^R-1 \\
\left(c_t^{j,s}\right)^{-\epsilon} =& \beta \mathbb{E}_t \left\{ (1+q_{t+1}) \left(c_{t+1}^{j,s+1}\right)^{-\epsilon} \right\} + \omega_{t+1}^{j,s+1} && s = 1, ..., T+T^R-1 \\
\omega_{t+1}^{j,s+1} b_{t+1}^{j,s+1} =& 0 \qquad \text{and} \quad (13), && (14)
\end{aligned}$$

where $\omega_{t+1}^{j,s+1}$ is the multiplier associated with the non-negativity constraint of bubbles.

2.2. Equilibrium

A competitive equilibrium consists of a sequence of individual consumption, capital, and bubbles $\left\{\left\{c_t^{P,s}, c_t^{U,s}, a_t^{P,s}, a_t^{U,s}, b_t^{P,s}, b_t^{U,s}\right\}_{s=1}^{T+T^R+1}\right\}_{t=0}^{\infty}$ satisfying the FOC's of the household optimization problem as given by (14), a sequence of prices $\{w_t, r_t, q_t^o\}_{t=0}^{\infty}$ satisfying (7) and (12), a sequence of shocks $\left\{z_t^a, z_t^b\right\}_{t=1}^{\infty}$ drawn from their respective distributions and initial values $\{a_0^{P,s}, a_0^{U,s}, b_0^{P,s}, b_0^{U,s}\}_{s=1}^{T+T^R-1}, z_0^a, z_0^b, q_0^o$ such that

- the labor market clears

$$L_t = \sum_{s=1}^{T} N_t^s = \frac{1}{1+\phi} N_t, \tag{15}$$

- the capital market clears

$$K_{t+1} = \sum_{s=1}^{T+T^R} N_{t+1}^{s+1} \left[\eta a_{t+1}^{P,s+1} + \sigma(1-\eta) a_{t+1}^{U,s+1}\right], \tag{16}$$

- the market for bubbles clears

$$(1+q_t)(B_t + B_t^N) = \sum_{s=1}^{T+T^R} N_{t+1}^{s+1} \left[\eta b_{t+1}^{P,s+1} + (1-\eta) b_{t+1}^{U,s+1}\right], \tag{17}$$

- the goods market clears[4]

$$Y_t = C_t + \Delta K_{t+1} + \delta K_t + \underbrace{\sum_{s=1}^{T+T^R} (1-\eta)(1-\sigma) N_t^s a_{t+1}^{U,s+1}}_{\text{output-loss due to the financial friction}}, \tag{18}$$

- capital does not become negative (or, equivalently, the bubble does not become too

[4] Although it is not necessary to state this equation – it is implied by the budget constraints – it is useful as a consistency check in the numerical solution. Note further that aggregate consumption is defined as $C_t = \sum_{s=1}^{T+T^R} N_t^s \left(\eta c_t^{P,s} + (1-\eta) c_t^{U,s}\right)$.

large)[5]

$$\sum_{s=1}^{T+T^R} \eta N_t^s a_{t+1}^{P,s+1} \geq 0 \qquad \text{and} \qquad \sum_{s=1}^{T+T^R} (1-\eta) N_t^s a_{t+1}^{U,s+1} \geq 0 \tag{19}$$

- and bubbles are freely disposable

$$B_t \geq 0. \tag{20}$$

3. Numerical Solution

The model economy grows asymptotically at a rate $g + n + gn$. Some variables have therefore to be normalized in order to yield a stationary system. The notation of normalized variables is as follows: A normalized aggregate variable X_t is defined by $x_t := \frac{X_t}{E_t N_t}$. A normalized individual-level variable $x_t^{j,s}$ is defined by $\widetilde{x}_t^{j,s} := \frac{x_t^{j,s}}{E_t}$. The normalized system of equations is derived in appendix A.2.

3.1. Calibration

The model is calibrated with respect to the postwar US economy. Whenever possible I use values common in the literature.

Demographics. Periods in the model correspond to years in real-time. I therefore set the number of years individuals spend working to 40 ($T = 40$) and the number of years in retirement to 20 ($T^R = 20$). The values for T and T^R are taken from Heer and Maußner (2012), who calibrate a life-cycle RBC model to quarters and set $T = 160$ and $T^R = 80$. The annual population growth rate n is set equal to 1 percent since the US was inhabited by 159 million individuals in 1950 and by 230 million in 2013 (UN, 2013, 61).

Preferences. The subjective discount factor is set below, but close to unity as common in the literature, i.e. $\beta = 0.9975$ (see, for instance, Heer and Maussner (2009)). The intertemporal elasticity of substitution $1/\epsilon$ cannot be inferred from time series of economic variables. According to a recent empirical meta-analysis by Havránek (2015) it should be far below unity. I choose a value of 0.25 such that $\epsilon = 4$.

[5] Due to the financial friction the total capital of unproductive or productive individuals cannot be negative. If, for example, total capital of unproductive individuals would be negative that would imply that unproductive individuals borrow resources from productive individuals at the return $\sigma(1 + r_t)$ contradicting the imposed financial friction.

Table 2: Set of parameters

Parameter	Value	Parameter	Value
T	40	ρ	0.814
T^R	20	σ^a	0.0142
n	0.01	σ	0.6
β	0.9975	η	0.05
ϵ	4	o	0.8
α	0.34	p	0.9
δ	0.08	θ	0.15
g	0.02		

Production. The capital income share and the capital depreciation rate are set in accordance with Prescott (1986) implying $\alpha = 0.34$ and $\delta = 0.08$. The growth rate g is determined by the population growth rate n and the growth rate of aggregate output. In the period 1960-2014 real US-GDP grew on average by approximately 3 percent as calculated from the US Bureau of Economic Analysis. Aggregate output in the model economy grows at the rate $n + g + ng$. The growth rate of E_t is then given by $g = (0.03 - n)/(1 + n) \approx 0.02$. The parameters for the AR(1) process for technology are set equal to $\rho = 0.814$ and $\sigma^a = 0.0142$. These parameters correspond to a quarterly AR(1) process with $\rho = 0.95$ and $\sigma^a = 0.00763$ as given in Prescott (1986).[6]

Financial friction. The parameters η and σ are not common in the literature on quantitative macroeconomic models and empirical time series for calibrating these parameters do not exist. I assume that 5 percent of the population is much more efficient in investing then the rest of the population. Further, σ is chosen such that the wedge between the interest rate of productive and unproductive individuals is sufficiently large for expansionary bubbles to exist. Therefore, $\eta = 0.05$ and $\sigma = 0.6$.

Bubbles. The choice of the parameters governing the duration, creation, bursting, and size of bubbles is difficult to infer from observed data. The parameters governing investors' sentiment are chosen to be $o = 0.8$ and $p = 0.9$ giving an average duration of a bubbly episode of 4 years and an average duration of a bubble-less episode of 9 years. The creation of new bubbles – governed by θ – is set such that the bubble-to-capital ratio is

[6]See Heer and Maussner (2009, Ch. 10, appendix 5) for how to derive parameters for the annual AR(1) process based on parameters of a quarterly AR(1) process.

approximately equal to 1 percent in the deterministic steady state, implying a value of $\theta = 0.15$. Large values of θ can result in bubbles becoming too large and violating the equilibrium conditions. Table 2 summarizes the calibration of the model.

3.2. Deterministic steady state

The deterministic steady state is a hypothetical trajectory where the stochastic exogenous state-variables z_t^a and z_t^b are equal to their expected values with probability one. The computation of the deterministic steady state is necessary for the numerical solution of the dynamic system in subsection 3.3.

The limiting distribution of the Markov process of investors' sentiment, π, is given by

$$\pi = \begin{bmatrix} \frac{1-p}{(1-o)+(1-p)} \\ \frac{1-o}{(1-o)+(1-p)} \end{bmatrix}. \tag{21}$$

The expected value of z_t^b is then given by $\bar{z} := \lim_{T\to\infty} \mathbb{E}_t[z_T^b] = \dfrac{1-p}{(1-o)+(1-p)}$. In the deterministic steady state $z_t^b = \bar{z}$, $z_t^a = 0$, and $A_t = 1$ for all t.

The stochastic Euler equations given by (33) become deterministic and read

$$\begin{aligned}
\left(\frac{(1+g)\widetilde{c}^{j,s+1}}{\widetilde{c}^{j,s}}\right)^{\epsilon} &= \beta R^j && s = 1, ..., T+T^R-1 \\
\left(\frac{(1+g)\widetilde{c}^{j,s+1}}{\widetilde{c}_t^{j,s}}\right)^{\epsilon} &= \beta(1+q) + \widetilde{\omega}^{j,s+1}\left(\widetilde{c}^{j,s+1}\right)^{\epsilon} && s = 1, ..., T+T^R-1 \\
\widetilde{\omega}^{j,s+1}\widetilde{b}^{j,s+1} &= 0 \qquad \text{and} \quad (29),
\end{aligned} \tag{22}$$

In what follows I assume $\theta > 0$, as in the calibration, such that new bubbles are created in the deterministic steady state. What is now the relation between the certain return to bubbles q and the return to capital r?

First, when $(1+q) = R^j$ individuals are indifferent with respect to the allocation of their wealth between productive capital $a^{j,s}$ and bubbles $b^{j,s}$, otherwise individuals hold only productive capital or only bubbles. The equilibrium condition for bubbles (17) necessitates that q is large enough such that some individuals in the economy choose to purchase bubbles, because new bubbles are created in each period. This rules out

$1 + q < \sigma(1 + r)$, because otherwise the newly created bubbles would not be purchased and the equilibrium condition (17) would be violated.

Second, assume $(1 + q) = R^U = \sigma(1 + r)$. Unproductive individuals hold bubbles and their non-negativity restriction does not bind, i.e. $\widetilde{\omega}^{U,s} = 0$. Productive individuals would not purchase bubbles because $1 + q < R^P = (1 + r)$ and their non-negativity restriction would bind, i.e. $\widetilde{\omega}^{P,s} > 0$, and $\widetilde{b}^{P,s} = 0$. Unproductive individuals are indifferent with respect to the amount of bubbles purchased. Any distribution of the existing bubble across all unproductive individuals would satisfy the household optimality and market equilibrium conditions. I assume that each individual holds the same amount of bubbles, i.e. $b^{U,s} = b^U$.

Third, only if $(1 + q) \geq (1 + r)$ all individuals in the economy would hold bubbles, i.e. $\widetilde{\omega}^{j,s} = 0$. The discussion is summarized by

$$1 + q = \begin{cases} = \sigma(1 + r) & \text{if } b < \sum_{s=1}^{T+T^R} (1 - \eta) N_0^{s+1} \widetilde{s}^{U,s} \\ \in [\sigma(1 + r), (1 + r)] & \text{if } b = \sum_{s=1}^{T+T^R} (1 - \eta) N_0^{s+1} \widetilde{s}^{U,s} \\ = (1 + r) & \text{if } b > \sum_{s=1}^{T+T^R} (1 - \eta) N_0^{s+1} \widetilde{s}^{U,s}, \end{cases} \tag{23}$$

where total individual wealth $\widetilde{s}^{j,s}$ is defined as $\widetilde{s}^{j,s} := \widetilde{a}^{j,s} + \widetilde{b}^{j,s}$ for $j \in \{U, P\}$.

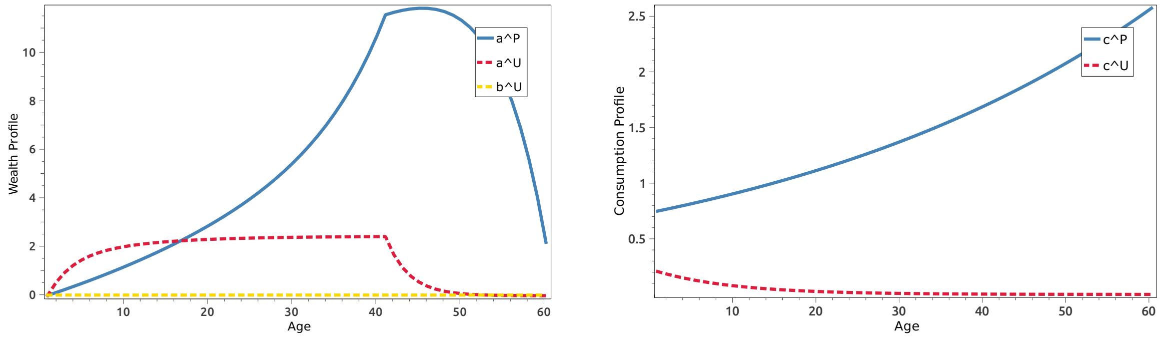
In the simulations I consider only the case where $1 + q = \sigma(1 + r)$, because the model would otherwise yield empirically implausible bubbles where the capital stock declines when bubbles emerge and increases when bubble burst. Therefore it is always checked that the wealth of the unproductive individuals is smaller or equal than the existing bubble such that (23) is satisfied. The size of individual bubbles is then given by $\widetilde{b}^{P,s} = 0$ and

$$\widetilde{b}^U = \frac{1}{(1 - \eta)(1 - N_0^1)} b. \tag{24}$$

The numerical procedure that has been employed to solve for the steady state is closely related to Auerbach and Kotlikoff (1987) and similar to the "direct computation method" as described in Heer and Maussner (2009). The solution algorithm is described in detail in appendix C.1.

In the steady state the aggregate (normalized) capital stock and the aggregate (nor-

Figure 1: Age profiles



malized) bubble are given by $k = 1.162$ and $b = 0.01201$. The returns to capital differ strongly between productive and unproductive individuals and are given by $R^P = 1.172$ and $R^U = 0.7032$. The bubble is positive and considerably smaller than the aggregate wealth of all unproductive individuals – which is equal to 1.511 – hence (23) is satisfied. Figure 1 depicts the individual wealth and consumption profiles. The consumption profile is increasing for productive individuals and decreasing for unproductive individuals. This is a direct result of the financial friction which draws a wedge between the interest rates faced by productive and unproductive individuals. Consumption of a productive individual grows at a constant rate of 2 percent over a life span while consumption of an unproductive individual declines at constant rate of 10 percent.[7] The increasing consumption profile of productive individuals therefore also implies a much steeper wealth profile, because productive individuals need to accumulate more wealth for higher consumption levels at an older age than unproductive individuals.

3.3. Dynamics

The model is solved with the deterministic extended path (DEP) method. This procedure was first applied to DSGE models by Gagnon (1990). As shown by Heer and Maußner (2008) the DEP method yields the highest accuracy in the computation of the standard business cycle model at the cost of longer computational time when compared with log-linear solution methods, second-order approximations, the parameterized expectations approach, Galerkin projetions, and value function iterations. In contrast to local

[7] The constant growth rate of individual consumption is obtained by rearranging the Euler equation such that $\frac{c^{j,s+1}}{c^{j,s}} - 1 = \frac{\left(\beta R^j\right)^{\frac{1}{\epsilon}}}{1+g} - 1$.

methods like the log-linear or higher-order approximations as well as linear-quadratic approximations the DEP method is a global method and therefore better suited for the present model, because the shocks due to evolving and bursting bubbles can shift the economy far away from its steady state.

The DEP method works as follows: For given endogenous and exogenous (stochastic) state variables in period t the conditional expectations of the stochastic state variables for the entire time span of the simulation is calculated and plugged into the stochastic Euler equations. The result is a deterministic dynamic system of equations. This deterministic system is solved for a time-span large enough for the economy to be very close to its deterministic steady state. From the solution the endogenous state variables at $t+1$ and the control variables at t are stored. In $t+1$ a new realization of the shocks is drawn and the whole procedure is repeated. Proceeding in this manner one obtains an approximation of a realized time path of the true stochastic model which – as shown by Heer and Maußner (2008) – can be very accurate.

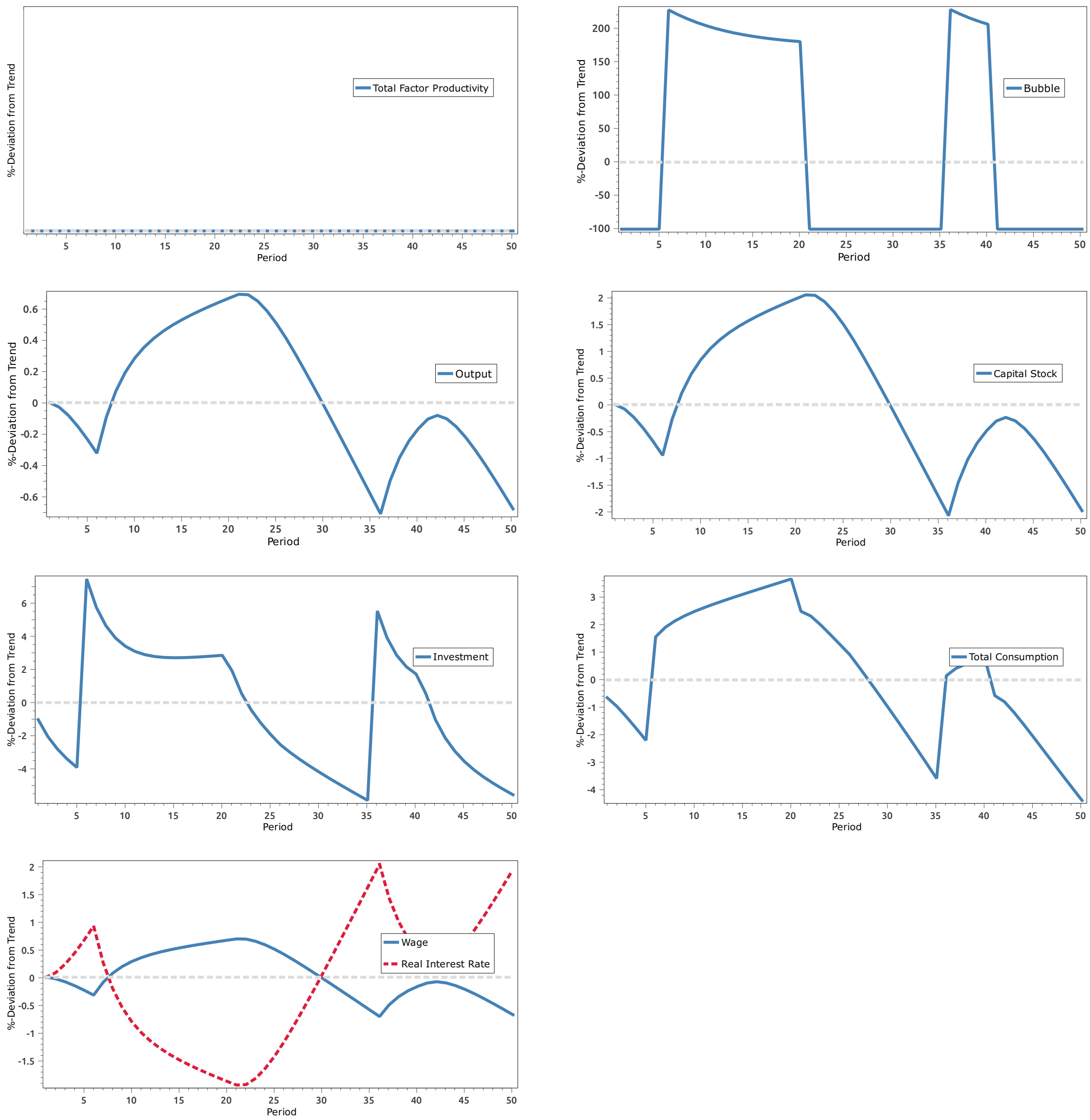
The deterministic counterpart of the true stochastic dynamic system of equations as well as the solution algorithm are described in appendix C.2.

I will compare three different numerical solutions in the following: i) an economy without bubbles, i.e. θ is set equal to zero, ii) an economy with bubbles but without TFP shocks, and iii) an economy with both types of shocks.

Figure 4 in the appendix depicts the impulse response functions for the model economy without bubbles. For all $t = 2, ...30$ z_t^a is equal to zero, except for $t = 1$, where a positive shock of one standard deviation occurs. When the positive TFP shock materializes in the first period output, wages, interest rates, consumption, and investment increase immediately, whereas capital reacts with a lag of one period. Consumption increases by 0.45 percent, output by 1.4 percent, and investment by more than 8 percent. All variables converge back to their steady states, but consumption converges very slowly reflecting the households' desire for distributing the positive income shock evenly over their life-span.

Figure 2 depicts a simulation for an economy with bubbles and without TFP shocks. During the 50 years simulated two episodes of asset price bubbles occur. When bubbles emerge capital stock, output, investment, and consumption increase and stay above their

Figure 2: Bubble-driven business cycles



respective trend. When bubbles burst all four variables decline. From the height of the first bubble-driven cycle to the subsequent trough output declines by approximately 1.5 percentage points, capital stock by four percentage points and consumption by almost eight percentage points. Note, aggregate consumption is much more volatile than in the bubble-less model economy depicted in Figure 4. The reason is that bursting bubbles destroy a part of individual wealth, whereas TFP shocks imply a reduction in wages or interest rates leaving current wealth unchanged.

How does the model fare with respect to second moments? To answer this question second moments of empirical time series of aggregate output, consumption, investment, and hours for the US are reported in Table 3 and second moments of the time series

Table 3: Business cycles statistics of the US-economy

Variable	Standard deviation	Relative standard deviation	First-order auto-correlation	Contemporaneous correlation with output
Output	0.014	1.000	0.221	1.000
Consumption	0.011	0.819	0.287	0.917
Investment	0.047	3.382	0.171	0.943
Hours	0.014	1.006	0.302	0.890

Table 4: Business cycles statistics of the model

Variable	Standard deviation	Relative standard deviation	First-order auto-correlation	Contemporaneous correlation with output
TFP-shocks				
Output	0.009	1.000	0.127	1.000
Consumption	0.002	0.241	0.172	0.985
Investment	0.057	6.263	0.020	0.933
Bubble-shocks				
Output	0.001	1.000	0.459	1.000
Consumption	0.007	12.100	0.052	-0.242
Investment	0.020	35.850	0.032	-0.447
Both shocks				
Output	0.009	1.000	0.115	1.000
Consumption	0.007	0.784	0.088	0.313
Investment	0.060	6.588	0.075	0.858

generated by the model are reported in Table 4. The empirical time series are from the US Bureau of Economic Analysis and cover the period 1954–2011. All variables – the empirical time series as well as the generated time series from the model – are expressed in logarithms and HP-filtered with an annual smoothing parameter of 6.25 as proposed by Ravn and Uhlig (2002). The time series generated by the model are obtained from simulating the model for 500 periods and dropping the first 100 observations in order to make sure that initial conditions do not effect the calculated second moments.[8]

The empirical time series reveal standard business cycle facts,[9] namely that i) investment is three times more volatile than output, ii) aggregate hours are as volatile as output, iii) consumption is less volatile than output, and iv) all four variables are procyclical (strong contemporaneous correlation with output). Since I am looking at annual data the first-order autocorrelation is of course lower than for quarterly data implying less persistent fluctuations at the annual frequency.

[8] On a Intel® Core™ i7 processor with four times 2.80GHz the computation of the model for 500 periods takes approximately 30 minutes.

[9] See, for instance, King and Rebelo (1999, ch. 2.2) for a description of stylized facts of US business cycles.

The results for the life-cycle RBC model without bubbles is depicted in the first four rows of Table 4. The model generates slightly insufficient volatility in output, slightly to strong volatility in investment and far to low volatility in consumption. The relative volatility of consumption with respect to output is almost four times smaller than in the empirical time series. The persistence of all variables is to small while their contemporaneous correlation with output matches the empirical moments quite well.

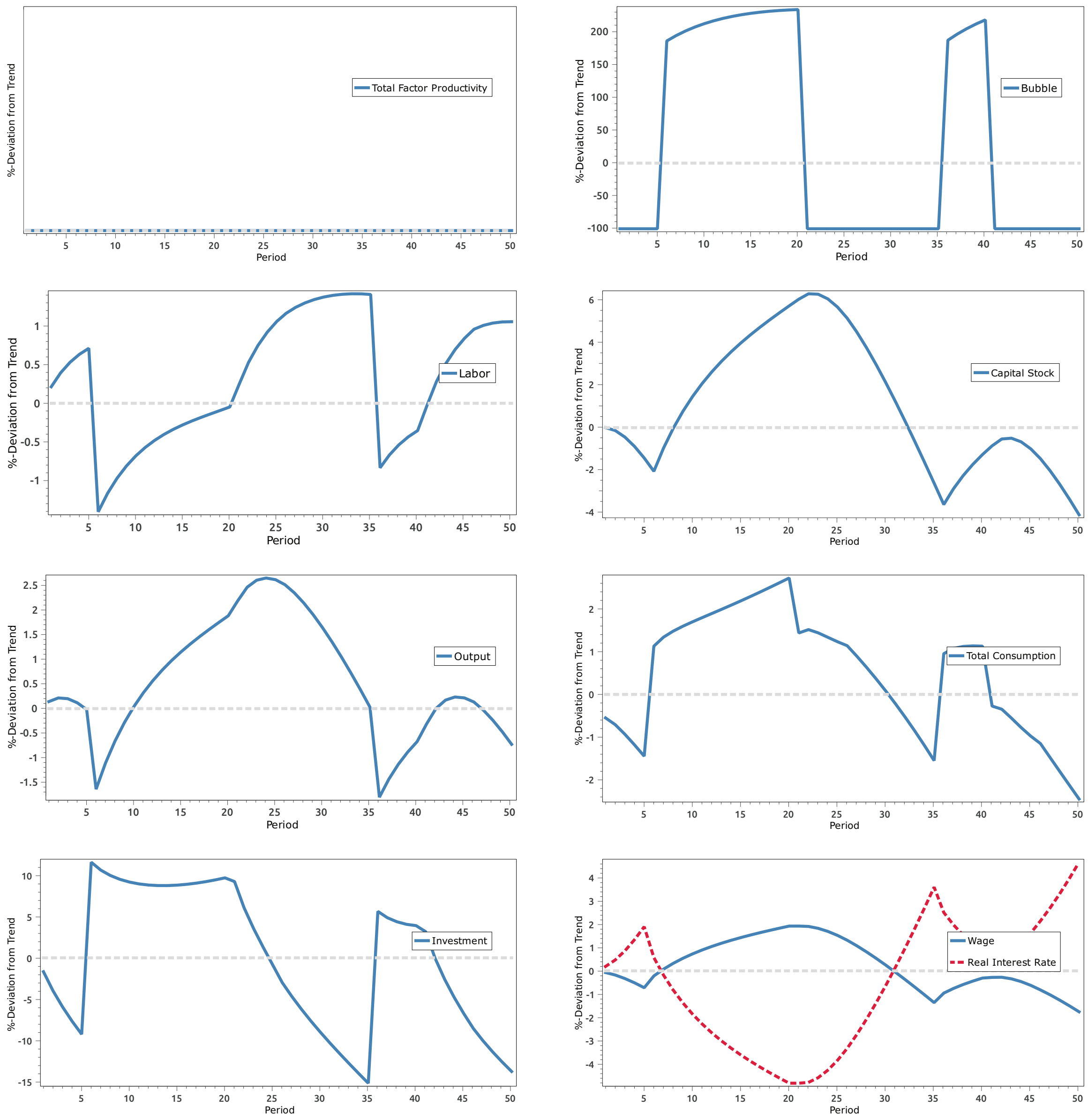
Considering the model economy with shocks from evolving and bursting bubbles and without TFP-shocks in Table 4 reveals implausible business cycle statistics. By construction of the rather small bubble (1 percent of the capital stock in the steady state) bubble-driven business cycles imply rather small deviations of output, consumption and investment from their respective trends. Consumptions and investment are way to volatile in relation to output. This is a result of the transmission channel of bubbles, which imply drastic changes in individual wealth. Taken as a sole source of business cycle fluctuations evolving and bursting bubbles seem rather implausible. Adding asset price bubbles to the model economy with TFP-shocks, however, results in second moments that better match their empirical counterparts. Most importantly, the model can generate much higher and empirically more plausible volatility of consumption. Relative to output the volatility of consumption in the model (0.784) is very close to its empirical counterpart (0.819). This better performance in terms of consumption volatility comes at the cost of lower contemporaneous correlation of consumption with output, as can be seen in the last column of Table 4.

4. The model with elastic labor supply

In RBC models, another important propagation mechanism besides capital accumulation is labor supply. I therefore loosen the assumption of an inelastic labor supply in this section. The changes to the model are shown in appendix B.

The computation of the steady state is similar to the model with inelastic labor supply, except that i) besides k also l has to be guessed and updated initially and ii) also $l^{j,s}$ has to be computed in the solution of the household problem. The solution algorithm is provided in appendix D. In the steady state $k = 0.4338$, $l = 0.399$, $b = 0.01474$,

Figure 3: Bubble-driven business cycles (endogenous labor supply)



$R^P = 1.2515$, $R^U = 0.7509$, and the bubble is considerably smaller than aggregate wealth of all unproductive individuals which is equal to 0.5814.

The impulse-response function for a TFP-shock in the model with elastic labor supply and without bubbles is shown in Figure 5 in the appendix. The graphs depict the difference of a time-path where the error term z_t^a is equal to zero for all t and a time-path where $z_t^a = 0$ for all t except for $t = 1$, where z_t^a is equal to one standard-deviation σ^a. Two qualitative differences to the model with inelastic labor supply are striking. First, the lagged increase of capital stock is much more pronounced such that output first jumps to a higher level and afterwards increases even further before converging back to its steady state level. Second, aggregate consumption drops initially, stays below its steady state for 15 years and stays above its steady state afterwards for a very long period before converging back to its steady state. Individuals smooth consumption and therefore even forgo higher consumption and leisure in the first periods in order to earn and save more and finance higher consumption in later periods of their lives. Compared to the model with inelastic labor supply individuals can now better smooth consumption via adjusting their labor supply which results in a flatter consumption curve. Although the drop in consumption is quantitatively very small (0.1 percent) it stands in contrast to empirical business cycle facts. All variables except consumption are much more volatile in the model with elastic labor supply.

Figure 3 depicts the results of a simulation with two bubble-episodes and without TFP shocks. When bubbles emerge capital, consumption, and investment increase, and when bubbles burst capital, consumption, and investment decline, similar to the model with inelastic labor supply. However, in this model individuals react to the shocks of evolving and bursting bubbles by adjusting their labor supply. The result is that labor and output respond slower to the bubble shocks and first increase when bubbles burst before decreasing. Consumption is slightly less volatile than output, output is more volatile than labor and capital and investment are most volatile. Similar to the TFP-shock depicted in Figure 5 in the appendix all variables except consumption are more volatile than in the model with elastic labor supply.

Second moments generated from the model with elastic labor supply are depicted in Table 5. The model without bubbles and with TFP-shocks generates volatility in output which is very close to its empirical counterpart, but consumption and working hours are far too stable and investment is far too volatile. Consumption is countercyclical and investment is acyclical. Both should be procyclical. The model with bubble-shocks and without TFP-shocks generates higher volatilities in consumption, investment and hours, lower volatility in output (due to the rather small bubbles) and countercyclical investment. Adding bubble-shocks to the economy with TFP-shocks yields higher, but still too low, volatilities in consumption and working hours. The contemporaneous correlation with output is also higher and consumption is now almost acyclical.

Table 5: Business cycles statistics for the model with elastic labor supply

Variable	Standard deviation	Relative standard deviation	First-order auto-correlation	Contemporaneous correlation with output
TFP-shocks				
Output	0.013	1.000	0.139	1.000
Consumption	0.0004	0.033	0.371	-0.749
Investment	0.113	8.645	0.026	0.922
Hours	0.001	0.470	0.145	0.100
Bubble-shocks				
Output	0.003	1.000	0.127	1.000
Consumption	0.005	1.604	0.059	-0.849
Investment	0.034	10.650	0.067	-0.904
Hours	0.004	1.201	0.066	0.962
Both shocks				
Output	0.013	1.000	0.168	1.000
Consumption	0.005	0.380	0.064	-0.276
Investment	0.118	8.948	0.113	0.774
Hours	0.007	0.541	0.116	0.938

5. Conclusion

Given their empirical relevance for business cycles it is important to integrate asset price bubbles into DSGE models. This paper presents an important step towards this goal. The computation of a life-cylce RBC model featuring rational asset price bubbles shows that under an empirically reasonable parameter constellation asset price bubbles lead to strong fluctuations in output, consumption, labor supply, capital stock and investment. The simulated time paths show that the model is able to depict i) evolving asset price bubbles together with increasing capital, investment, consumption, labor and output and ii) bursting asset price bubbles leading to decreasing capital, investment, consumption, labor and output. However, when labor supply is endogenous bursting bubbles lead to first increasing labor supply before it starts to decline, such that output even shortly increases before decreasing in the subsequent periods.

The model did not consider a government and hence also no taxes, pensions and monetary policy reacting to evolving and bursting bubbles. Adding a social security system with pay-as-you-go pensions to the model would result in lower aggregate savings and a lower capital stock, increasing the interest rate and possibly making the existence of rational bubble impossible. However, only a part of the US-pension system is based on governmental pay-as-you-go transfers, while the major part of pension is either occupational or private. Further, since World War II investment in the US has been heavily driven by capital imports as the US run mostly current account deficits. The model economy is a closed economy abstracting from international capital flows. Considering international capital flows would increase the capital stock and make bubbles more likely. Including further relevant macroeconomic aspects into the model to see how bubbles can exist and conducting a welfare analyses remains therefore of future interest.

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Appendix

A. The basic model

A.1. Timing of events

Within a period t the exact timing of events is as follows:

- The sequence of capital $\left\{a_t^{P,s}, a_t^{U,s}\right\}_{s=1}^{s=T+T^R}$ and bubbles $\left\{b_t^{P,s}, b_t^{U,s}\right\}_{s=1}^{s=T+T^R}$ as well as the return to bubbles in the optimistic state q_t^o are predetermined from the previous period.
- The two shocks z_t^a and z_t^b materialize.
- Given total factor productivity A_t production takes place and labor and capital are rewarded with their respective factor prices.
- If $z_t^b = 1$ bubbles from the previous period and newly created bubbles are traded. If $z_t^b = 0$ the bubble bursts.
- Given capital income and labor income as well as the income from selling bubbles individuals choose consumption $c_t^{j,s}$ and adjust their stock of capital $a_{t+1}^{j,s+1}$ and bubbles $b_t^{j,s+1}$.

A.2. Normalization

Firms. Normalized output, wages, and interest rates are given by

$$
\begin{aligned}
y_t &= A_t \left[\alpha k_t^{\kappa} + (1-\alpha) \left(\frac{1}{1+\phi} \right)^{\kappa} \right]^{\frac{1}{\kappa}} \\
\widetilde{w}_t := \frac{w_t}{E_t} &= (1-\alpha) A_t^{\kappa} \left[(1+\phi) y_t \right]^{1-\kappa} \\
r_t &= \alpha A_t^{\kappa} \left(\frac{y_t}{k_t} \right)^{1-\kappa} - \delta.
\end{aligned}
$$

Bubbles. The equations for bubbles in normalized variables change as follows

$$b_t = \sum_{s=1}^{T+T^R} N_0^s \left[\eta \widetilde{b}_t^{P,s} + (1-\eta)\widetilde{b}_t^{U,s}\right] \tag{25}$$

$$\widetilde{b}_t^N := \frac{\hat{b}_t^N}{E_t} = z_r^b \theta \tag{26}$$

$$b_t^N = \frac{\eta}{1+\phi}\widetilde{b}_t^N \tag{27}$$

$$1 + q_t = z_t^b(1+q_t^o). \tag{28}$$

Note, $\frac{N_t^s}{N_t} = \frac{N_0^s}{N_0} = N_0^s$ is the time-constant population share of cohort s.

Household optimum. The budget constraints now read

$$\begin{aligned}
&\widetilde{c}_t^{j,s} + (1+g)\left(\widetilde{a}_{t+1}^{j,s+1} + \widetilde{b}_{t+1}^{j,s+1}\right) = \widetilde{w}_t^j + R_t^j \widetilde{a}_t^{j,s} + (1+q_t)\widetilde{b}_t^{j,s} && s = 1, ..., T \\
&\widetilde{c}_t^{j,s} + (1+g)\left(\widetilde{a}_{t+1}^{j,s+1} + \widetilde{b}_{t+1}^{j,s+1}\right) = R_t^j \widetilde{a}_t^{j,s} + (1+q_t)\widetilde{b}_t^{j,s} && s = T+1, ..., T+T^R \\
&0 = \widetilde{a}_t^{j,1} = \widetilde{b}_t^{j,1} = \widetilde{a}_t^{j,T+T^R+1} = \widetilde{b}_t^{j,T+T^R+1},
\end{aligned} \tag{29}$$

with $\widetilde{w}_t^P := \widetilde{w}_t + \widetilde{b}_t^N$ and $\widetilde{w}_t^U := \widetilde{w}_t$.

The FOCs in normalized variables are given by

$$\begin{aligned}
(1+g)^\epsilon \left(\widetilde{c}_t^{j,s}\right)^{-\epsilon} &= \beta \mathbb{E}_t \left\{R_{t+1}^j \left(\widetilde{c}_{t+1}^{j,s+1}\right)^{-\epsilon}\right\} && s = 1, ..., T+T^R-1 \\
(1+g)^\epsilon \left(\widetilde{c}_t^{j,s}\right)^{-\epsilon} &= \beta \mathbb{E}_t \left\{(1+q_{t+1}) \left(\widetilde{c}_{t+1}^{j,s+1}\right)^{-\epsilon}\right\} + \widetilde{\omega}_{t+1}^{j,s+1} && s = 1, ..., T+T^R-1 \\
\widetilde{\omega}_{t+1}^{j,s+1}\widetilde{b}_{t+1}^{j,s+1} &= 0 \qquad \text{and } (29),
\end{aligned} \tag{30}$$

with $\widetilde{\omega}_{t+1}^{j,s+1} := E_{t+1}^\epsilon \omega_{t+1}^{j,s+1}$.

Equilibrium. The equilibrium conditions in normalized variables are given by

$$\frac{L_t}{N_t} = \frac{1}{1+\phi}$$

$$k_{t+1} = \sum_{s=1}^{T+T^R} N_0^{s+1} \left[\eta \widetilde{a}_{t+1}^{P,s+1} + \sigma(1-\eta)\widetilde{a}_{t+1}^{U,s+1}\right]$$

$$(1+q_t)(b_t + b_t^N) = (1+g)(1+n) \sum_{s=1}^{T+T^R} N_0^{s+1} \left[\eta \widetilde{b}_{t+1}^{P,s+1} + (1-\eta)\widetilde{b}_{t+1}^{U,s+1}\right]$$

$$y_t + (1-\delta)k_t = c_t + (1+n)(1+g)k_{t+1} + (1+g)(1-\eta)(1-\sigma)\sum_{s=1}^{T+T^R} N_0^s \widetilde{a}_{t+1}^{U,s+1}$$

$$\sum_{s=1}^{T+T^R} \eta N_0^s \widetilde{a}_{t+1}^{P,s+1} \geq 0 \quad \text{and} \qquad \sum_{s=1}^{T+T^R} (1-\eta) N_0^s \widetilde{a}_{t+1}^{U,s+1} \geq 0.$$

B. The model with elastic labor supply

Each young individual has an time endowment of one unit per period. Individual leisure is given by $l_t^{j,s}$, individual labor supply by $n_t^{j,s}$ and $1 = l_t^{j,s} + n_t^{j,s}$. The instantaneous utility function is given by

$$u(c, l) = \frac{\left(cl^{\psi}\right)^{1-\epsilon} - 1}{1 - \epsilon}, \tag{31}$$

where $\psi > 0$ is the relative weight given to leisure and ϵ is the inverse of the intertemporal elasticity of substitution.

The budget constraints now read

$$\begin{aligned}
\widetilde{c}_t^{j,s} + (1+g)\left(\widetilde{a}_{t+1}^{j,s+1} + \widetilde{b}_{t+1}^{j,s+1}\right) &= \widetilde{w}_t^j n_t^{j,s} + R_t^j \widetilde{a}_t^{j,s} + (1+q_t)\widetilde{b}_t^{j,s} && s = 1, ..., T \\
\widetilde{c}_t^{j,s} + (1+g)\left(\widetilde{a}_{t+1}^{j,s+1} + \widetilde{b}_{t+1}^{j,s+1}\right) &= R_t^j \widetilde{a}_t^{j,s} + (1+q_t)\widetilde{b}_t^{j,s} && s = T+1, ..., T+T^R \\
0 = \widetilde{a}_t^{j,1} = \widetilde{b}_t^{j,1} &= \widetilde{a}_t^{j,T+T^R+1} = \widetilde{b}_t^{j,T+T^R+1}, &&
\end{aligned} \tag{32}$$

and the FOC now also contain an intra-temporal condition reflecting the leisure-consumption trade-off:

$$\begin{aligned}
\psi \frac{c_t^{j,s}}{l_t^{j,s}} &= w_t \\
(1+g)^{\epsilon} \left(\widetilde{c}_t^{j,s}\right)^{-\epsilon} \left(l_t^{j,s}\right)^{\psi(1-\epsilon)} &= \beta \mathbb{E}_t \left\{ R_{t+1}^j \left(\widetilde{c}_{t+1}^{j,s+1}\right)^{-\epsilon} \left(l_{t+1}^{j,s+1}\right)^{\psi(1-\epsilon)} \right\} && s = 1, ..., T+T^R-1 \\
(1+g)^{\epsilon} \left(\widetilde{c}_t^{j,s}\right)^{-\epsilon} \left(l_t^{j,s}\right)^{\psi(1-\epsilon)} &= \beta \mathbb{E}_t \left\{ (1+q_{t+1}) \left(\widetilde{c}_{t+1}^{j,s+1}\right)^{-\epsilon} \left(l_{t+1}^{j,s+1}\right)^{\psi(1-\epsilon)} \right\} + \widetilde{\omega}_{t+1}^{j,s+1} && s = 1, ..., T+T^R-1 \\
\widetilde{\omega}_{t+1}^{j,s+1} \widetilde{b}_{t+1}^{j,s+1} &= 0 \qquad \text{and } (32),
\end{aligned} \tag{33}$$

Output, wages, and interest rates can be written as

$$\begin{aligned}
y_t &= A_t \left[\alpha k_t^{\kappa} + (1-\alpha) l_t^{\kappa}\right]^{\frac{1}{\kappa}} \\
\widetilde{w}_t &= (1-\alpha) A_t^{\kappa} \left(\frac{y_t}{l_t}\right)^{1-\kappa} \\
r_t &= \alpha A_t^{\kappa} \left(\frac{y_t}{k_t}\right)^{1-\kappa} - \delta
\end{aligned}$$

where $l_t := \frac{L_t}{N_t}$ is the average labor supply. Furthermore, the labor-market clearing condition changes to:

$$l_t = \sum_{s=1}^{T+T^R} N_0^s \left[\eta n_t^{P,s} (1-\eta) n_t^{U,s} \right]. \tag{34}$$

All other equations remain unchanged.

Based on Ríos-Rull (1996), the new utility parameter capturing the weight of leisure in the utility function, ψ, is set equal to 2 in the computation.[10]

[10] Ríos-Rull (1996) uses a slightly different instantaneous utility function. By defining $\psi := \frac{1-\alpha_{\text{Ríos-Rull}}}{\alpha_{\text{Ríos-Rull}}}$ and $\epsilon := 1 - \alpha_{\text{Ríos-Rull}}(1 - \sigma_{\text{Ríos-Rull}})$, where $\alpha_{\text{Ríos-Rull}}$ and $\sigma_{\text{Ríos-Rull}}$ are parameters of the utility function in Ríos-Rull (1996), the utility function in Ríos-Rull (1996) can be transformed into the utility function of this paper. In Ríos-Rull (1996) α is set equal to 0.33 resulting in $\psi = 2.03$.

C. Computation methods

C.1. Solution algorithm for the steady state

1. Define all parameter values, set A equal to one, and compute $\bar{z}$ as well as N_0^1.

2. Provide an initial guess for k.

3. Given k obtain the values for (obey the order) $r, \widetilde{w}, q, q^1$ and

$$\begin{aligned} b =& \frac{\theta(1+q)\eta}{(1+\phi)\left[(1+n)g+n-q\right]} \\ b^N =& \frac{\eta\theta}{1+\phi} \\ \widetilde{b}^U =& \frac{1}{(1-\eta)(1-N_0^1)} b, \qquad \text{and} \qquad \widetilde{b}^P = 0. \end{aligned}$$

4. Solve for $\{\widetilde{s}^{P,s}, \widetilde{s}^{U,s}\}_{s=1}^{T+T^R-1}$ and by using the FOC conditions (22). For each $j \in \{P, U\}$:

   - Guess $\widetilde{c}^{j,1}$
   - Compute all $\{\widetilde{c}^{j,s}\}_{s=2}^{T+T^R}$ by iterating over the Euler equation
   - Given $\{\widetilde{c}^{j,s}\}_{s=1}^{T+T^R}$ obtain all $\{\widetilde{s}^{j,s+1}\}_{s=1}^{T+T^R}$ from the budget constraints, where $s^{j,1}0$ is set equal to zero, but $s^{j,T+T^R+1}$ is not predetermined
   - If $\widetilde{s}^{j,T+T^R+1}$ is not close enough to zero as defined by chosen tolerance criterion repeat the last three steps with an updated $\widetilde{c}^{j,1}$ according to the Secant Method (Newton's Method does also work, but it is slower):

$$\widetilde{c}_{i+2}^{j,1} = \widetilde{c}_{i+1}^{j,1} - \frac{\widetilde{c}_{i+1}^{j,1} - \widetilde{c}_{i}^{j,1}}{\widetilde{s}_{i+1}^{j,T+T^R+1} - \widetilde{s}_{i}^{j,T+T^R+1}} \widetilde{s}_{i+1}^{j,T+T^R+1} \tag{35}$$

5. Compute $\{\widetilde{a}^{j,s+1}\}_{s=1}^{T+T^R-1}$ by $\widetilde{a}^{j,s} = \widetilde{s}^{j,s} - \widetilde{b}^j$.

6. Compute k' by making use of the capital and labor market equilibrium condition

$$k' = \frac{1}{1+n} \sum_{s=1}^{T+T^R} N_0^s \left[\eta \widetilde{a}^{P,s+1} + \sigma(1-\eta)\widetilde{a}^{U,s+1}\right]$$

7. Compare both k' with k. If the difference is close to zero (defined by a tolerance criterion) stop. Otherwise update the initial guess of k and go back to step 3.

In the last step of the solution algorithm the updating of the initial guess for k is linear, i.e. a linear combination of k' and the previous initial guess k (l). The steady state solution is robust to small changes in parameters and initial guesses and takes only a few seconds.

C.2. Solution of the dynamic stochastic system

The conditional expectation of the bubble shocks is equal to the condition probability that $z^b_{t+k} = 1$ (the other possibility, $z^b_{t+k} = 0$, drops out due to the zero):

$$\tilde{z}^b_{t+k} := \mathbb{E}_t[z^b_{t+k}] = (1 - z^b_t)P^{(k)}_{21} + z^b_t P^{(k)}_{11}, \tag{36}$$

where $P^{(k)}_{ij}$ is the entry in the i-th row and j-th column of P^k.

The conditional expectation of the TFP-shock is equal to zero, i.e. $\tilde{z}^a_{t+k} := 0$ for all $k > 0$. The conditional expectation of A from period t on is given by

$$\tilde{A}_{t+k} := A_t^{\rho^k}. \tag{37}$$

This deterministic dynamic system is now obtained by replacing all z^b_{t+k} by $\tilde{z}_{t+k}$ and all A_{t+k} by $\tilde{A}_{t+k}$ for $k > 0$. In the firm sector A_{t+k} is replaced by $\tilde{A}_{t+k}$ for all $k > 1$ in the equations for output, wages, and interest rates (25).

The households' optimization problem is now a problem under certainty with the FOC:

$$\begin{aligned}
(1+g)^\epsilon \left(\widetilde{c}^{j,s}_t\right)^{-\epsilon} =& \beta R^j_{t+1} \left(\widetilde{c}^{j,s+1}_{t+1}\right)^{-\epsilon} && s = 1, ..., T + T^R - 1 \\
(1+g)^\epsilon \left(\widetilde{c}^{j,s}_t\right)^{-\epsilon} =& \beta(1 + q_{t+1}) \left(\widetilde{c}^{j,s+1}_{t+1}\right)^{-\epsilon} + \widetilde{\omega}^{j,s+1}_{t+1} && s = 1, ..., T + T^R - 1 \\
\widetilde{\omega}^{j,s+1}_{t+1}\widetilde{b}^{j,s+1}_{t+1} =& 0 \qquad \text{and (29)},
\end{aligned} \tag{38}$$

where the investment-efficiency-specific returns $R^P_{t+1} = (1 + r_{t+1})$ and $R^U_{t+1} = \sigma(1 + r_{t+1})$

are deterministic, as well as the return to bubbles as given by

$$1 + q_{t+1} = \tilde{z}_{t+1}(1 + q^o_{t+1}). \tag{39}$$

The same arguments as in the deterministic steady state apply with respect to the return to bubbles (see (23)). I consider only bubbles that potentially crowd-in capital such that the return to bubbles is given by

$$\tilde{z}^b_{t+k}(1 + q^o_{t+k}) = \sigma(1 + r_{t+k}) \tag{40}$$

for all $k > 0$.

Solution Algorithm

1. Initialization:
   - Define parameter values and initial values $\{\widetilde{a}_1^{P,s}, \widetilde{b}_1^{P,s}, \widetilde{a}_1^{U,s}, \widetilde{b}_1^{U,s}\}_{s=1}^{T+T^R-1}, q^o_1$
   - Choose nn numbers of periods for the simulation
   - Draw a random sequence of the shocks $\{z^a_t, z^b_t\}_{t=1}^{nn}$
   - Choose a number of transitional periods N large enough for the (deterministic) system to be close to its steady state
   - Calculate the deterministic steady state as explained in subsection 3.2
2. At each point in time $t = 1, ..., nn$:
   - Calculate the conditional expectations $\tilde{z}^a_{t+k}$ and $\tilde{z}^b_{t+k}$ $\forall k = 1, ..., N$ as described by (37) and (36)
   - Transform the whole dynamic stochastic equation system into a deterministic system by replacing $\left(z^a_{t+k}, z^b_{t+k}\right)$ with their expected values $\left(\tilde{z}^a_{t+k}, \tilde{z}^b_{t+k}\right)$ and solve this deterministic model for the N periods by applying the direct computation method, i.e. guess $\{k_{t+k}\}_{k=1}^{T}$ (that gives also $\widetilde{w}_t$, r_t, q_t, q^1_t, b_t, $\widetilde{b}^U_t$, and b^N_t), solve for the individual problem, and update the initial guess $\{k_{t+k}\}_{k=1}^{T}$ until convergence.[11]

[11]The household problem is solved similarly by guessing $c_t^{j,1}$, computing all other c's by iterating over the

- From the solution store $\{\widetilde{a}_{t+1}^{P,s}, \widetilde{a}_{t+1}^{U,s}, \widetilde{b}_{t+1}^{P,s}, \widetilde{b}_{t+1}^{U,s}, \widetilde{c}_t^{P,s}, \widetilde{c}_t^{U,s}\}_{s=1}^{T+T^R}$ and q_{t+1}^o
- Use $\{\widetilde{a}_{t+1}^{P,s}, \widetilde{a}_{t+1}^{U,s}, \widetilde{b}_{t+1}^{P,s}, \widetilde{b}_{t+1}^{U,s}\}_{s=1}^{T+T^R}$ and q_{t+1}^o as initial values for period $t+1$.

C.3. Solution algorithm for the steady state with elastic labor supply

1. Define all parameter values, set A equal to one, and compute $\bar{z}$ as well as N_0^1.
2. Provide an initial guess for k and l.
3. Given k and l obtain the values for (obey the order) $r, \widetilde{w}, \widetilde{d}, q, q^1$ and

$$
\begin{aligned}
b =& \frac{\gamma_1(1+q)\eta}{(1+n)(1+g)(1+\phi)-(1+q)\left(1+\phi+\gamma_2\eta\right)} \\
b^N =& \frac{\eta}{1+\phi}\left(\gamma_1+\gamma_2 b\right) \\
\widetilde{b}^U =& \frac{1}{(1-\eta)(1-N_0^1)} b, \qquad \text{and} \qquad b^P = 0.
\end{aligned}
$$

4. Solve for $\{\widetilde{s}^{P,s}, \widetilde{s}^{U,s}\}_{s=1}^{T+T^R-1}$ and $\{\widetilde{l}^{P,s}, \widetilde{l}^{U,s}\}_{s=1}^{T}$ by using the Euler equations. For each $j \in \{P, U\}$:
   - Guess $\widetilde{c}^{j,1}$
   - Compute all $\{\widetilde{c}^{j,s}\}_{s=2}^{T+T^R}$ by iterating over the Euler equations
   - Obtain all $\{\widetilde{s}^{j,s+1}\}_{s=1}^{T+T^R}$ from the budget constraints
   - If $\widetilde{s}^{j,T+T^R+1}$ is not close enough to zero (as defined by some chosen tolerance) repeat the last 3 steps with an updated $\widetilde{c}^1$ (here the Secant Method, Newton's Method does also work):

$$
\widetilde{c}_{i+2}^{j,1} = \widetilde{c}_{i+1}^{j,1} - \frac{\widetilde{c}_{i+1}^{j,1} - \widetilde{c}_i^{j,1}}{\widetilde{s}_{i+1}^{j,T+T^R+1} - \widetilde{s}_i^{j,T+T^R+1}} \widetilde{s}_{i+1}^{j,T+T^R+1} \tag{41}
$$

5. Compute $\{\widetilde{a}^{s+1,U}\}_{s=1}^{T+T^R-1}$ by $\widetilde{a}^{s,U} = \widetilde{s}^{s,U} - \widetilde{b}^U$ ($\widetilde{a}^{s,P} = \widetilde{s}^{s,P}$).
6. Compute $\{\widetilde{l}^{P,s}, \widetilde{l}^{U,s}\}_{s=1}^{T}$ from the intra-temporal FOC

Euler equation, and computing all s's by making use of the budget constraints. The last $s_t^{j,T+T^R+1}$ should then be equal to zero, otherwise the initial guess of $c_t^{j,1}$ is updated until convergence.

7. Compute k' and l' by making use of the capital and labor market equilibrium condition

$$k' = \frac{1}{1+n} \sum_{s=1}^{T+T^R} N_0^s \left[\eta \widetilde{a}^{P,s+1} + \sigma(1-\eta)\widetilde{a}^{U,s+1}\right]$$
$$l' = \sum_{s=1}^{T} N_0^s \left[\eta(1-l^{P,s}) + (1-\eta)(1-l^{U,s})\right].$$

8. Compare both k' and l' with k and l', respectively. If the largest of both differences is close to zero stop. Otherwise update the initial guess of k and l and go back to step 3.

D. Solution algorithm for the dynamic stochastic system with elastic labor supply

1. Initialize
   - Define parameter values and initial values $\{\widetilde{a}_1^{s,P}, \widetilde{b}_1^{s,P}, \widetilde{a}_1^{s,U}, \widetilde{b}_1^{s,U}\}_{s=1}^{T+T^R-1}, q_1^1$ (I choose the deterministic steady state)
   - Choose nn numbers of periods for the simulation
   - Draw a random sequence of the shocks $\{z_t^a, z_t^b\}_{t=1}^{nn}$
   - Choose a number of transitional periods N large enough for the (deterministic) system to be close to its steady state
   - Calculate the deterministic steady state for z_t^a and z_t^b equal to the stationary expected value of the Markov process, $\bar{z}^a$ and $\bar{z}^b$

2. At each point in time $t = 1, ..., nn$:
   - Calculate the conditional expectations $\tilde{z}_{t+k}^a$ and $\tilde{z}_{t+k}^b$ $\forall k = 1, ..., N$ as given by (37) and (36)
   - Transform the whole dynamic stochastic equation system into a deterministic system by replacing $\left(z_{t+k}^a, z_{t+k}^b\right)$ with their expected values $\left(\tilde{z}_{t+k}^a, \tilde{z}_{t+k}^b\right)$ and solve this deterministic model for the N periods by applying the direct computation method, i.e. guess $\{k_{t+k}, l_{t+k}\}_{k=1}^{T}$ (that gives also $\widetilde{w}_t$, r_t, q_t, q_t^1, b_t, $\widetilde{b}_t^U$, b_t^N,

$\widetilde{d}_t$), and solve for the individual problem, update the initial guess $\{k_{t+k}, l_{t+k}\}_{k=1}^{T}$ until convergence.[12]

- From the solution store $\{\widetilde{a}_{t+1}^{s,P}, \widetilde{a}_{t+1}^{s,U}, \widetilde{b}_{t+1}^{s,P}, \widetilde{b}_{t+1}^{s,U}, \widetilde{c}_t^{s,P}, \widetilde{c}_t^{s,U}\}_{s=1}^{T+T^R}$ and q_{t+1}^1
- Use $\{\widetilde{a}_{t+1}^{s,P}, \widetilde{a}_{t+1}^{s,U}, \widetilde{b}_{t+1}^{s,P}, \widetilde{b}_{t+1}^{s,U}\}_{s=1}^{T+T^R}$ and q_{t+1}^1 as initial values for period $t+1$.

[12] The household problem is solved similarly by guessing $c_t^{j,1}$, computing all other c's by iterating over the Euler equation where l is substituted, computing all s's by making use of the budget constraints. The last $s_t^{j,T+T^R+1}$ should then be equal to zero, otherwise the initial guess of $c_t^{j,1}$ is updated until convergence.

E. Figures

Figure 4: Impulse response functions for a TFP shock in a bubble-less economy

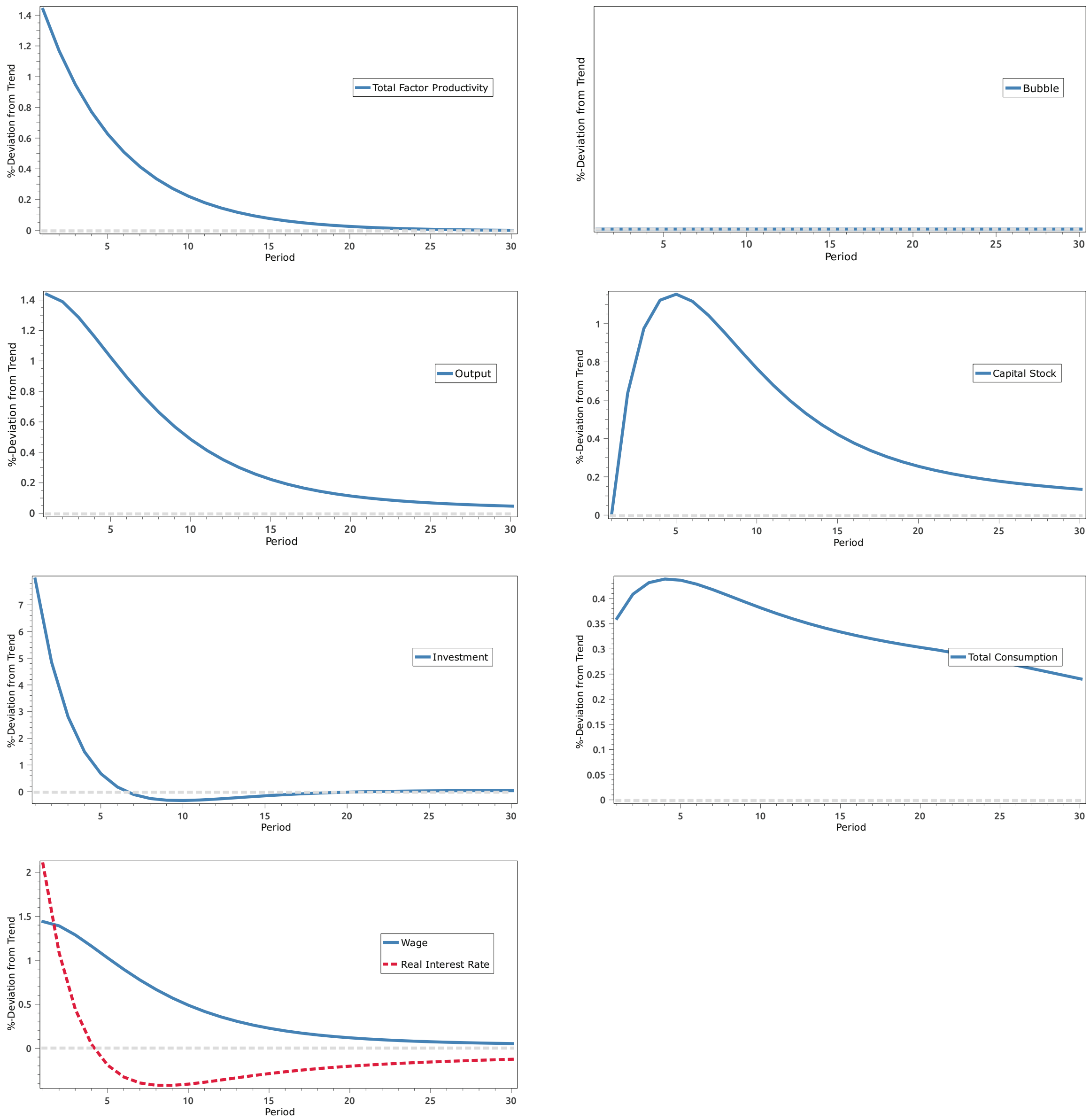
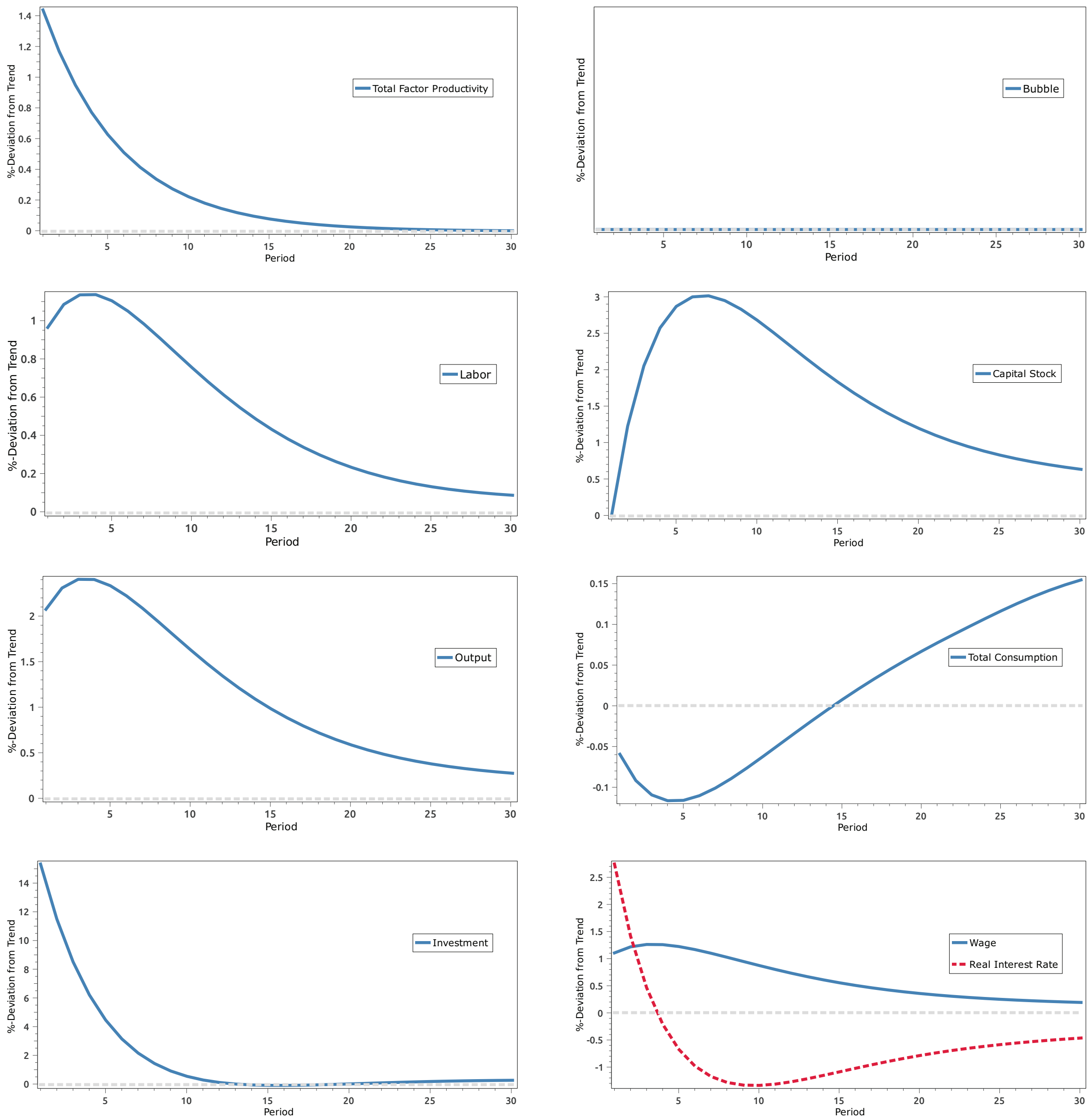


Figure 5: Impulse response functions for a TFP shock in a bubble-less economy with elastic labor supply



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